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The Averaging Principle for Stochastic Slow-Fast Systems Driven by G-Brownian Motion

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INTRODUCTION & AIM

The averaging principle reduces slow-fast dynamical systems to an effective slow equation, but classical theory assumes known noise distributions.

In many applications (climate economics, systemic risk, epidemiology), volatility is uncertain—only known to belong to a set of possible values. No averaging theory exists for slow-fast systems where the driving noise has *distributional ambiguity*.

In this work, we establish an averaging principle for slow-fast stochastic systems driven by G-Brownian motion (volatility uncertainty). Then, we introduce a notion of G-invariant measure and averaged coefficients in a worst-case sense and finally we prove convergence of the slow component to the solution of an averaged equation,

METHOD

We consider the following system driven by G-Brownian motion:

$$dX_t^\epsilon = f_1 dt + f_2 d\langle B\rangle_t + f_3 dB_t,$$
$$dY_t^\epsilon = \frac{1}{\epsilon} g_1 dt + \frac{1}{\sqrt{\epsilon}} g_2 dB_t.$$

For each frozen slow variable x, the fast process has a *set* of invariant measures (one for each possible volatility scenario). The associated sublinear expectation is

$$\mu^x(\Phi) = \sup_{\mu \in \mathcal{M}_x} \int \Phi(y)\,\mu(dy).$$

The Averaged coefficients (in worst-case sense) are given by:

$$\overline{f_i}(x) = \mu^x(f_i(x,\cdot)), \quad i = 1,2,3.$$

We use Khasminskii time discretization + G-Itô calculus + G-BDG inequality to prove the desired results.

RESULTS & DISCUSSION

Under suitable assumption we obtain:

1. **Convergence in capacity:**

$$\lim_{\epsilon\to 0} \mathbb{C}\left(\sup_{0\le t\le T} |X_t^\epsilon - \overline{X}_t| > \kappa \right) = 0 \quad \forall \kappa > 0.$$

2. **Convergence in mean-square :**

$$\lim_{\epsilon\to 0} \mathbb{E}\left[\sup_{0\le t\le T} |\Gamma(X_t^\epsilon) - \Gamma(\overline{X}_t)|^2 \right] = 0.$$

The G-invariant measure set replaces the classical unique invariant measure – averaging is done in a **worst-case** manner. Uniform moment bounds and tightness hold independently of ϵ. The averaged equation is well-posed (linear growth and local Lipschitz continuity of the coefficients).

What is new compared to classical averaging?

- Classical theory: unique invariant measure μ^x → averaged coefficient $\int f_i(x,y)\mu^x(dy)$.
- This work: **set of invariant measures** $\mathcal{M}_x$ → worst-case average $\sup_{\mu\in\mathcal{M}_x} \int f_i(x,y)\mu(dy)$.
- Convergence holds **simultaneously for all possible volatilities** (robust).

Why two types of convergence?

- **Capacity convergence** = worst-case analogue of convergence in probability.
- **Mean-square (L_G^2) convergence** = stronger, implies capacity convergence via Markov inequality. The paper proves both.

CONCLUSION

We introduced a **G-invariant measure** and **worst-case averaged coefficients** for slow-fast systems under volatility uncertainty. Under natural ergodicity and regularity assumptions, we proved that the slow component converges to the solution of an averaged equation – both **in capacity** and **in mean square.** This bridges multiscale dynamics and GG-expectation theory, providing robust model reduction when noise statistics are not precisely known.

FUTURE WORK / REFERENCES

Numerical verification of the G-averaged equation remains an open challenge (how to simulate worst-case expectations?).

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