IOCMA 2026 Conference

The 2nd International Online Conference on Mathematics and Applications

10–12 June 2026 | Online

Fixed-Point and Common Fixed-Point Theorems for Rational and Almost Contractions in θ_R-Metric Space

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INTRODUCTION & AIM

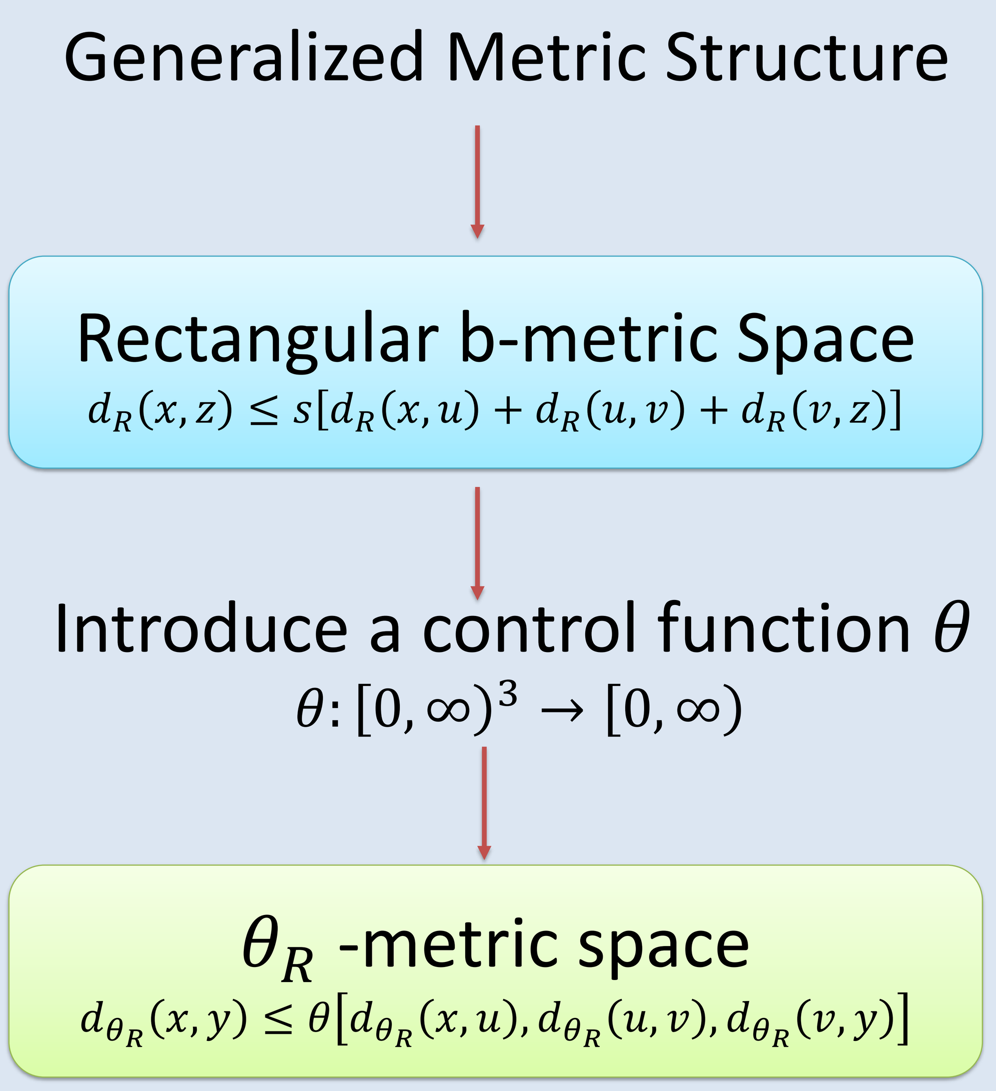
❖ Since the pioneering work of Banach on contraction mappings, metric fixed-point theory has witnessed extensive development through the introduction of various generalized metric structures. One of the primary objectives of these generalizations has been to enlarge the class of spaces in which fixed-point results remain valid, particularly when the associated metric is not necessarily continuous. In this direction, Bakhtin and subsequently Czerwik introduced the concept of a b-metric space, obtained by weakening the classical triangle inequality. Later, Branciari defined a generalized metric space by replacing the triangle inequality with a quadrilateral inequality. Motivated by these developments, George et al. unified the ideas of b-metric and generalized metric spaces and introduced the concept of a rectangular b-metric space.

