

Convexity, Robustness, and Stability:

A Mathematical Investigation of MSE and Huber Loss in Machine Learning

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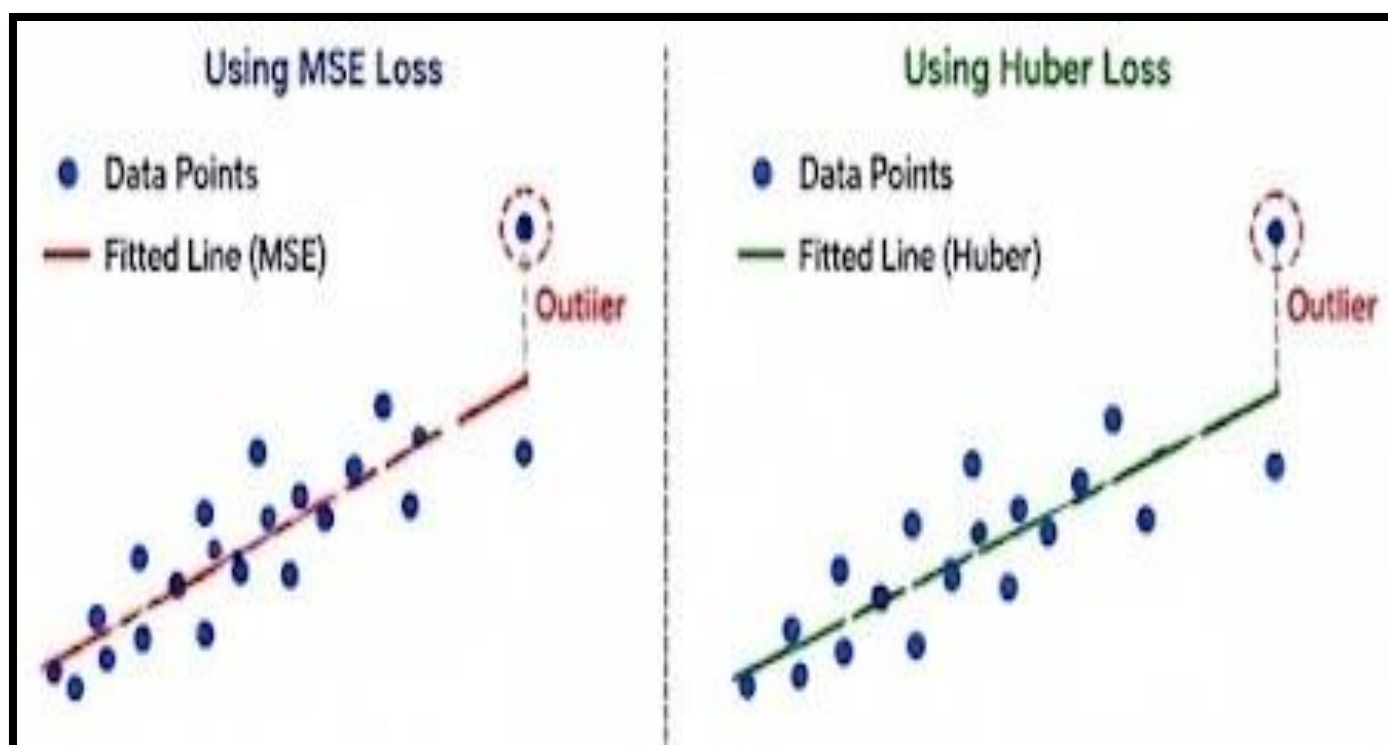
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INTRODUCTION & AIM

- Raw data is processed by a model to generate predictions, and the resulting errors are used to optimize model performance.

