STABILITY ANALYSIS OF A HOTEL MODEL AND ASSESSMENT USING ADDITIVE FUNCTIONAL EQUATIONS

Mohan Arunkumar[1], Elumalai Sathya[2], Thirumalai Namachivayam[3], Vadamalai Chandiran[4], Velmurugan Alexpandiyan[5], Tamilselvan Velmurugan[6]

[1,2,3,4,5] Department of Mathematics, Kalaignar Karunanidhi Government Arts College, Tiruvannamalai, Tamil Nadu, India

[6] Department of Mathematics, MRK College of Arts and Science, Pazhanchanallur, Kattumannarkoil, Tamil Nadu, India

1. MODEL: ADDITIVE FUNCTIONAL EQUATIONS

We introduce a new class of systems of additive functional equations of the form:

$$\begin{aligned} H_k(p_k + r_k + f_k + q_{k1} + q_{k2} + s_k + t_k) &= H_k(p_k) + H_k(r_k) + H_k(f_k) + H_k(q_{k1}) \\ &\quad + H_k(q_{k2}) + H_k(s_k) + H_k(t_k), \quad k = 1, \ldots, 8, \end{aligned} \tag{1}$$

where:

- p_k : Parking availability
- r_k : Restroom facilities
- f_k : Food availability
- q_{k1}, q_{k2} : Quality & Quantity of food
- s_k : Service (serves attitude)
- t_k : Tips

2. THEOREM (Uniqueness and Stability)

Theorem 2.

Assume a function $H_k : T \to U$ satisfies the system of FIS

$$\| H_k(p_k; r_k; f_k; q_{k1}; q_{k2}; s_k; t_k) \| \le F_k(p_k, r_k, f_k, q_{k1}, q_{k2}, s_k, t_k); \; K = 1, \ldots, 8, \tag{2}$$

where $F_k : T \to [0, \infty)$ with

$$\lim_{E \to \infty} \frac{F_k(A_k^{ES} p_k, A_k^{ES} r_k, A_k^{ES} f_k, A_k^{ES} q_{k1}, A_k^{ES} q_{k2}, A_k^{ES} s_k, A_k^{ES} t_k)}{A_k^{ES}} = 0, \quad A_k^{ES} \begin{cases} 7; & K = 1 \\ 5; & K = 2, \\ 4; & K = 3, 4, 5, \\ 3; & K = 6, 7, \\ 2; & K = 8. \end{cases}$$

Then, there exist unique additive functions $G_k : T \to U$ given by

$$G_k(t_k) = \lim_{E \to \infty} \frac{H_k(A_k^{ES} t_k)}{A_k^{ES}}, \quad K = 1, \ldots, 6 \tag{4}$$

such that for all $t_k \in T$,

$$\| G_k(t_k) - H_k(t_k) \| \le \frac{1}{A_k^{ES}} \sum_{\frac{D=1-s}{2}}^{\infty} \frac{F_k(A_k^{DS} t_k)}{A_k^{DS}}, \quad K = 1, \ldots, 6. \tag{5}$$

3. COROLLARY (Simplified Stability Bounds)

Corollary 3.

Under the conditions of Theorem 2, we have for all $t_k \in T$,

$$\| G_k(t_k) - H_k(t_k) \| \le \begin{cases} \dfrac{N}{|A_k - 1|}; & K = 1, \ldots, 8 \\ \dfrac{N A_k \| t_k \|^m}{|A_k - A_t^{t,m}|}; & K = 1, \ldots, 8 \\ \dfrac{N \| t_k \|^m}{|A_1 - A_1^{t,m}|}; & K = 1 \\ \dfrac{N (A_k + 1) \| t_k \|^{h,m}}{|A_1 - A_t^{t,m}|}; & K = 1 \end{cases} \tag{6}$$

where $N > 0$, $m \ne 1$, $A_1 \ne 1$, and A_k are constants as in Theorem 2.