❖ **Aim**

- Introduce a new generalized metric structure, called a $\boldsymbol{\theta_R}$**-metric space**, extending rectangular b-metric spaces through a control function θ.
- Establish existence and uniqueness results for fixed points of **rational contractions** and **almost contractions** in θ_R-metric spaces.
- Derive **common fixed-point theorems** for pairs of self-mappings.
- Demonstrate that θ_R-metric spaces provide a broader framework that unifies and extends several existing fixed-point results.

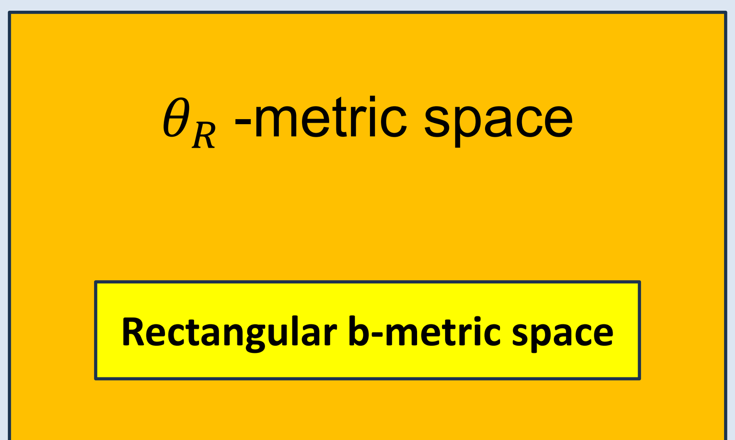
METHOD

The framework of θ_R-metric spaces is analyzed through special cases, examples, and fundamental topological properties. Conditions on the control function θ are established and subsequently used to derive fixed point and common fixed-point theorems for rational and almost contractions.



❖ **Assumptions on control function θ**

- Monotonicity: $a_i \le b_i, i = 1,2,3$
  $\Rightarrow \theta(a_1,a_2,a_3) \le \theta(b_1,b_2,b_3)$.
- Continuity at origin: $\theta(a_n,b_n,c_n) \to 0$
  if $\{a_n\},\{b_n\},\{c_n\}$ converges to 0.
- Compositional stable: $\forall\, a,b,c,d,e \ge 0$
  $\theta(a,\theta(b,c,d),e) \le \theta(a+b,c,d+e)$.
- Vanishing at origin: $\theta(0,0,0) = 0$.



Example: $(\mathbb{R}, e^{|x-y|} - 1)$ is θ_R -metric space but do not belong to the class of rectangular b-metric space.

RESULTS & DISCUSSION

It is established that a self-mapping $T: X \to X$ satisfying the rational contractive condition,

$$d_{\theta_R}(Tx,Ty) \le k\frac{d_{\theta_R}(y,Ty)\left[1+d_{\theta_R}(x,Tx)\right]}{1+d_{\theta_R}(x,y)} + \beta\left[d_{\theta_R}(x,Tx)+d_{\theta_R}(y,Ty)\right] + \delta d_{\theta_R}(x,y), \qquad (1)$$

where $k,\beta,\delta \ge 0$ and $k+2\beta+\delta < 1$, has a unique fixed point in a complete θ_R-metric space, provided that θ satisfies $\theta(0,t,0) = t\ \forall t \ge 0$, together with all the assumptions stated in the previous section.