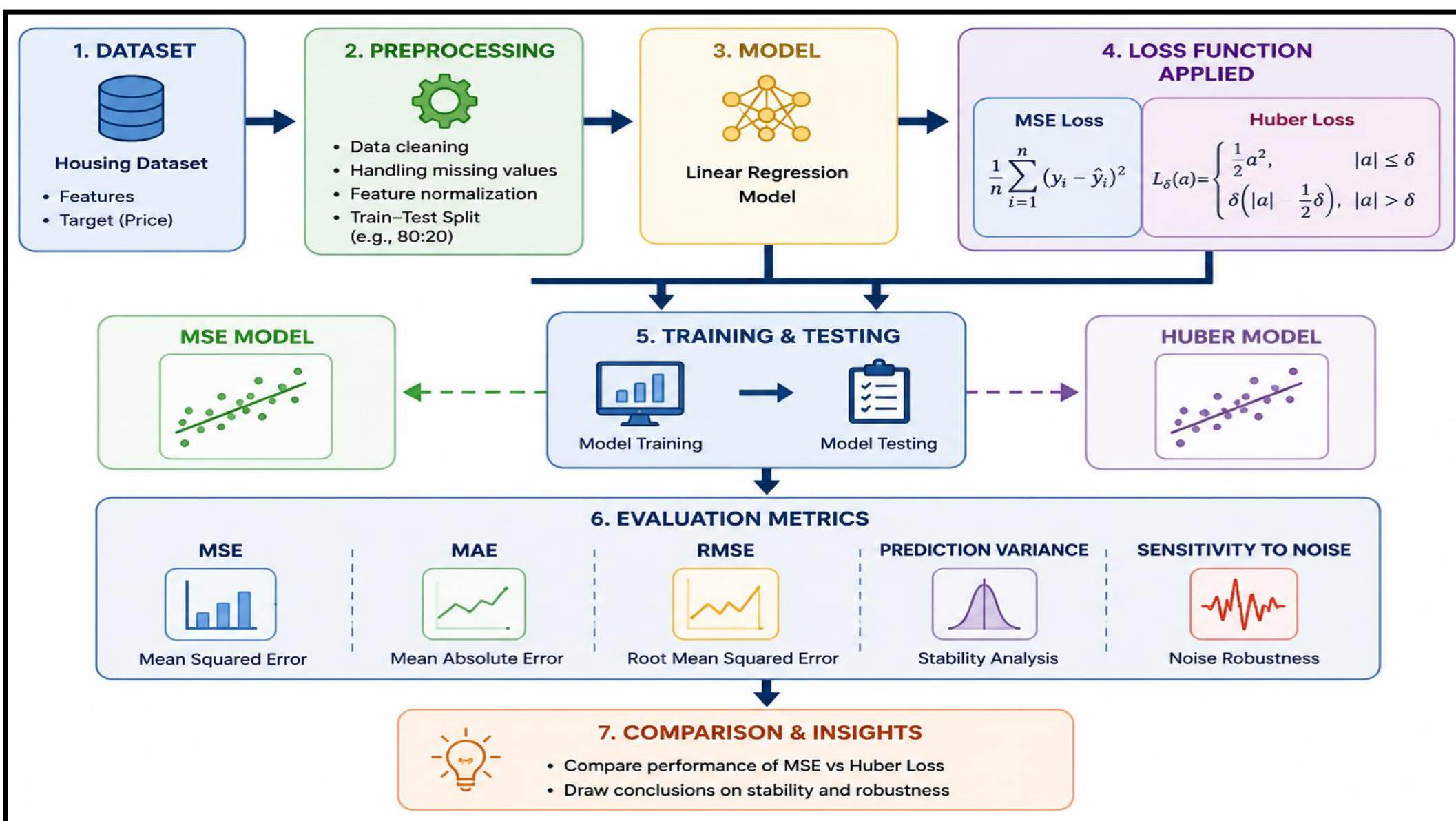


- Different loss functions respond differently to outliers, where Huber Loss provides a more stable fitted line than MSE Loss.

Aim

- To reduce the influence of outliers in predictive modeling
- To improve model robustness and prediction accuracy
- To compare Huber Loss performance with MSE Loss

METHOD



Huber Loss Equation

$$L_{\delta}(a) = \begin{cases} \frac{1}{2}a^2 & |a| \leq \delta \\ \delta(|a| - \frac{1}{2}\delta) & |a| > \delta \end{cases}$$

- Combines quadratic and linear error behavior.
- Reduces sensitivity to outliers.
- Improves robustness in noisy datasets.

