Cosmological and Thermodynamic Insights into Barrow–Ricci–Gauss–Bonnet Holographic Dark Energy in the Symmetric Teleparallel Framework of $f(Q)$ Gravity

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Introduction

Observations from SNe Ia, CMB and BAO confirm accelerated cosmic expansion.

Problems with ΛCDM:

- Fine-tuning problem
- Cosmological constant issue
- UV–IR inconsistency
- Dark energy origin

Holographic dark energy:

$$\rho_{DE} = 3c^2L^{-2}$$

Barrow entropy deformation:

$$S_B = \left(\frac{A}{A_0}\right)^{1+\Delta/2}$$

BRGB-HDE density:

$$\rho_{BRGB} = 3c^2(\alpha R + \beta G)^{1-\Delta/2}$$

Motivation of the Present Work

The present study is motivated by the following cosmological and theoretical issues:

- Understanding the origin of late-time cosmic acceleration beyond the standard ΛCDM paradigm.
- Incorporating quantum gravitational corrections through Barrow entropy deformation.
- Constructing a unified holographic dark energy model using Ricci and Gauss–Bonnet geometric invariants.
- Exploring whether symmetric teleparallel $f(Q)$ gravity can provide a stable and thermodynamically consistent cosmological framework.

The combined BRGB-HDE framework provides a geometrically rich description of dark energy evolution and modified gravity dynamics.

$f(Q)$ Gravity Framework

In symmetric teleparallel gravity, gravity originates from non-metricity:

$$Q$$

Advantages:

- Second-order field equations
- Simpler cosmological evolution
- Stable late-time behavior
- No curvature/torsion complications

Hybrid scale factor:

$$a(t) = a_0 t^n e^{\lambda t}$$

$$H = \frac{n}{t} + \lambda$$

Model Equations

Ricci scalar:

$$R = 6(\dot{H} + 2H^2)$$

Gauss–Bonnet invariant:

$$G = 24H^2(\dot{H} + H^2)$$

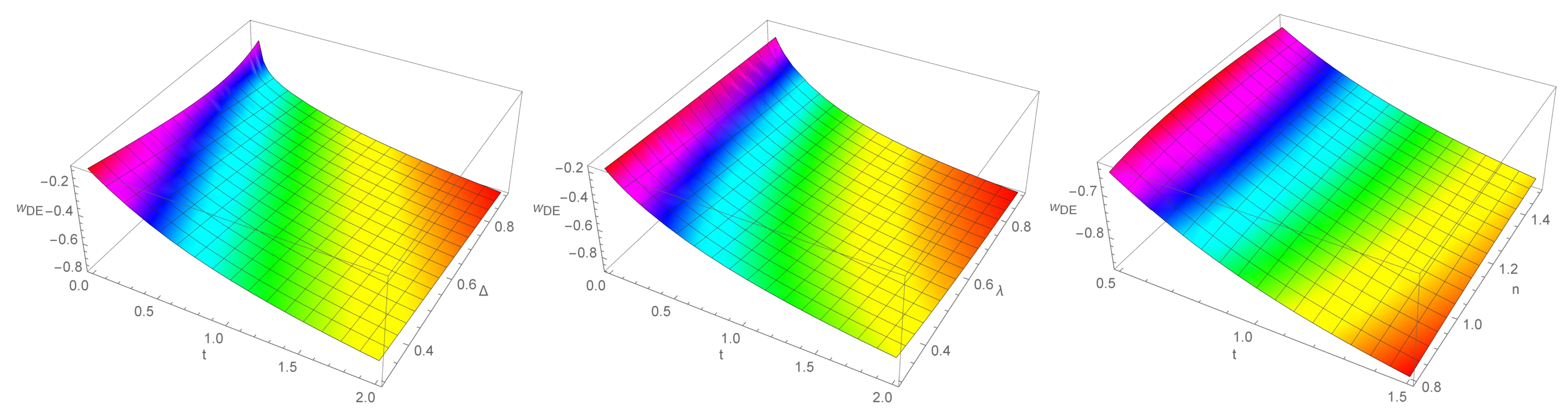
Equation of state:

$$w_{DE} = -1 - \frac{\dot{\rho}_{BRGB}}{3H\rho_{BRGB}} + \frac{b^2\rho_m}{\rho_{BRGB}}$$

Interaction term:

$$\mathcal{Q} = 3Hb^2\rho_m$$

Evolution of Equation of State



The EoS parameter smoothly evolves toward $w = -1$.