4. COROLLARY (Simplified Stability Bounds) CONTINUATION

$$\| G_k(t_k) - H_k(t_k) \| \le \begin{cases} \dfrac{N}{|A_k - 1|}; & K = 1, \ldots, 8 \\ \dfrac{N A_k \| t_k \|^m}{|A_k - A_t^{t,m}|}; & K = 1, \ldots, 8 \\ \dfrac{N \| t_k \|^m}{|A_1 - A_1^{t,m}|}; & K = 1 \\ \dfrac{N (A_k + 1) \| t_k \|^{h,m}}{|A_1 - A_t^{t,m}|}; & K = 1 \end{cases} \tag{7}$$

where $N > 0$, $m \ne 1$, $A_1 \ne 1$, and A_k are constants as in Theorem 2.

5. STABILITY BOUNDS FOR 8 HOTELS

$$\| G_1(t_1) - H_1(t_1) \| \le \frac{N}{|A_1 - 1|} = \frac{N}{|7 - 6|} = \frac{N}{6}, \tag{8}$$

$$\| G_2(t_2) - H_2(t_2) \| \le \frac{N}{|A_2 - 1|} = \frac{N}{|5 - 1|} = \frac{N}{4}, \tag{9}$$

$$\| G_3(t_3) - H_3(t_3) \| \le \frac{N}{|A_3 - 1|} = \frac{N}{|4 - 1|} = \frac{N}{3}, \tag{10}$$

$$\| G_4(t_4) - H_4(t_4) \| \le \frac{N}{|A_4 - 1|} = \frac{N}{|4 - 1|} = \frac{N}{3}, \tag{11}$$

$$\| G_5(t_5) - H_5(t_5) \| \le \frac{N}{|A_5 - 1|} = \frac{N}{|4 - 1|} = \frac{N}{3}, \tag{12}$$

$$\| G_6(t_6) - H_6(t_6) \| \le \frac{N}{|A_6 - 1|} = \frac{N}{|3 - 1|} = \frac{N}{2}, \tag{13}$$

$$\| G_7(t_7) - H_7(t_7) \| \le \frac{N}{|A_7 - 1|} = \frac{N}{|3 - 1|} = \frac{N}{2}, \tag{14}$$

$$\| G_8(t_8) - H_8(t_8) \| \le \frac{N}{|A_8 - 1|} = \frac{N}{|2 - 1|} = N. \tag{15}$$

6. APPLICATION: HOTEL ASSESSMENT

Binary Digits for Good (1) and Bad (0) in All Hotels

Hotel	Parking	Rest Rooms	Food	Quality	Quantity	Service	Tips	Total	Preference from highest to lowest (1 is highest)
H_1	1	1	1	1	1	1	1	7	1*
H_2	1	1	1	1	1	0	0	5	2*
H_3	1	1	1	0	1	0	0	4	3
H_4	1	1	1	1	0	0	0	4	3
H_5	1	1	0	1	1	1	0	4	3
H_6	1	1	1	0	0	0	0	3	4
H_7	0	0	1	1	1	1	0	3	4
H_8	0	0	0	1	1	1	0	2	5

Preference from highest to lowest (1 is highest)

7. DISCUSSION

- In (8) and (9), we obtain the best possible upper bound stability analysis in the first two hotels only.
- Also in (10), (11), (12), we obtain some bounds.
- In (13), (14), we have half bounds.
- In the remaining situation, as in (15), we cannot arrive at a bound.

8. CONCLUSIONS

- Maximum people prefer hotels with parking availability, clean rest rooms and tasty food.
- Some people like the non-favorite food hotels and some may leave to another.
- Specific people prefer self-service hotels.
- People can't prefer to go inside non-parking and unclean rest rooms hotels.

KEYWORDS
Additive FEs, Ulam Stability, UH Stability, UHR Stability, GHUR Stability, Banach space, hotel.

CORRESPONDING AUTHOR
darun4maths@gmail.com

AFFILIATION
Kalaignar Karunanidhi Government Arts College, Tiruvannamalai, Tamil Nadu, India

EMAILS
sathya24mathematics@gmail.com
namachisiva1968@gmail.com
valexpandiyan98@gmail.com
tamilselvanvel0@gmail.com