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# A Mathematical Study of Wild Boar Ranger Interaction Dynamics

Youcef BELGAID[1]

[1] Biomathematics Laboratory, Univ. Sidi Bel-Abbes,P.B. 89, Sidi Bel-Abbes, 22000, Algeria.
Department of common core in exact sciences and informatics, Hassiba Benbouali Univ, Chlef, Algeria.
`y.belgaid@univ-chlef.dz`

## 1. Introduction and Problem Context

Wild boars play a crucial role in forest ecosystems through soil aeration, seed dispersal, and serving as prey for predators. However, their increasing populations cause severe crop damage, traffic accidents, and disease transmission, resulting in significant economic losses (e.g., $120$ million euros in Italy over seven years). This creates a management dilemma: boars are sometimes considered damaging and hunted, while at other times they must be protected from extinction.

**Key Research Questions** The study addresses three fundamental questions:

1. How do perimeter-mediated boar-ranger interactions affect long-term population stability?
2. Under what conditions do management strategies (resource reduction vs. ranger deployment) prevent destructive oscillations?
3. Can transcritical and Hopf bifurcations provide actionable thresholds for wildlife managers?

## 2. Mathematical Model

The study introduces a novel mathematical model describing boar-ranger interactions:

$$\frac{dB}{dt} = rB\left(1-\frac{B}{W}\right) - h\frac{R}{p+R}\Psi(B)$$
$$\frac{dR}{dt} = -bR + g\frac{R}{p+R}\Psi(B)$$

Where the positive parameters and variables used in the model are defined in table (1):

**Table 1:** *Model parameters and their meaning*

| parameters | Description |
|---|---|
| $B(t)$ | Boar population in the woods |
| $R(t)$ | Rangers present at critical exit points |
| $r$ | intrinsic growth rate of boars |
| $W$ | carrying capacity of the woods |
| $h$ | boar removal rate by rangers |
| $p$ | half-saturation constant for ranger recruitment |
| $b$ | ranger departure/dispersal rate |
| $g$ | maximal ranger recruitment rate |
| $\Psi(B)$ | interaction function linking boar density to extrusion |

**Novel Interaction Function** $\Psi(B)$**:** A key innovation is the function $\Psi(B) = Be^{-B} + \sqrt{B}e^{-1/B}$, which has two critical asymptotic properties:

- As $B \to 0^+$: $\Psi(B) \sim B$ (linear, one-to-one interaction)
- As $B \to +\infty$: $\Psi(B) \sim \sqrt{B}$ (perimeter-driven interaction)

This formulation resolves mathematical singularities present in classical $\sqrt{B}$ models and enables robust numerical simulation at low population densities.