- Quintessence phase
- Stable accelerated expansion
- Dependence on Δ
- Smooth ΛCDM convergence

Reconstruction of $f(Q)$ Gravity

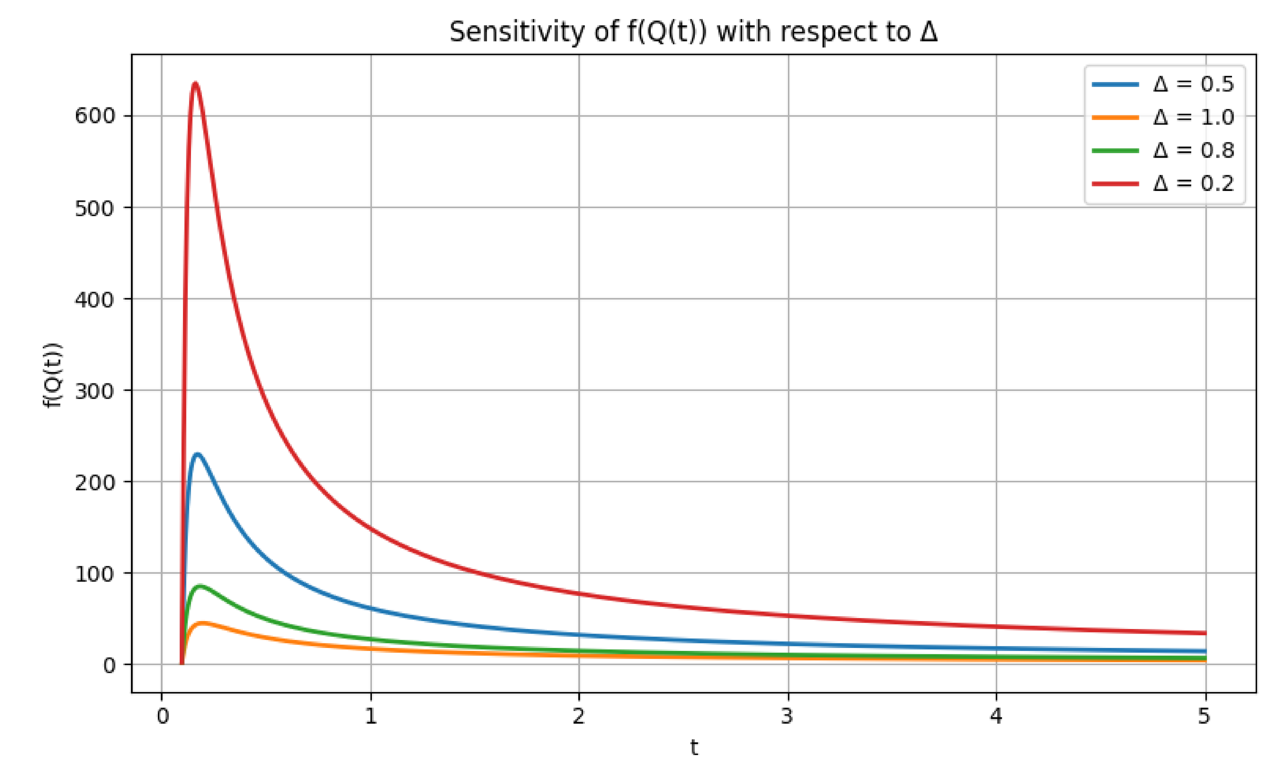
Correspondence relation:

$$\rho_{BRGB} = \frac{1}{2}(f - Qf_Q)$$

Reconstruction equation:

$$Qf_Q - f = -2\rho_{BRGB}(Q)$$

The nonlinear differential equation is solved numerically using the Runge–Kutta method.



