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Modelling vaccination decisions in heterogeneous populations

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INTRODUCTION & AIM

Background: Most behavioral vaccination frameworks, such as Bauch and Earn (2004), assume a homogeneous population where all individuals assess disease and vaccine risks identically.

The Problem: During a vaccine scare, the perceived risk of vaccination rises. Homogeneous models fail to account for variance introduced by unequal socio-economic factors or geographical differences. Consequently, standard public health responses often fail to restore pre-scare vaccine coverage levels.

Our Approach: We treat individual vaccination costs as a stochastic quantity (a random variable) to capture diverse public fears regarding infection and safety.

Aim: To develop an analytical game-theoretic framework incorporating behavioral heterogeneity to forecast post-scare recovery endpoints (Nash equilibria) and evaluate how risk distribution alters societal resilience.

METHOD

Game-Theoretic Setup: We generalize the infinite-population vaccination game by introducing decisions (vaccinate vs. not vaccinate) based on individual perceived costs, while adjusting for imperfect vaccine efficacy (e) and breakthrough infections.

Epidemiological Risk: The average risk of infection for an unvaccinated individual, $\overline{r}(p)$, is modeled as a function of coverage p and basic reproduction number R_0:

$$\overline{r}(p) = 1 - \frac{1}{(1-p)R_0}$$

when $p \leq 1 - \frac{1}{R_0} =: \mathrm{p}_{crit}$, and $\overline{r}(p) = 0$ otherwise.

Expected Payoffs:

- Unvaccinated individual: $E_N(p) = -S\overline{r}(p)C_D$,
- Vaccinated individual: $E_V(p) = -C_V - (1-e)S\overline{r}(p)C_D$,

where S is individual susceptibility, and C_D and C_V are perceived costs of disease and vaccination.

Quantifying Heterogeneity: We define the net relative cost of vaccination as the random variable

$$C := \frac{C_V}{e \cdot S \cdot C_D}.$$

Population heterogeneity is characterized using its cumulative distribution function $F_C(c)$ and corresponding quantile function $Q_C(p)$.

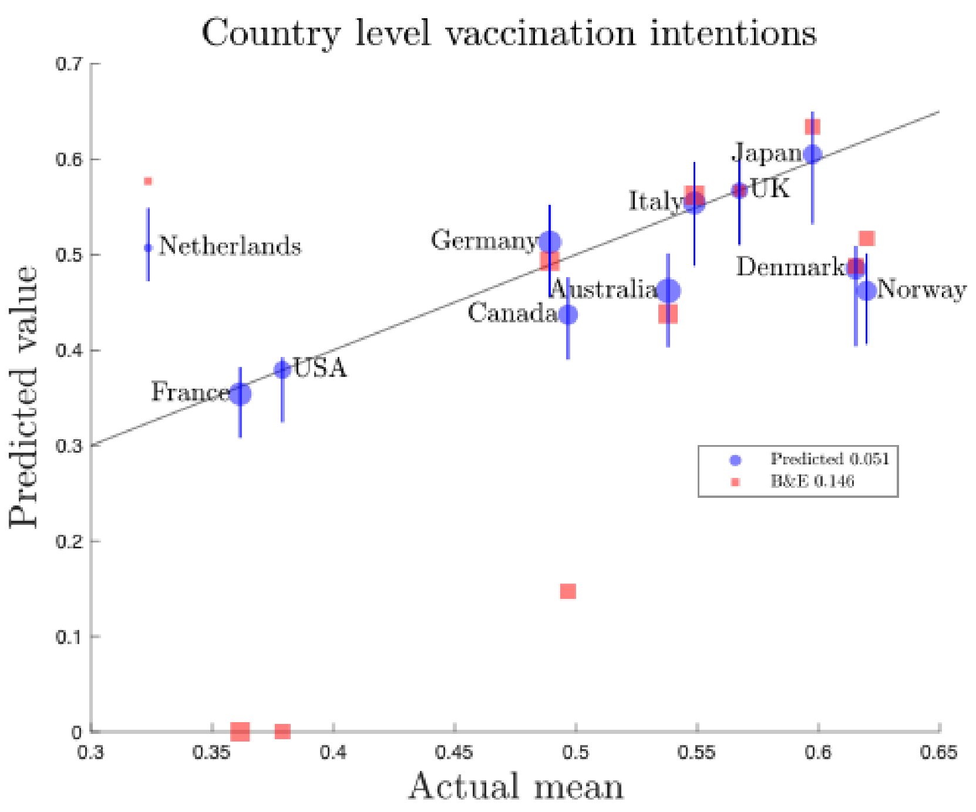
Nash Equilibrium: $p_{NE} := \max\{p \in [0,1] \mid Q_C(p) \leq \overline{r}(p)\}$.

