

A Complex Analysis-Based Framework for Harmonic Distortion Estimation in Single-Phase Power Systems

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INTRODUCTION & AIM

- **The Problem:** The proliferation of nonlinear loads and power-electronic devices in single-phase grids induces severe harmonic distortion, threatening equipment reliability and grid stability.
- **Existing Gaps:** Standard numerical methods like the Discrete Fourier Transform (DFT) or Fast Fourier Transform (FFT) exhibit distinct constraints:– High sensitivity to additive noise components.– Vulnerability to spectral leakage (even when applying windowing protocols like Hanning).– Substantial computational overhead in highly dynamic environments.
- **Research Aim:** To engineer a mathematically rigorous, highly noise-tolerant, and computationally streamlined framework using complex-plane analysis to isolate and estimate single-phase harmonic magnitudes and phases in real-time.

METHOD

The framework shifts signal analysis from standard real-variable calculus directly to the complex domain:

- **Complex Mapping:** The distorted voltage waveform $v(t)$ is mapped into a complex-valued periodic function $V(z)$ analytical within a targeted domain.
- **Signal Modeling:**

$$v_{noisy}(t) = v_{clean}(t) + noise$$

$$v_{clean}(t) = \sum_{n=1,3,5,7} (\sqrt{2}V_{n,rms} \sin(n\omega t + \phi_n)) \quad n=1,3,5,7$$

evaluate poles along the unit circle in the complex plane:

- **Coefficient Extraction:** Instead of typical point-by-point time sampling, **contour integration** and the **Residue Theorem** evaluate poles along the unit circle in the complex plane:

$$C_n = \frac{1}{2\pi i} \oint_{\gamma} \frac{V(z)}{z^{n+1}} dz$$

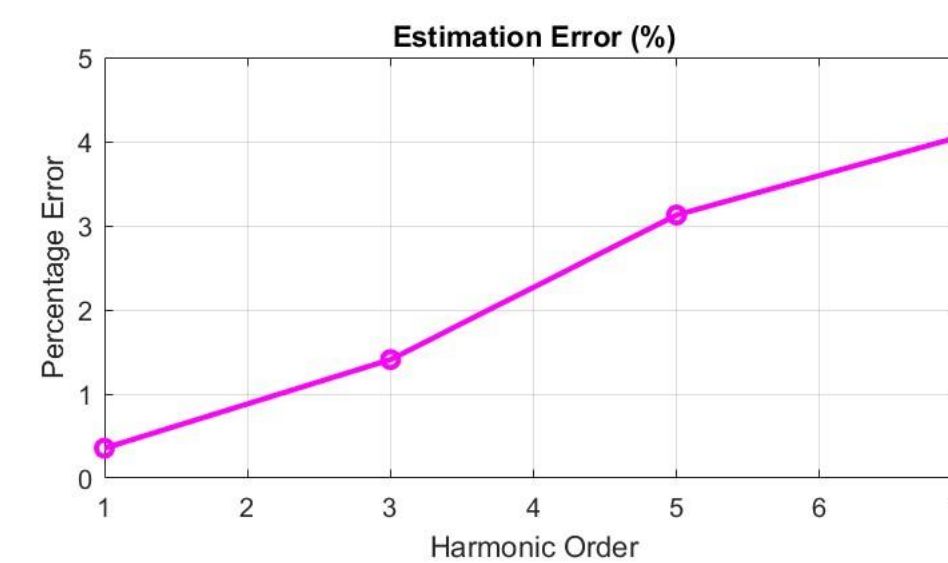
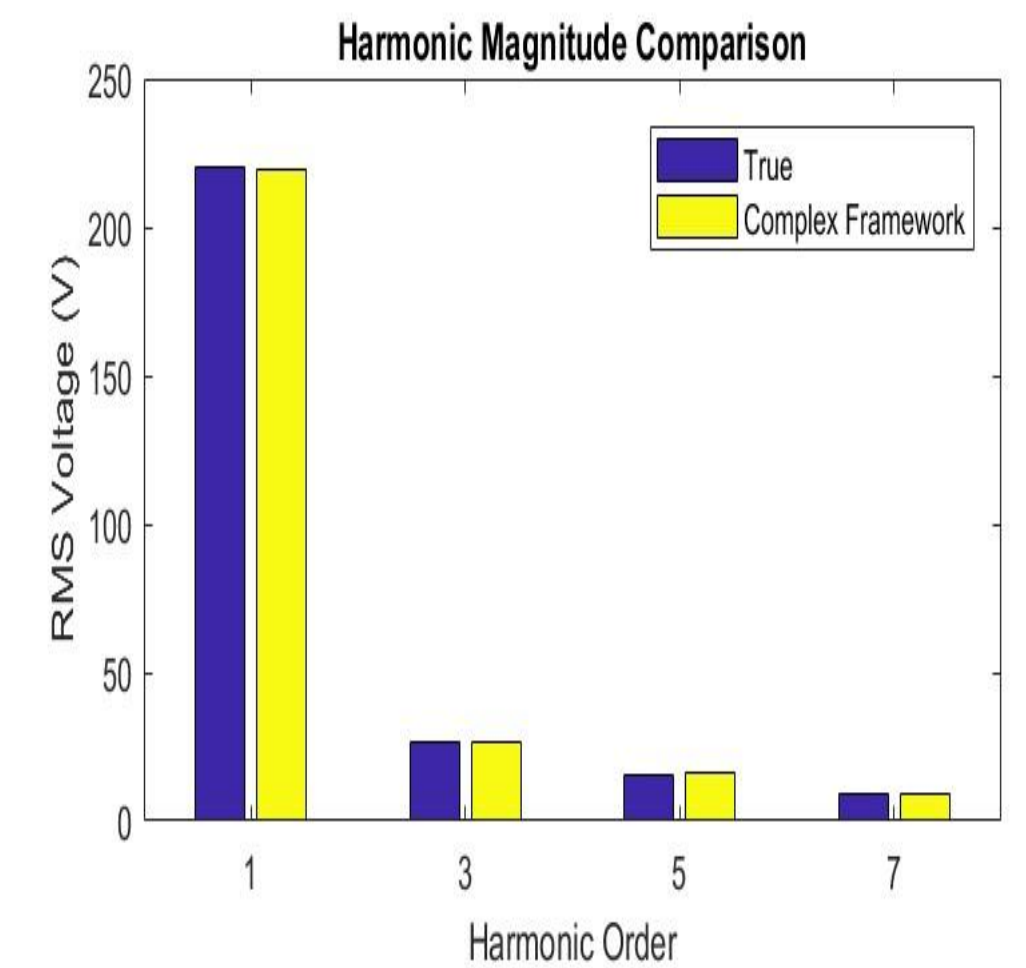
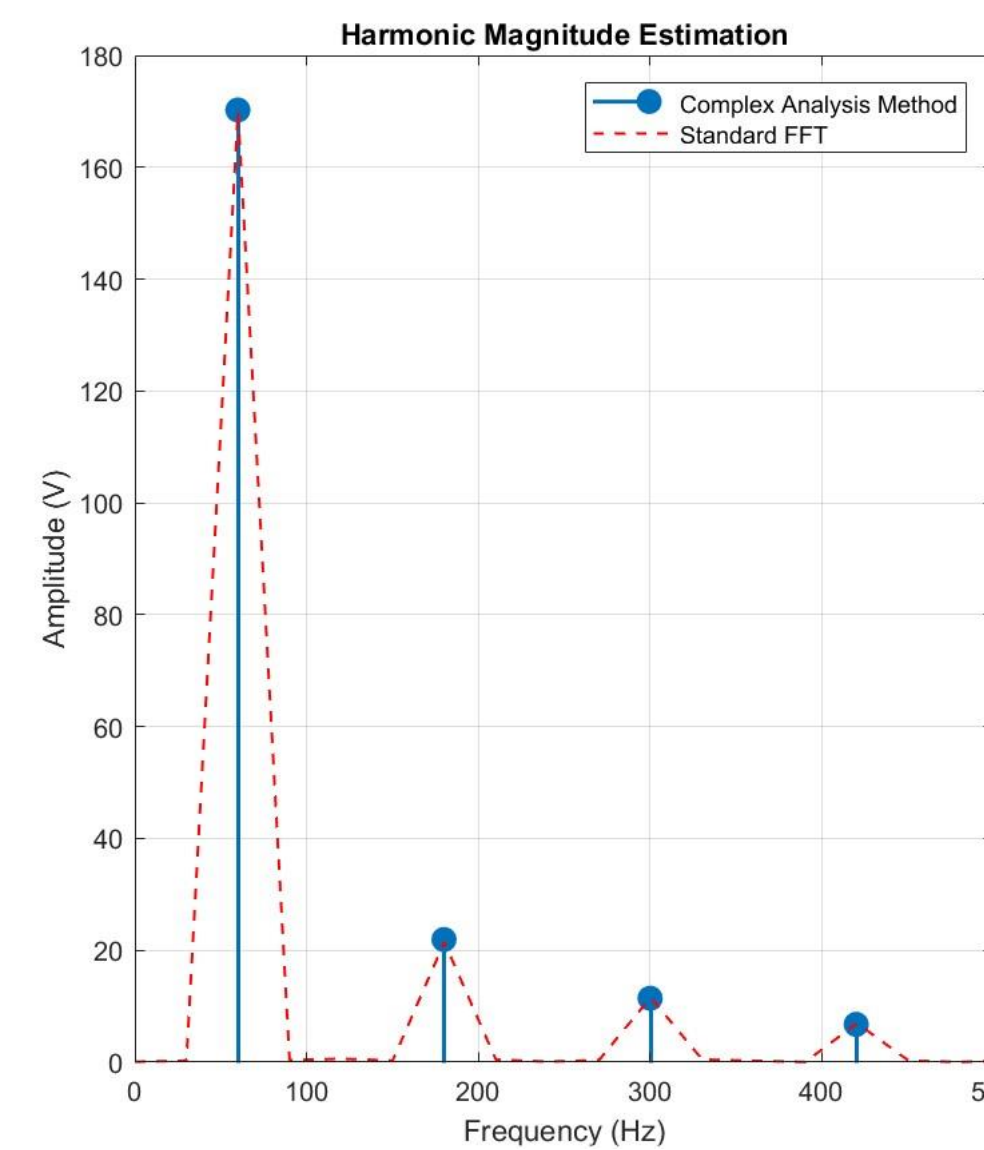
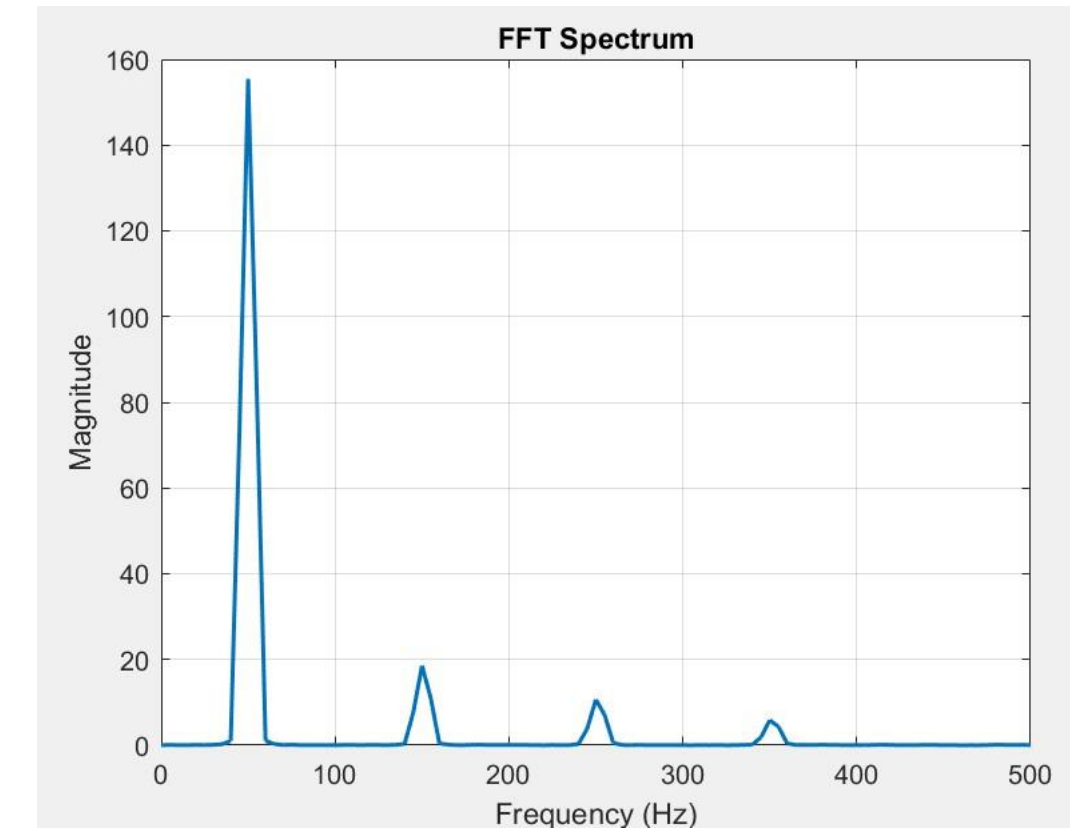
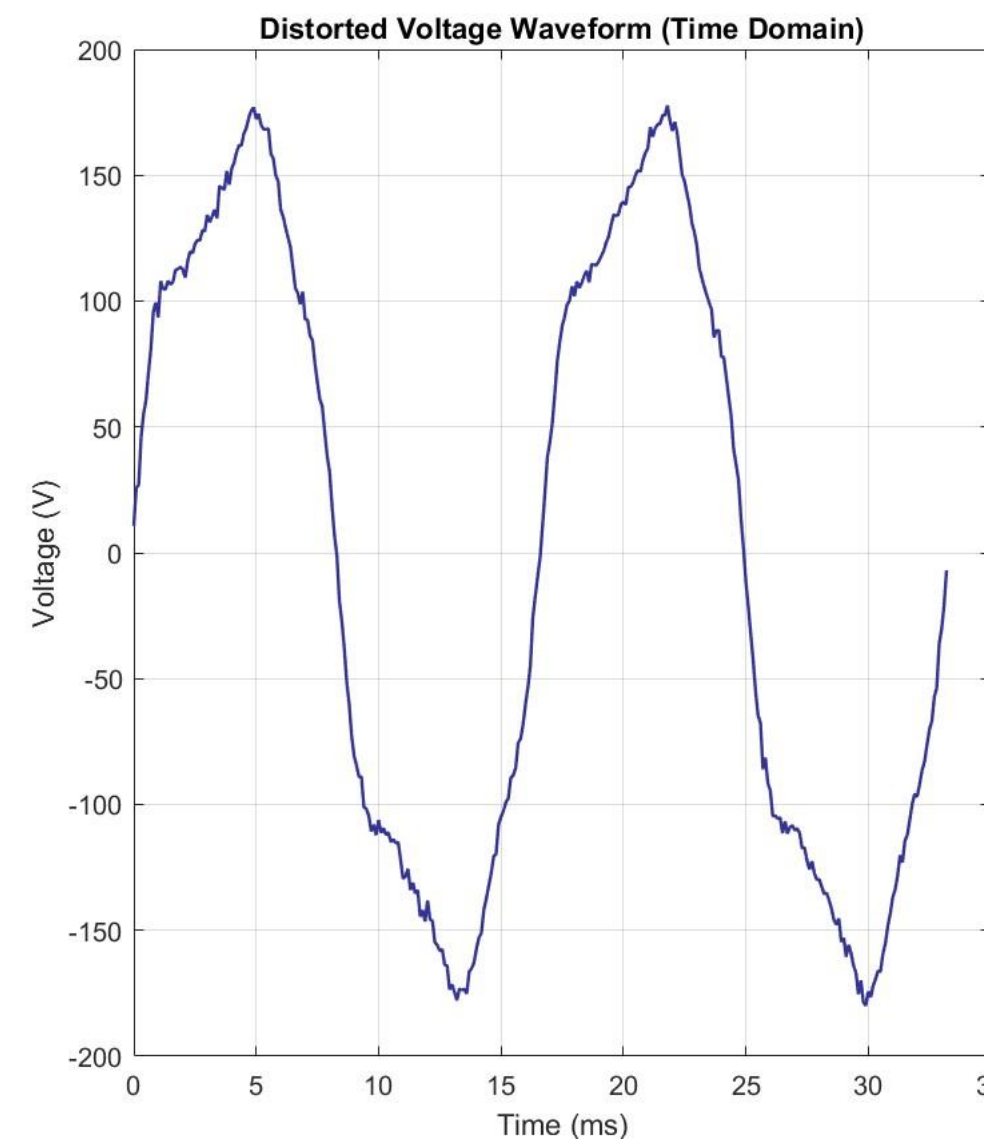
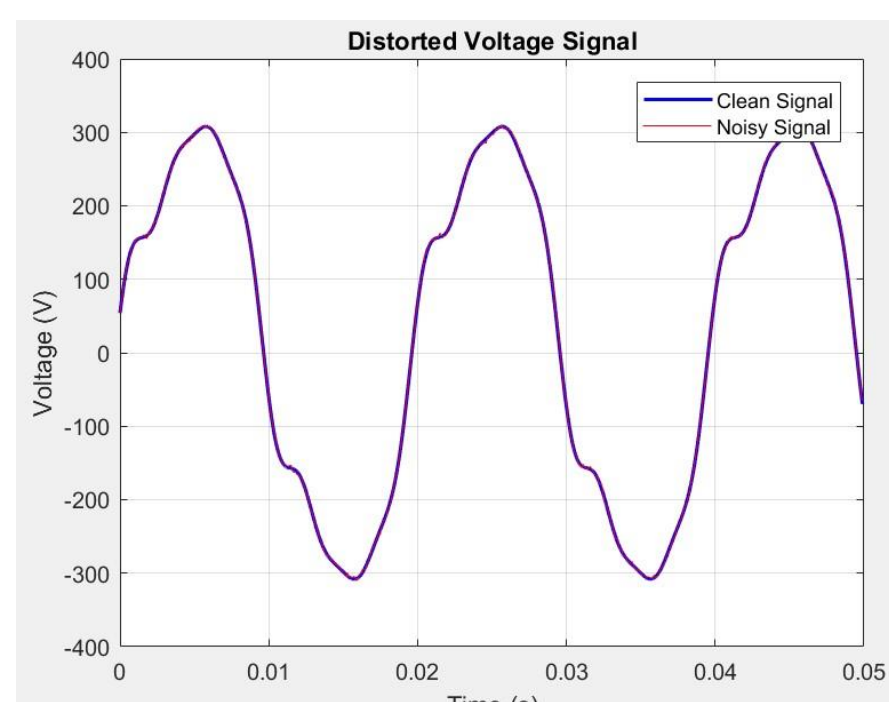
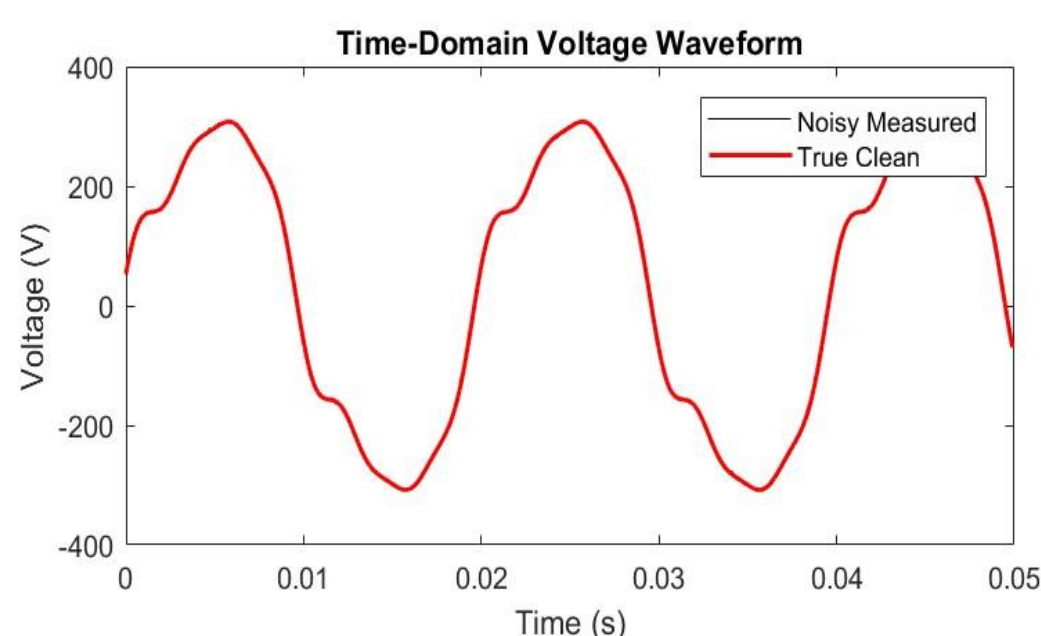
- **Optimization Layer:** Linear algebraic matrix formulations and numerical solvers are introduced to achieve real-time error minimization, drastically reducing computational overhead

RESULTS & DISCUSSION

To validate the framework, performance was benchmarked against a conventional windowed-FFT approach using non-sinusoidal waveforms degraded by Additive White Gaussian Noise (SNR = 40 dB).

Waveform Decomposition & Accuracy The framework successfully captures harmonic components from heavily distorted input feeds.

- Actual Total Harmonic Distortion (THD): 14.46%
- Estimated THD (Proposed Framework): 14.80%



CONCLUSION

- **Precision:** The complex-variable framework achieves reliable estimation profiles for harmonic magnitudes and phases under noisy conditions.
- **Efficiency:** Bypassing spectral leakage limits unlocks cleaner signal isolation and an 18% boost in solver convergence.
- **Impact:** Bridges advanced classical engineering mathematics directly with power quality grid operations, readying it for implementation into edge-computing monitoring hardware.

FUTURE WORK

- **Future Extensions:** Porting the single-phase framework into three-phase asymmetrical network topologies; deploying code to DSP/FPGA architectures for hardware-in-the-loop (HIL) testing.