A Unified Survey of Grand Challenges and Open Problems at the Intersection of Physics, Mathematics, and Computation

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INTRODUCTION

Modern science operates at the intersection of physics, mathematics, and computation, where many of the most fundamental open questions remain unresolved.

Landmark problem sets such as Hilbert's problems, the Millennium Prize problems, and Smale's list have shaped scientific progress for more than a century.

These challenges are typically studied within disciplinary boundaries, which limits systematic cross domain analysis and obscures shared structural foundations.

Fundamental physics confronts problems such as quantum gravity, dark matter, and nonlinear physical systems.

METHODOLOGY

This study follows a structured comparative analytical design to construct a unified framework across physics, mathematics, and computation.

Representative grand challenges were compiled from foundational problem sets and canonical literature in three domains: Physics, Mathematics, and Computation.

Each problem was analyzed to extract its underlying structural components, including symmetry properties, dynamical behavior, computational requirements, and proof structure.

Problems were grouped according to recurring structural motifs such as nonlinearity, emergence, complexity growth, and intractability.

Structural similarities across domains were mapped to identify shared mathematical and computational patterns that transcend disciplinary boundaries.

These mappings supported the development of a unified conceptual classification model that organizes open problems according to structural complexity and computational accessibility.

This methodology enables systematic comparison and provides a coherent analytical lens for interpreting scientific unknowns.

An additional layer of analysis examined epistemic constraints associated with each problem, including limits of simulation, measurability, formal provability, and computational feasibility.

Workflow for Unified Transdisciplinary Framework Construction

Grand Challenge Compilation
Select representative foundational problems in physics, mathematics, and computation

Epistemic Constraints Analysis
Assess feasibility limits

Conceptual Decomposition
Extract structural properties like symmetry and complexity

Comparative Pattern Analysis
Compare structural motifs

Thematic Clustering
Group problems by motifs such as nonlinearity and intractability

Refinement Checks
Verify consistency and logic

Cross Domain Mapping
Identify shared mathematical and computational patterns

Model Construction
Develop unified classification based on structural complexity

Model Construction
Develop unified firation

Figure 1. Workflow for constructing the trans-disciplinary classification model across physics, mathematics, and computation.