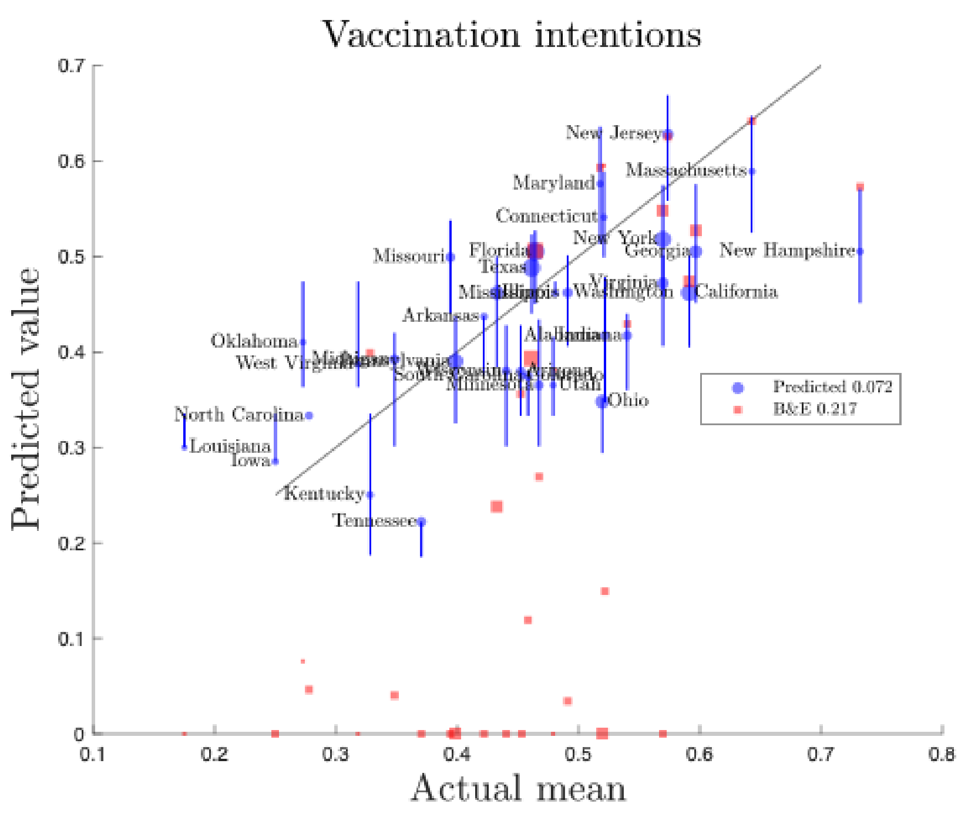
Real-World Validation: The model was tested against empirical COVID-19 behavioral survey data (Jones et al., 2020) spanning 11 countries and individual US states, using population density as an operational proxy for susceptibility (S).

RESULTS & DISCUSSION

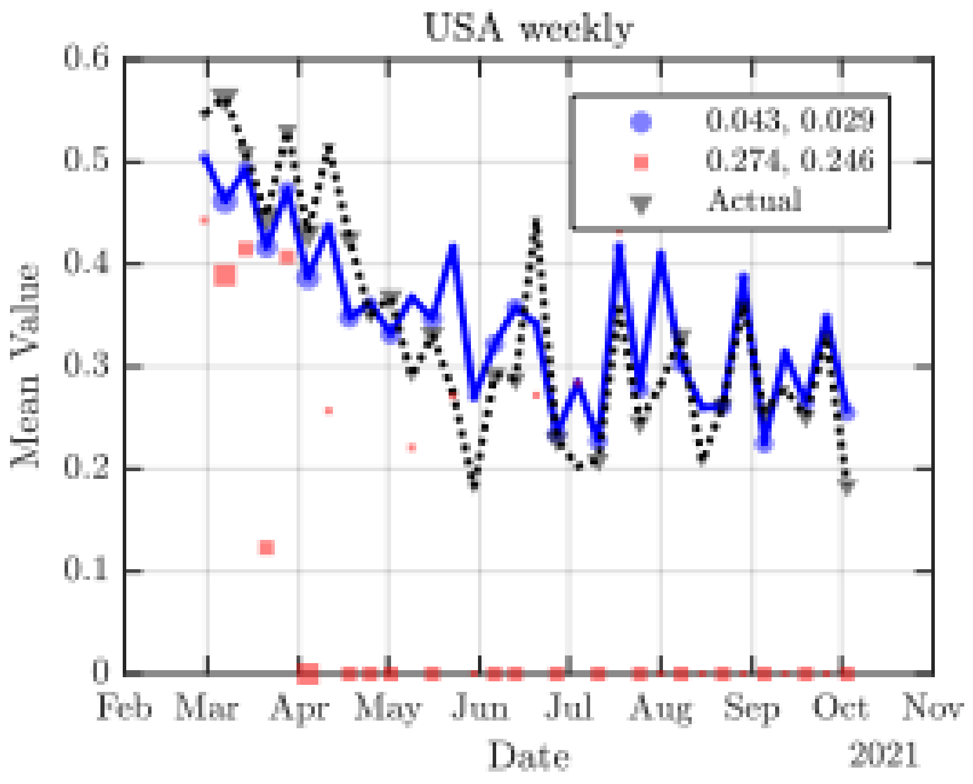
The Herd Immunity Gap: We prove the existence of a unique Nash equilibrium. When $Q_C(p_{crit}) > 0$, voluntary vaccination coverage under equilibrium falls short of the herd immunity threshold ($p_{NE} < p_{crit}$).

Model Performance & Robustness Comparisons:





Our model (blue) performs better than Bauch and Earn (2004) on country level (top left) as well as on the state level (top right) with small sample sizes. It also performs very well with very small sample sizes such as during a weekly surveys (bottom right).



CONCLUSIONS

- Incorporating behavioral and cost heterogeneity provides a more accurate, mathematically robust framework for projecting post-scare population recovery than homogeneous assumptions.
- Because a disease's basic reproduction number (R_0) governs whether behavioral heterogeneity helps or hurts coverage, public health bodies must avoid "one-size-fits-all" communication strategies after a vaccine scare.
- Utilizing quantile distribution functions allows public health officials to reliably forecast intervention endpoints even when restricted to sparse or small-scale regional survey data.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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