MSE Equation

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

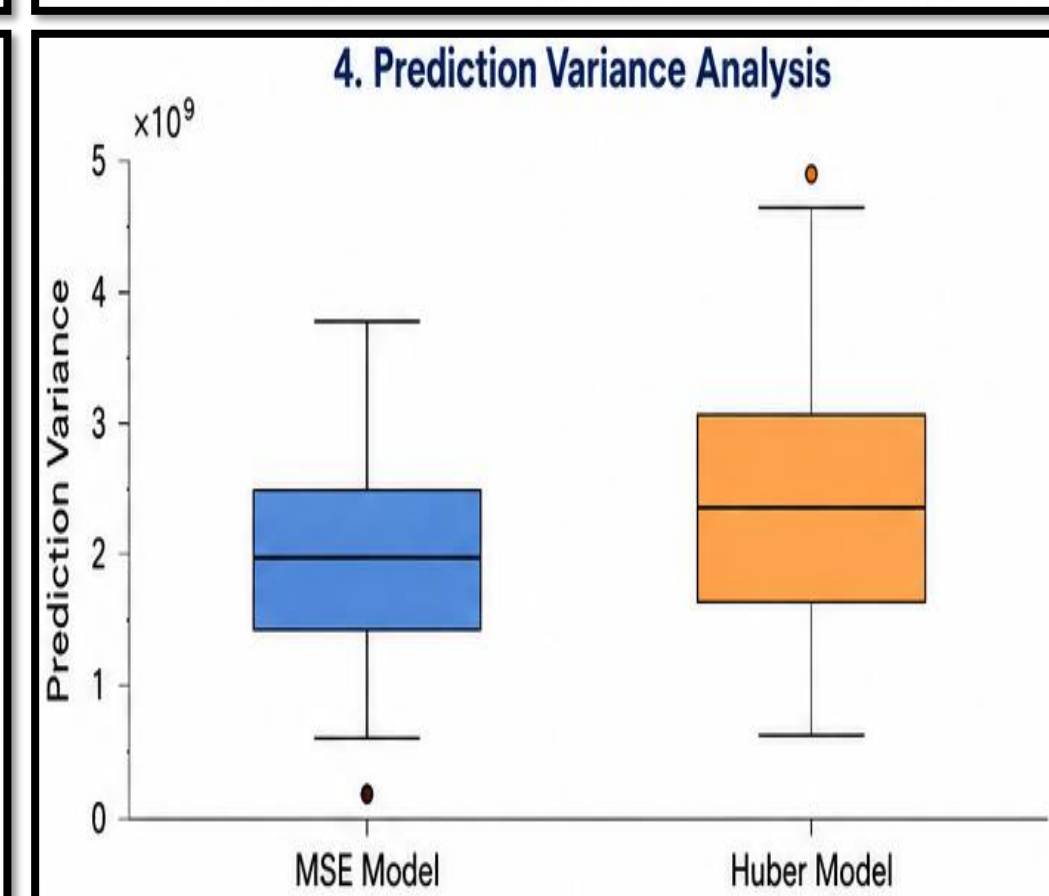
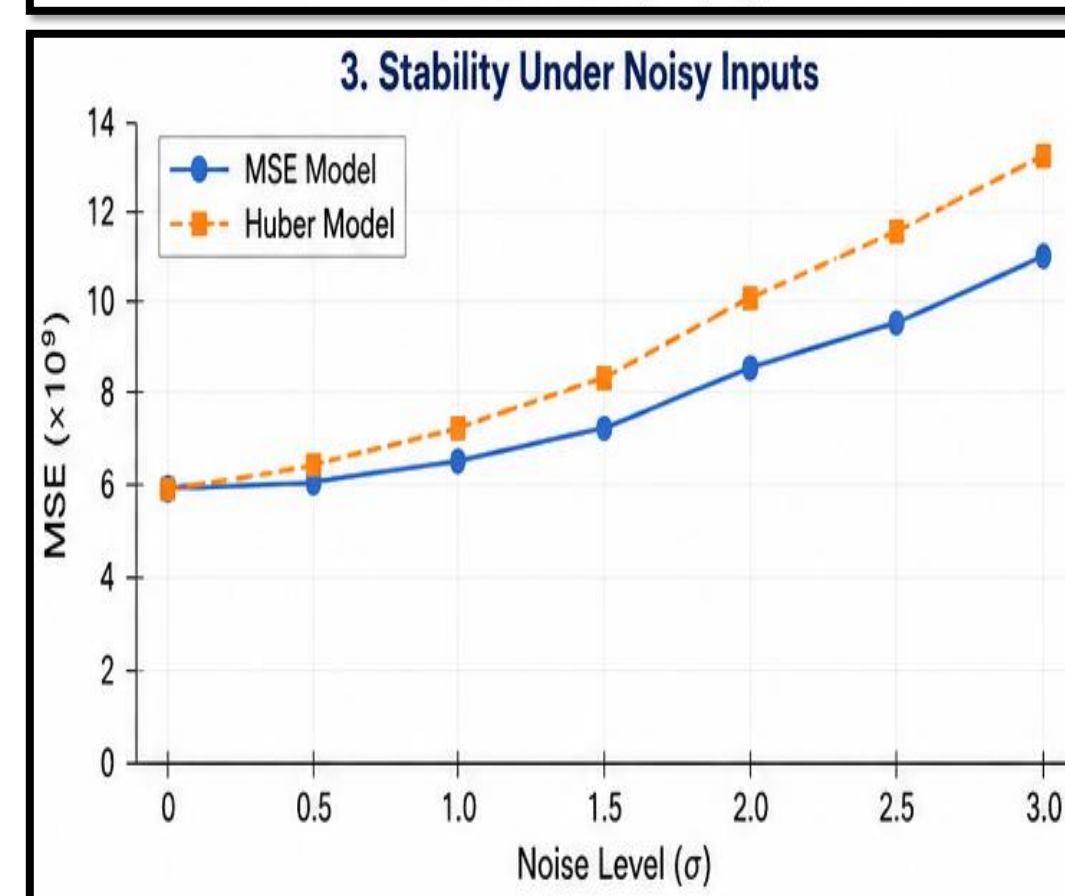
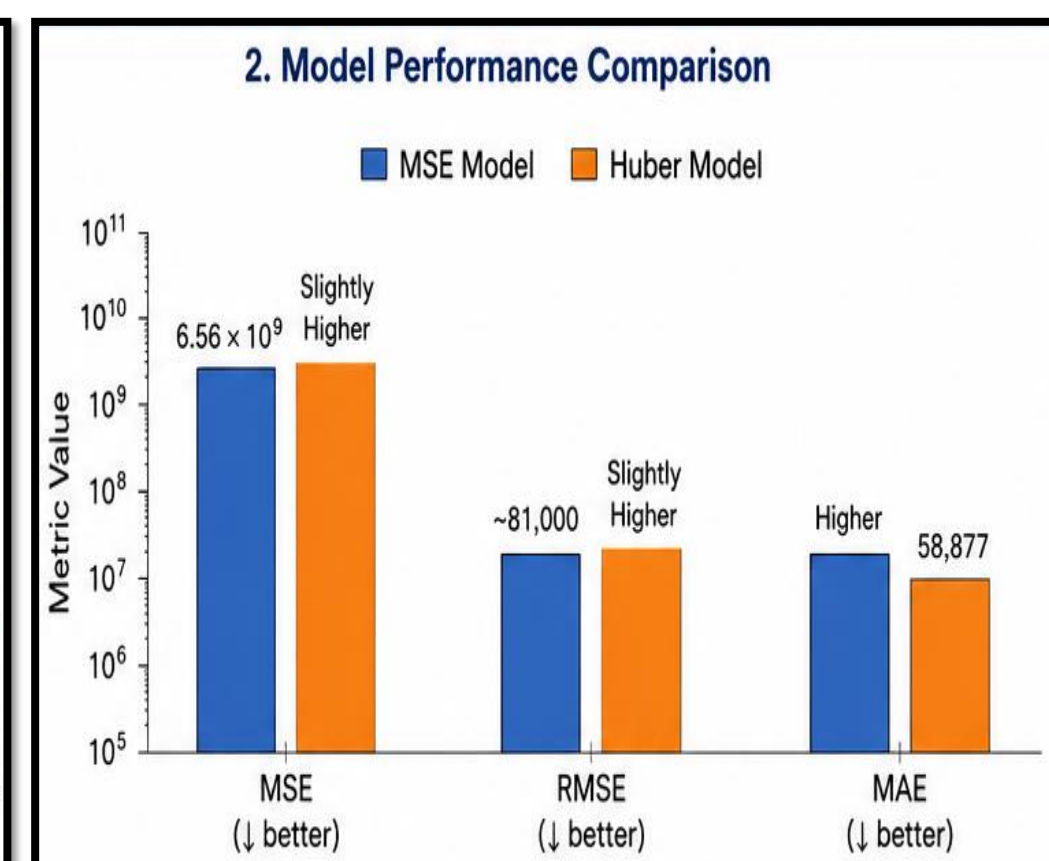
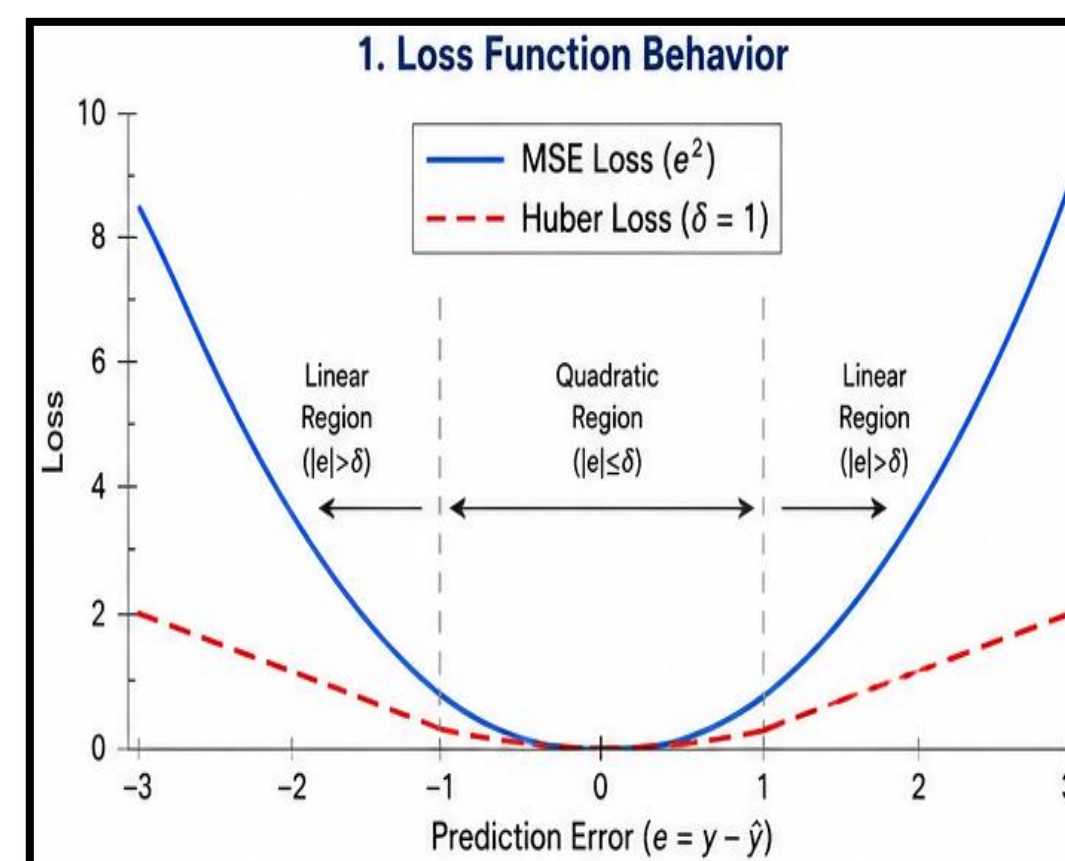
- Penalizes large errors quadratically.

Experimental Setup

- Housing dataset → regression analysis
- Linear regression models trained using MSE & Huber Loss.
- Performance evaluation → MSE, MAE, RMSE.
- Prediction variance & Noise sensitivity analysis.

RESULTS & DISCUSSION

- MSE produced lower overall prediction error and better stability.
- Huber Loss achieved lower MAE, indicating stronger robustness to outliers.
- Both models (MSE/ Huber Loss) showed performance degradation under noisy conditions.
- Huber Loss reduced the influence of large prediction errors through its piecewise structure.
- Mathematical properties of loss functions affect model behavior.



CONCLUSION/ FUTURE WORK

Conclusion

- Loss functions significantly affect model accuracy, robustness, and stability.
- MSE performs well on datasets with lower prediction error.
- Huber Loss provides better resistance to noise and outliers, improving model reliability.

References

- Deep Learning, MIT Press, 2016.
- Pattern Recognition and Machine Learning, Springer, 2006.
- Robust Estimation of a Location Parameter, Annals of Mathematical Statistics, 35(1), 73-101.

Future Work

- Explore additional robust loss functions (Tukey, Cauchy).
- Extend the study to deep learning models under noisy conditions.
- Develop adaptive or hybrid loss functions for improved performance on real-world datasets.