STRUCTURAL CLASSIFICATION MODEL

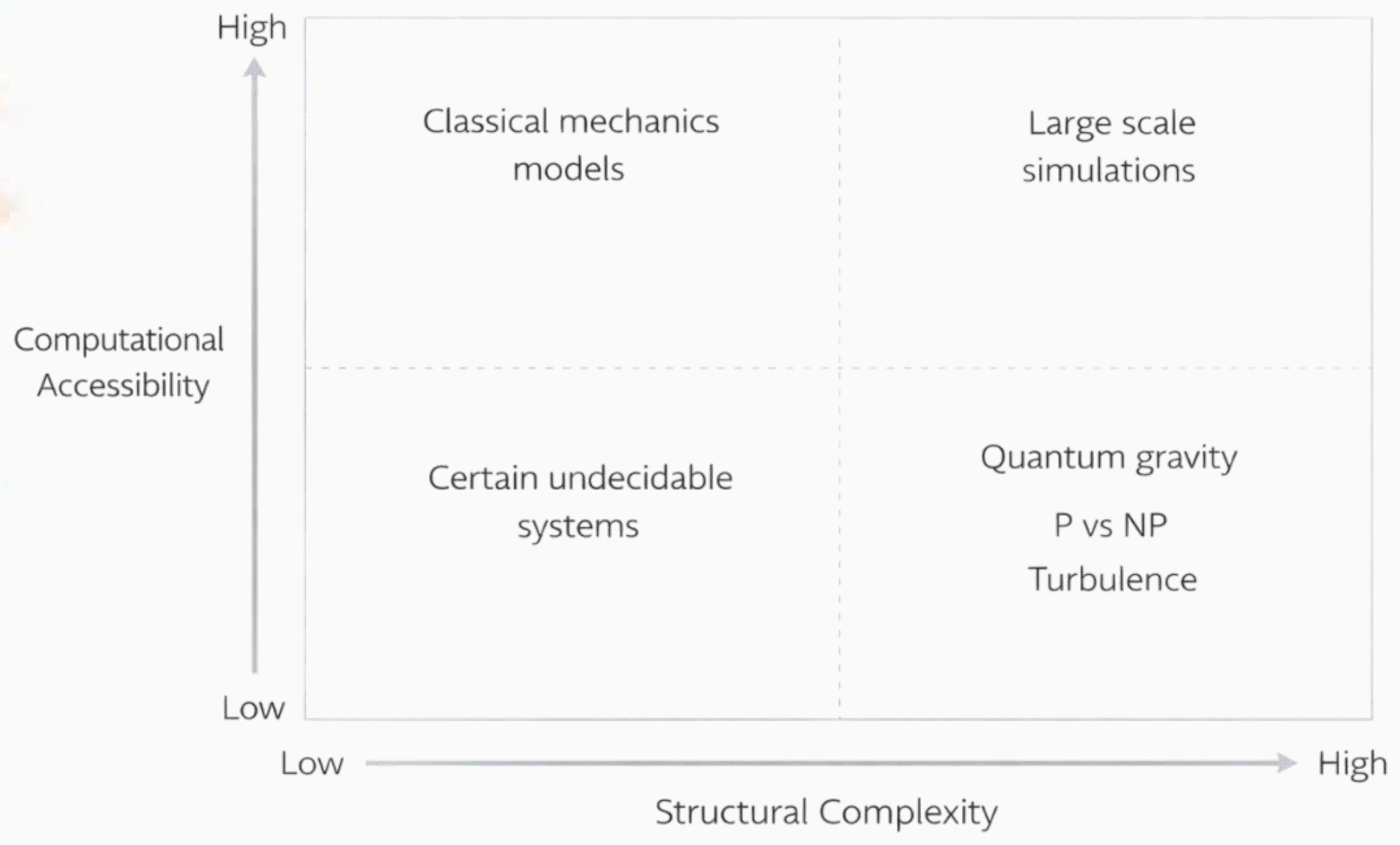

Figure 2. Conceptual classification of grand challenges based on structural complexity and computational accessibility.

RESULTS AND DISCUSSION

The analysis reveals that major open problems across physics, mathematics, and computation share recurring structural patterns. These patterns transcend disciplinary boundaries and reflect common mathematical foundations rather than isolated domain specific difficulties.

Four dominant motifs emerge consistently: symmetry, emergence, nonlinearity, and computational intractability. These motifs function as structural constraints that shape both theoretical formulation and practical solvability.

Symmetry appears as a unifying principle in gauge theories, conservation laws, group theory, and algorithmic optimization. It constrains admissible solutions and reduces structural degrees of freedom. Emergence characterizes multi-scale systems in condensed matter physics, dynamical systems theory, and distributed computational models, where macroscopic order arises from local interactions. Nonlinearity governs turbulence, chaotic dynamics, nonlinear partial differential equations, and non convex optimization landscapes, generating sensitivity to initial conditions and analytical instability. Intractability defines formal computational limits in complexity theory, high dimensional simulations, combinatorial explosion, and undecidability phenomena.

Cross domain comparison shows that many problems traditionally viewed as unrelated occupy structurally similar regions when classified by complexity and computational accessibility. For example, turbulence in fluid dynamics and hardness results in computational complexity both exhibit exponential growth of solution space under nonlinear constraints. Similarly, questions of quantum gravity and formal proof complexity both involve limits of represent-ability within existing mathematical frameworks.

The unified classification space defined by structural complexity and computational accessibility provides a systematic mapping of scientific unknowns. Problems with low structural complexity and high computational accessibility are typically resolved through analytic or algorithmic refinement. Problems with high structural complexity and low computational accessibility define persistent frontiers of research.

This framework highlights that computational constraints are intrinsic components of scientific theory. Limits of simulation, provability, and algorithmic feasibility directly influence what can be experimentally tested or theoretically verified. The boundary between solvable and unsolved problems often reflects structural growth rates and resource scaling rather than purely conceptual gaps.

Grand challenges reflect deeper mathematical and computational structures.

Shared motifs enable coherent interdisciplinary research.

CONCLUSIONS

- Grand challenges in physics, mathematics, and computation exhibit shared structural foundations.
- A unified analytical framework enables systematic comparison and reveals recurring mathematical and computational motifs.

References:
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[2] Clay Mathematics Institute. Millennium Prize Problems.
[3] Smale, S. Mathematical Problems for the Next Century.
[4] Foundational literature in quantum gravity, complexity theory, nonlinear dynamics, and computational limits.