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The geometry, topology, group theory and harmonics of Kaluza-Klein unification

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OBJECTIVE & STARTING POINT

Objective: Find a description of the curvature of higher-dimensional spacetime which includes:

- field multiplets which transform as spinors with the quantum numbers of known fermions, above the electroweak (EW) scale
- 4D gravity and the known gauge fields above the EW scale: EW & strong interactions

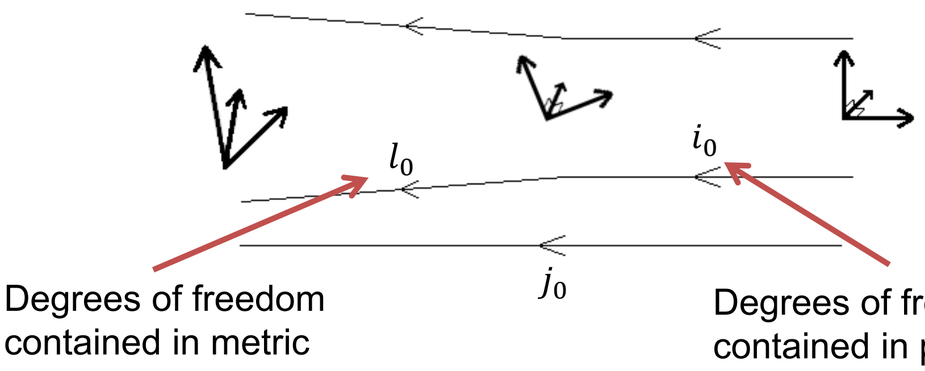
Starting point: General Relativity describes 4D gravity as manifestation of curvature. Build out from there. Kaluza-Klein theories provide models – but don't assume correct in all details.

STEPS (S), OBSERVATIONS & CONCLUSIONS (C)

S1: consider transformation groups induced by changes of 4D coordinates.
Observations:

- Lorentz group preserves lengths of vectors – or equivalently, preserves orthonormality of frames on tangent space
- changes of coordinates induce GL(4,R) transformations on vectors or on frames – of which, SO(1,3) is subgroup

C1: changes of frame can be given polar decomposition



S2: consider the action of gauge transformations on fermion fields
Observation:

- strong and EW gauge groups are unitary (SU(3) & SU(2) x U(1) respectively), changing basis of Hilbert spaces of complex fields

C1: orthogonal groups act on **frames**; but do not act directly on **field configurations**
Further observations:

- homomorphisms: U(1) → O(2), SU(2) → SO(3), SU(4) → SO(6)
- in each case, Lie algebra of unitary group is same as Lie algebra of orthogonal group
- gauge fields are vector field-valued elements of Lie algebras

S3: consider Levi-Civita (L-C) connection on product spaces and relations to gauge fields
Observations:

- in appropriate coordinates, metric has block for each factor
- components of L-C connection fall into sets, e.g. $\Gamma_{\mu\nu}{}^{\rho}, \Gamma_{\mu Y}{}^{X}, \Gamma_{XY}{}^{Z}, \ldots$
- one set ($\Gamma_{\mu Y}{}^{X}$) has spin connection which is vector field-valued element of SO(N_2) Lie algebra, i.e. gauge fields

C2: N_2 = 2 → U(1) gauge fields; N_2 = 3 → SU(2); N_2 = 6 → SU(4)

S4: consider limit of zero-curvature

Observations:

- In limit, all spatial dimensions appear the same
- SO(N_2) is unified with SO(1,3)
- Reverse this: larger symmetry groups are non-linearly realised

C3: known gauge fields can occur alongside gravity in product spacetime, as manifestations of curvature
C4: higher-dimensional tensors decompose on factor spaces
C5: 'unification' only occurs in non-physical limit of zero curvature

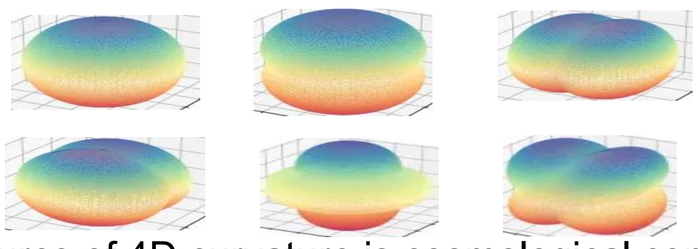
S5: consider quantum numbers of known fermions and use to determine total dimensionality and shape of extra dimensions
Observations:

- three generations, distinguished by mass – which is norm of momentum vector in 4D spacetime
- each generation has 32 states – compile into 32-spinor
- only 10D or 11D (pseudo-)orthogonal groups have 32-spinors
- EW symmetry doesn't commute with parity (a spacetime transformation), but colour SU(3) ⊂ SU(4) does

C6: Need N_2 = 6; then get SU(3) gauge fields in spin connection; harmonics include multiplets containing three quarks + lepton

S6: consider how to get spinors with known quantum numbers from harmonics on N-spheres and deformations of them
Observations:

- Periodicity results in quantum numbers for field configurations



- if only source of 4D curvature is cosmological constant, at instant in cosmic time, 3-space is S^3
- under diffeomorphic action of SO(N+1) on N-sphere, harmonics form multiplets, “dragged” into each other in unitary transformations
- multiplets and components distinguished by quantum numbers
- diffeomorphisms: SO(4) on S^3 induces $\mathrm{SU(2)_L} \otimes \mathrm{SU(2)_R}$ action on its harmonics; SO(6) on S^6 induces SU(4) action on its harmonics

C8: At instant in cosmic time, background geometry is S^3 x S^6
Further observation:

- Stabiliser of right-handed neutrino is $\mathrm{SU(3)} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$

C9: Harmonics form basis of Hilbert space of spinors, with quantum numbers of known fermions above EW scale
C10: Right-handed neutrino is background/deep space harmonic

RELEVANT OUTPUTS BY THE AUTHOR

Tangent space symmetries in general relativity and teleparallelism, *Int. J. Geom. Meth. Mod. Phys* (2021)**, 18**, 2140008
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Covariant Compactification: progress towards a Theory of Everything Fundamental, https://www.youtube.com/@WarpedBroken