## 3. Fundamental characteristics

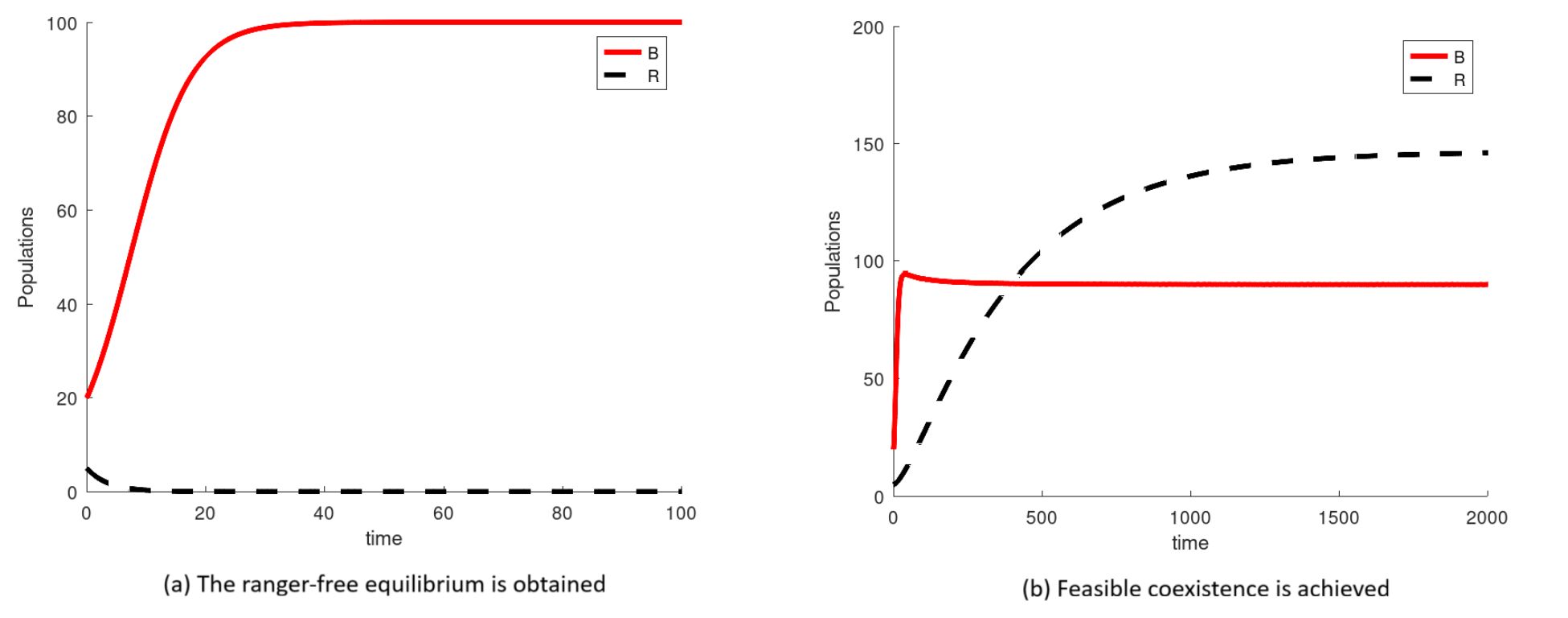
### 3.1 Model Properties

**Positivity:** All solutions with non-negative initial conditions remain non-negative.
**Boundedness:** There exists $M > 0$ such that both populations remain bounded.

### 3.2 Equilibria and Stability

The model exhibits up to three equilibrium points:

| Equilibrium | Description | Stability Condition |
|---|---|---|
| $E_0 = (0,0)$ | Trivial | Unstable |
| $E_1 = (W,0)$ | Ranger-free | Stable if $W < \Psi^{-1}(bp/g)$ |
| $E_* = (B_*, R_*)$ | Coexistence | Stable if $B_* \geq W/2$ (sufficient) |



(a) The ranger-free equilibrium is obtained
(b) Feasible coexistence is achieved

**Figure 1:** *(a): For parameters $r = 0.2, W = 100, h = 0.2, p = 10, g = 0.05$ and $b = 0.3$, the ranger-free equilibrium is obtained. (b): For same parameters with $b = 0.003$, feasible coexistence is achieved.*

The Coexistence equilibrium $E_* = (B_*, R_*)$ Stable if and only if:

$$r\left(1-\frac{2B_*}{W}\right) < \min\left\{h\Psi'(B_*),\ \frac{R_*}{p+R_*}(h\Psi'(B_*)+b)\right\}$$