The reconstructed function strongly deviates from GR at early times and gradually converges toward GR at late times.

Physical Interpretation

The proposed framework combines:

- Holographic dark energy
- Barrow entropy correction
- Ricci geometry
- Gauss–Bonnet invariant
- Symmetric teleparallel gravity

Main implications:

- Stable cosmic evolution
- Thermodynamic consistency
- Quantum gravity corrections
- Geometrically unified cosmology

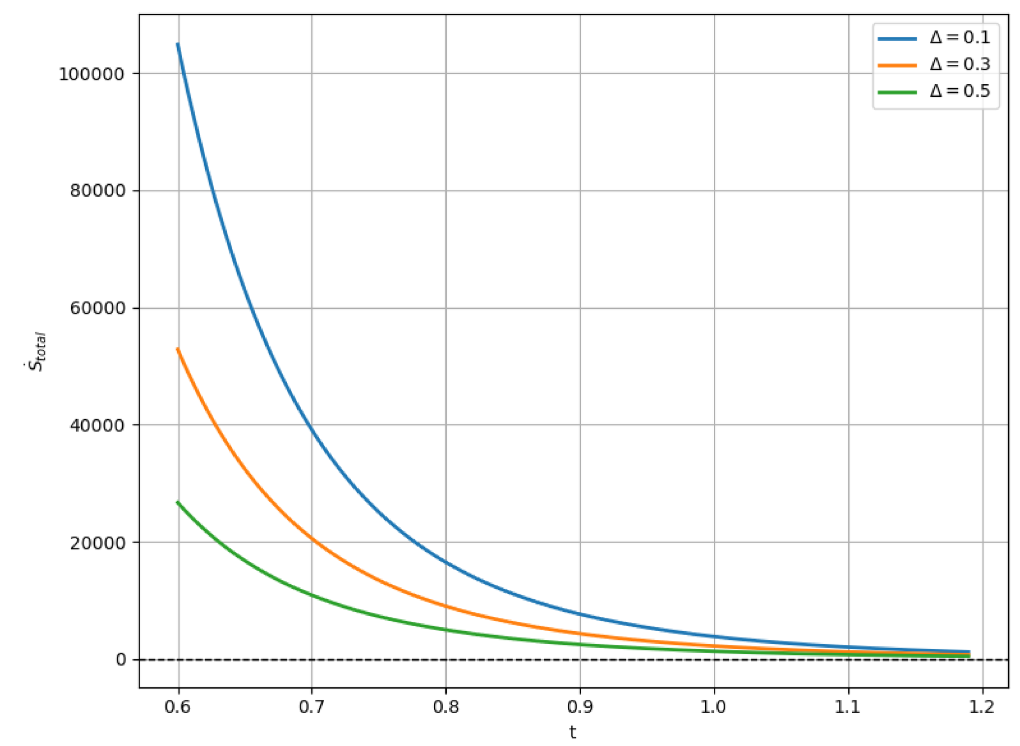
Thermodynamic Analysis

Apparent horizon radius:

$$R_A = \frac{1}{H}$$

Generalized second law:

$$\dot{S}_{tot} = \dot{S}_B + \dot{S}_{in} \geq 0$$



Entropy remains positive during the complete cosmic evolution.

Cosmological Implications

- Early-time modified gravity dominance
- Late-time GR convergence
- Stable holographic dark energy evolution
- Viable thermodynamic behavior
- Dynamically stable cosmic expansion