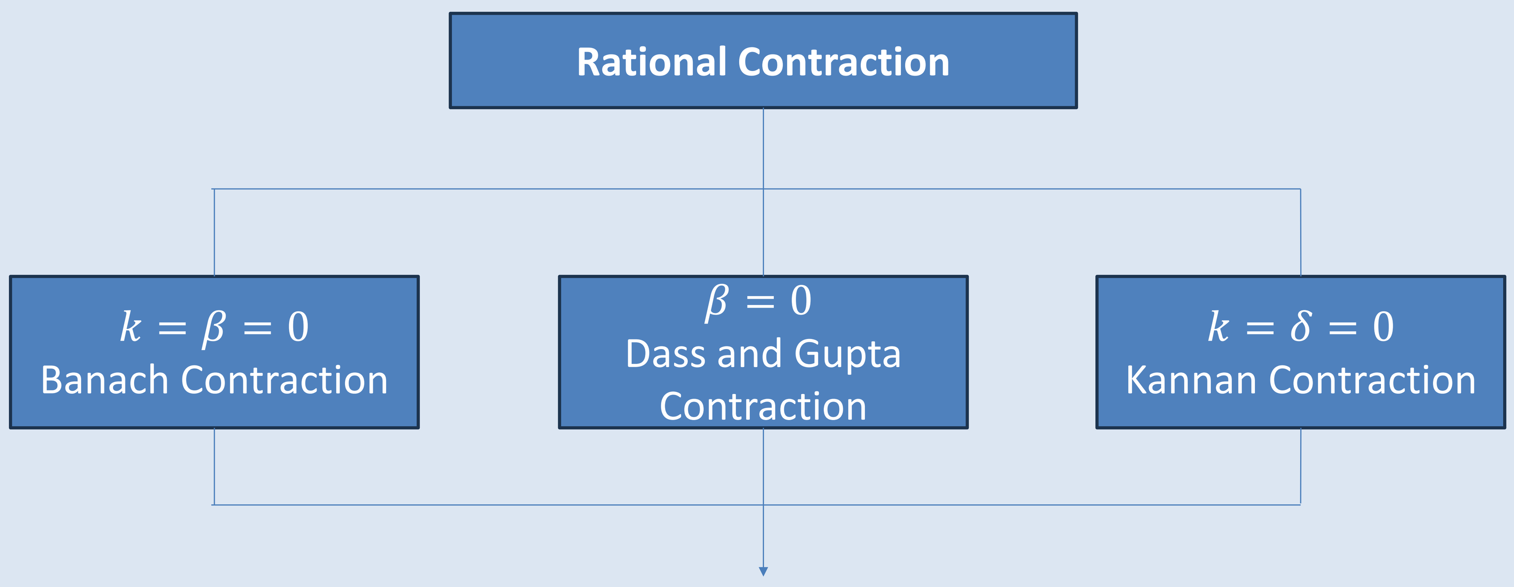
Moreover, under the same hypotheses on the control function θ, the existence and uniqueness of fixed point are obtained for a self-mapping $T: X \to X$ satisfying the almost contractive condition,

$$d_{\theta_R}(Tx,Ty) \le \alpha d_{\theta_R}(x,y) + L\min\{d_{\theta_R}(y,Tx), d_{\theta_R}(x,Ty)\}, \qquad (2)$$

where $\alpha \in [0,1)$ and $L \ge 0$.

❖ **Discussion:**

- Special cases of proposed rational contraction



Fixed point result in θ_R- metric spaces

- Since Banach, Kannan, and Chatterjea contractions are special cases of the proposed almost contraction, the obtained fixed-point results extend the corresponding classical fixed-point theorems to θ_R- metric spaces.

- Special Cases $\longleftarrow$ θ_R-Metric Space

Choice of θ	Obtained Space
$\theta(a,b,c) = a+b+c$	Branciari Generalized Metric Space
$\theta(a,b,c) = s(a+b+c), s \ge 1$	Rectangular b-Metric Space
General θ	θ_R -Metric Space

❖ **Application:** Common fixed-point theorems for pairs of self-mappings are obtained as consequences of the main results.

CONCLUSION

The proposed θ_R-metric framework successfully accommodates fixed point theory beyond the scope of rectangular b-metric spaces. The presented results demonstrate the effectiveness of control-function-based metric structures in obtaining fixed point and common fixed-point conclusions under weaker assumptions. Consequently, the range of spaces in which fixed point techniques can be applied is significantly enlarged.

FUTURE WORK / REFERENCES

- Investigate fixed point results in θ_R-metric spaces under weaker assumptions on the control function θ, particularly by relaxing the compositional stability condition.
- Extend the present framework to other classes of contractions, such as Reich, Ćirić, Geraghty, and multivalued contractions in θ_R-metric spaces.

Key References: Branciari (2000); George et al. (2015).