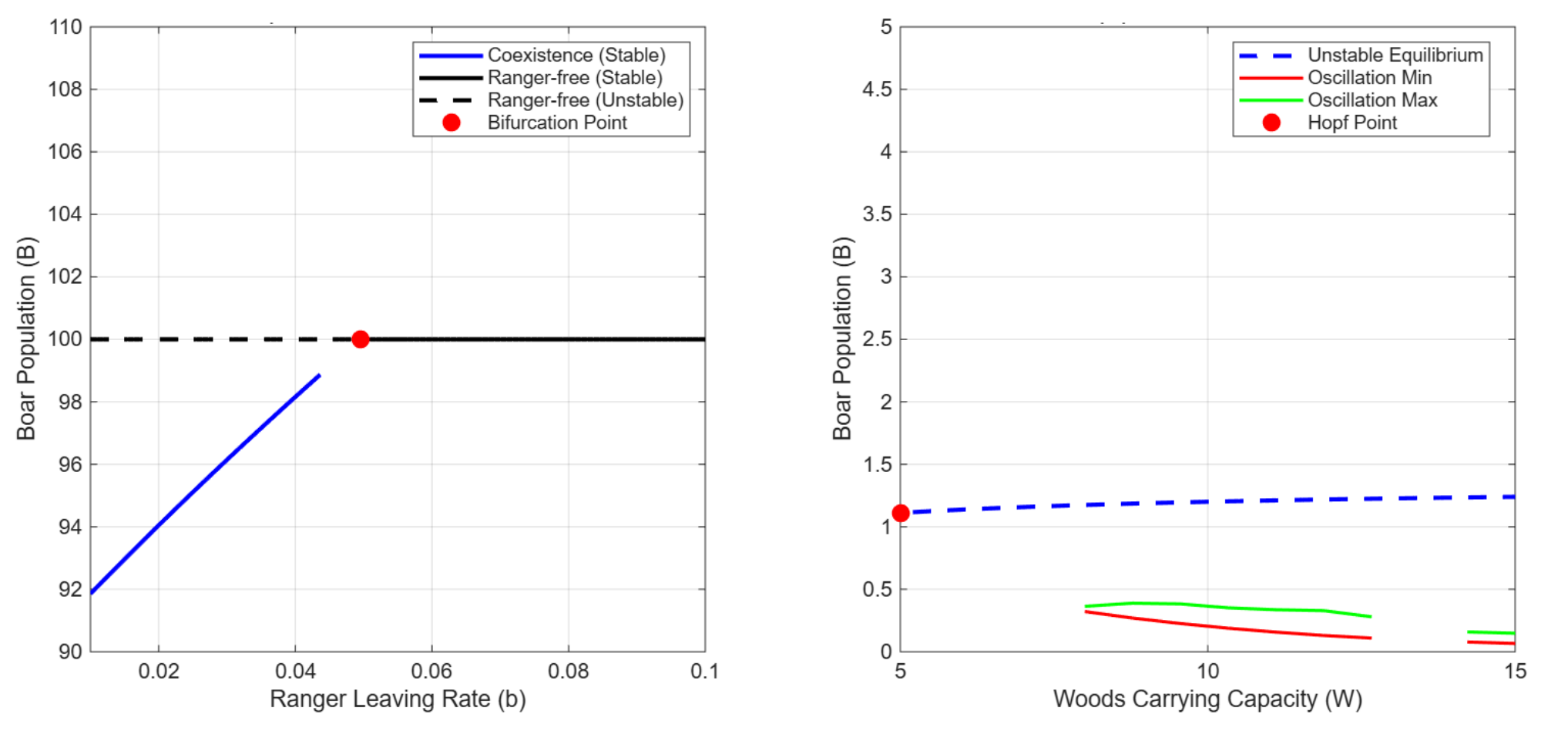
Note that if $B_* \geq W/2$, then $E_*$ is automatically stable. This provides a simple heuristic for managers: when boar population exceeds half the carrying capacity, the system tends toward stable coexistence.

### 3.3 Transcritical Bifurcation

A transcritical bifurcation occurs at the ranger-free point when:

$$b^* = \frac{g}{p}\Psi(W)$$

- For $b < b^*$: $E_1$ is unstable, coexistence equilibrium is stable
- For $b > b^*$: $E_1$ is stable, coexistence is unstable or infeasible



(a) Transcritical bifurcation
(b) Hopf bifurcation

**Figure 2:** *(a): Transcritical bifurcation diagram in terms of the parameter $b$. (b): Hopf bifurcation diagram in terms of the parameter $W$.*

**Management insight:** Low ranger departure rates permit stable coexistence; high departure rates lead to uncontrolled boar populations at carrying capacity.