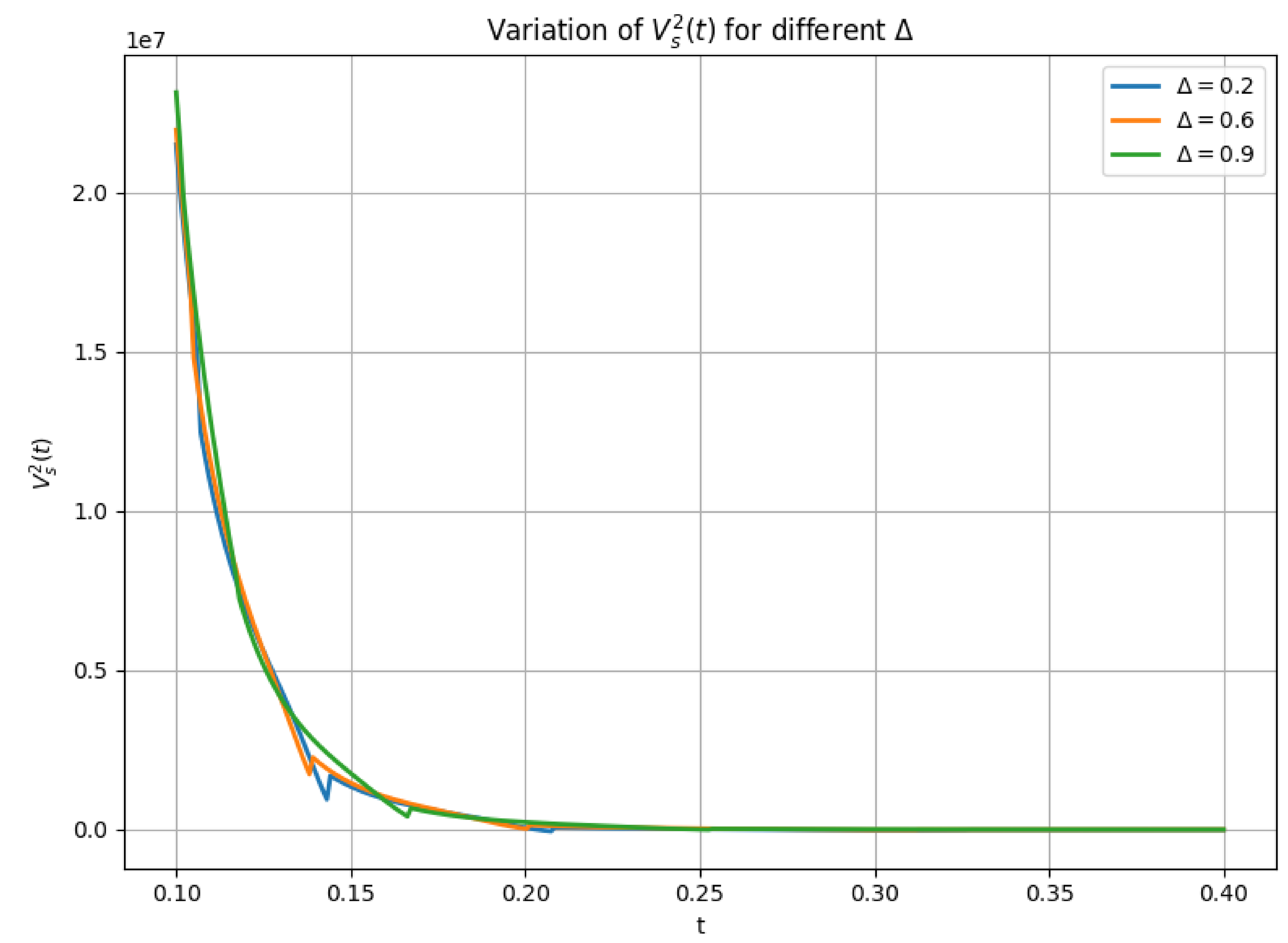
Stability Analysis

Squared sound speed:

$$v_s^2 = \frac{\dot{p}_{DE}}{\dot{\rho}_{DE}}$$

Stability criterion:

$$v_s^2 > 0$$

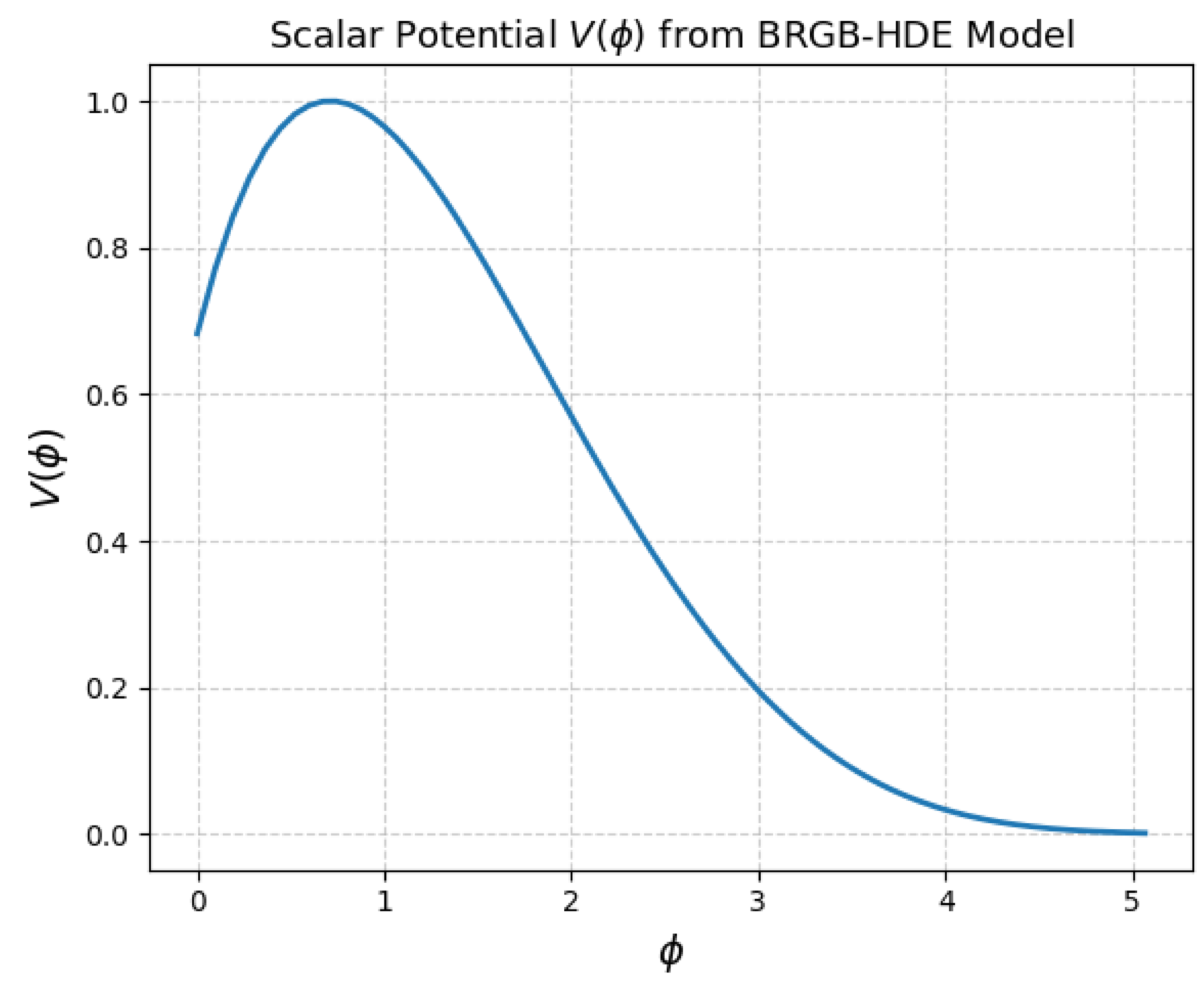


Positive squared sound speed confirms stable late-time evolution.

Scalar Field Correspondence

Scalar potential:

$$V(\phi) = \frac{1}{2}\rho_{DE}(1 - w_{DE})$$



The reconstructed scalar field behaves like quintessence dark energy.

Conclusions

- BRGB-HDE model successfully constructed in $f(Q)$ gravity.
- Explains late-time acceleration naturally.
- Reconstruction smoothly approaches GR.
- GSLT validity confirms thermodynamic consistency.
- Stability analysis confirms viable cosmology.
- Scalar field reconstruction supports quintessence behavior.

Future Scope:
Quantum gravity corrected holographic cosmology may provide deeper insight into dark energy evolution and modified gravity dynamics.