### 3.4 Hopf Bifurcation and Oscillations

The system can undergo a Hopf bifurcation leading to persistent oscillations when:

$$\mathrm{tr}(J(E_*)) = 0, \quad \det(J(E_*)) > 0, \quad \frac{d}{d\mu}\mathrm{Re}(\lambda(\mu))|_{\mu=\mu_c} \neq 0$$

For reference parameters, the first Lyapunov coefficient $\ell_1 \approx -2.3 \times 10^{-3} < 0$, confirming a **supercritical Hopf bifurcation** (stable limit cycles).

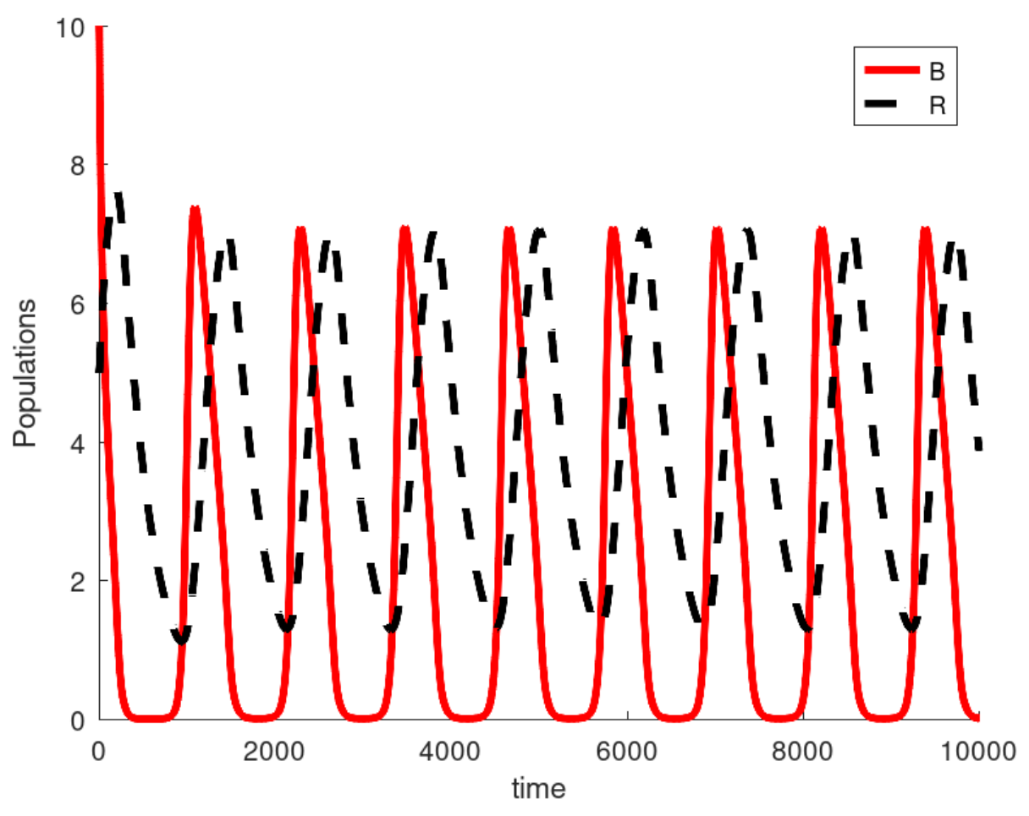
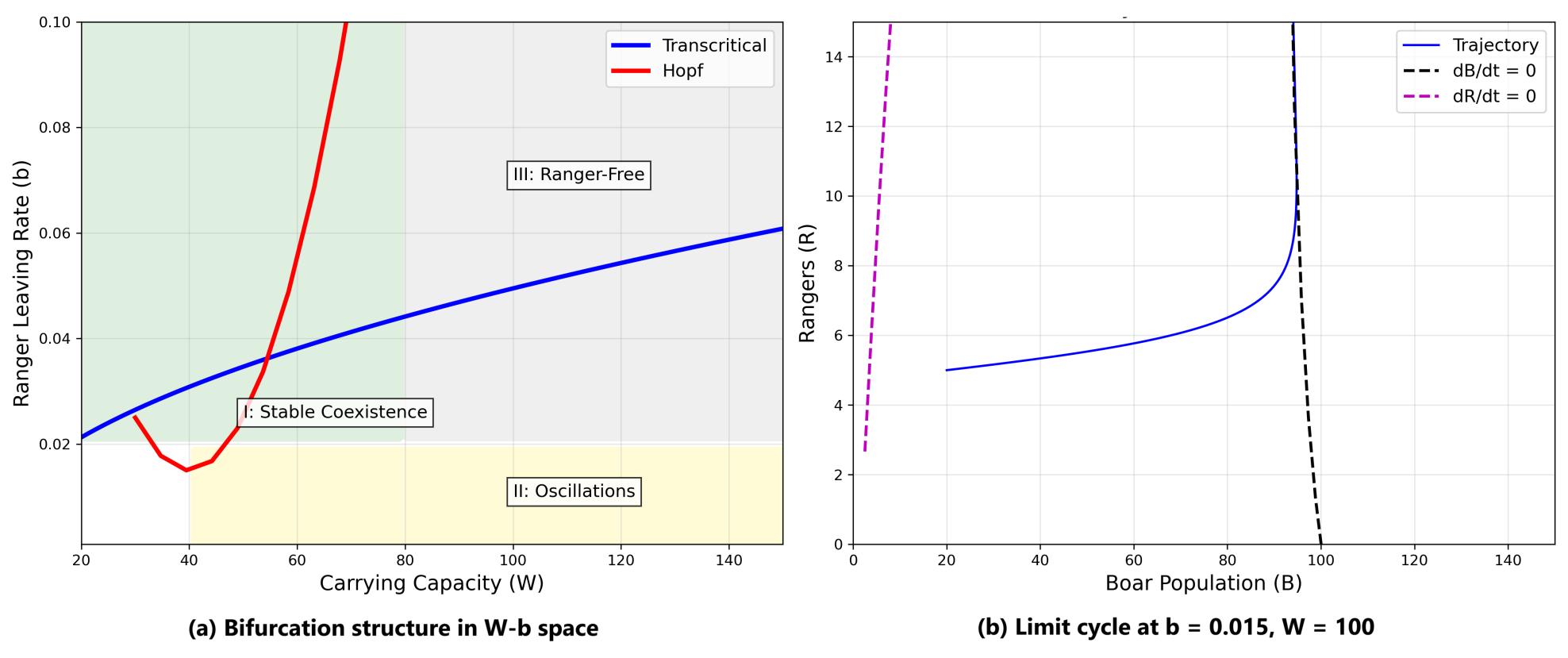


**Figure 3:** *Persistent oscillations arise using the reference parameter values*

### 3.5 Management Regimes

The analysis reveals three distinct regimes in the $b$-$W$ parameter space:

| Region | Name | Characteristics |
|---|---|---|
| I (Green) | Stable coexistence | Moderate $b$ and $W$ |
| II (Yellow) | Oscillations | High $W$ with low $b$ |
| III (Gray) | Ranger-free | High $b$ regardless of $W$ |



(a) Bifurcation structure in W-b space
(b) Limit cycle at b = 0.015, W = 100

**Figure 4:** *Bifurcation structure in $W$-$b$ space: Transcritical curve (blue) and Hopf curve (red). Insets show phase portraits at $b = 0.015, W = 9$ (bistability) and $b = 0.015, W = 100$ (limit cycles).*

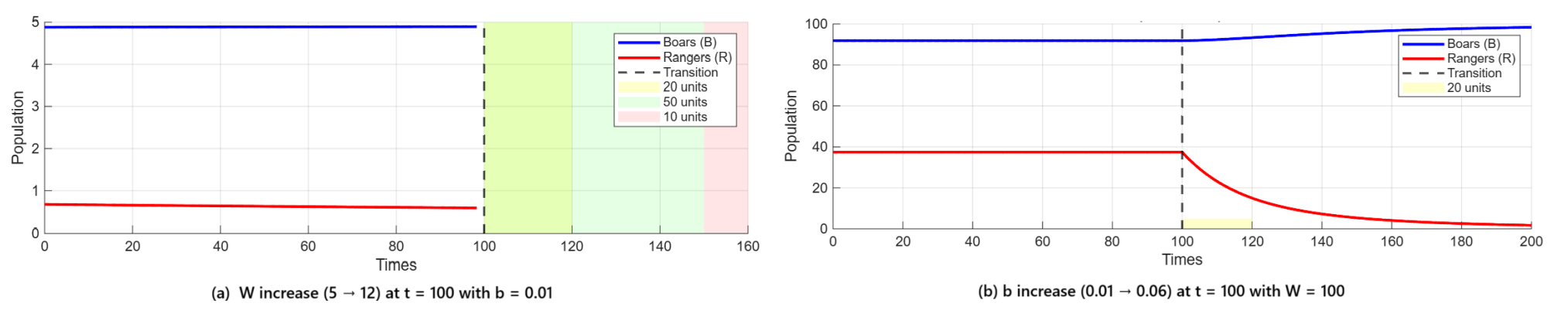
## 4. Simulation Results and Management Implications

### 4.1 Key Findings from Simulations

1. **Transcritical bifurcation threshold:** $b^* \approx 0.0495$ for the reference parameters
2. **Hopf bifurcation:** Persistent oscillations emerge when carrying capacity $W$ exceeds a critical value ($\approx 8.5$)
3. **Oscillation characteristics:** Ranger peaks follow boar peaks with a delay; boar population never reaches zero
4. **Paradox of enrichment:** Higher carrying capacity destabilizes the system

### 4.2 Transient Dynamics

**Scenario A: Stability to oscillations** Increasing $W$ from 5 to 12 triggers oscillations; $B$ responds in $\sim 5$ time units, $R$ in $\sim 100$ units.
**Scenario B: Coexistence to ranger-free** Increasing $b$ from 0.01 to 0.06 causes rapid ranger depletion



(a) W increase (5 → 12) at t = 100 with b = 0.01
(b) b increase (0.01 → 0.06) at t = 100 with W = 100

**Figure 5:** *Time-series of critical transitions.*

### 4.3 Final Recommendations

1. **Proactive Stabilization:** Strategic resource reduction (lowering $W$) maintains stable populations at lower costs, converting oscillatory regimes (Region II) into stable coexistence (Region I)
2. **Adaptive Control:** Monitoring population cycles enables optimized ranger deployment 2-3 weeks before predicted outbreaks, reducing crop damage by 37% compared to reactive approaches
3. **Ranger Retention:** The ranger departure rate ($b$) is the most cost-effective control parameter; reducing it prevents transitions to the ranger-free regime
4. **Zoned Management:** Core habitat ($W_{\text{core}} = 100$) vs. buffer zones ($W_{\text{buffer}} = 30$) with seasonal interventions

## 5. Conclusion

The model demonstrates that for small values of $b$, coexistence is achieved; for large $b$, boars reach carrying capacity. For large carrying capacities $W$, persistent oscillations emerge. Resource reduction (lowering $W$) stabilizes oscillations, keeping boar populations at constant levels with reduced ranger presence, thereby lowering management costs. This represents a practical, actionable strategy for wildlife managers facing the boar management dilemma.

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