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Propagation of KdV-DIA and mKdV-DIA Solitons in an Unmagnetized Dusty Plasma with Kaniadakis Distributed Electrons

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INTRODUCTION & AIM

The investigation of nonlinear electrostatic structures in dusty plasma has gained substantial interest due to their occurrence in a wide range of space and astrophysical environments. The presence of micron to submicron-sized charged dust particles along with electrons and ions introduces novel low-frequency wave modes, such as dust acoustic wave (DA) and dust ion-acoustic wave (DIA). The theoretical foundation for the existence of DIA was initially proposed by Shukla and Silin, and their existence, along with several key properties, was subsequently confirmed through experimental investigations by Barkan et al.

Over the past few decades, numerous generalized entropic frameworks such as Renyi entropy, Tsallis entropy, Havrda-Charvat entropy, etc., have been proposed to extend the conventional Boltzmann-Gibbs-Shannon (BGS) entropy. Another theoretical framework known as κ (kappa)-deformed statistics emerging from Kaniadakis entropy has been introduced. The κ-deformed formalism has attracted growing interest due to its successful application across various physical contexts, including the formation of quark-gluon plasma, the energy spectra of cosmic rays, etc.

Our primary objective is to investigate the propagation of small-amplitude dust ion-acoustic (DIA) solitary waves in an unmagnetized dusty plasma consisting of Kaniadakis distributed electrons, negative mobile dusts, and ions. For this, we first derive the KdV equation and then, for higher-order nonlinearity (where KdV equation fails), find the mKdV equation by using Reductive Perturbation Technique (RPT). We also examine the influence of various plasma parameter viz., the Kaniadakis deformation parameter (κ), the dust charge number (Z_d), and the initial streaming speed of ions (u_{i0}) and dusts (u_{d0}), on the evolution of DIA solitary waves.

BASIC GOVERNING EQUATIONS

In this plasma model, to study the propagation of DIA solitary waves in an unmagnetized dusty plasma, we consider the following normalized fluid continuity and momentum equations for ions and dusts, along with Kaniadakis distributed electrons and Poisson's equation, as given below: for dust

$$\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial u_d}{\partial t} + u_d \frac{\partial u_d}{\partial x} = Z_d \frac{\partial \phi}{\partial x} \qquad (2)$$

for positive ions

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i u_i)}{\partial x} = 0 \qquad (3)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{1}{Q}\frac{\partial \phi}{\partial x} \qquad (4)$$

Kaniadakis κ-deformed electrons

$$\kappa = \left(\sqrt{1+\kappa^2\phi^2} + \kappa\phi\right)^{\frac{1}{\kappa}} \qquad (5)$$

and the Poisson's equation

$$\frac{\partial^2 \phi}{\partial x^2} = Z_d n_d + n_e - n_i \qquad (6)$$

where u_j and n_j are velocities and densities of the ions, electrons and dust particles, respectively, here $j = d, e, i$. Moreover, Z_d is the dust grain and $Q = \frac{m_i}{m_d}$ is the ion-to-dust mass ratio.

METHOD (KDV AND MKDV EQUATIONS)

To examine the salient features of small amplitude DIA solitary waves, we derive the KdV and mKdV equations using **Reductive perturbation technique (RPT).** For KdV eq, we use stretched variables as

$$\xi = \varepsilon^{1/2}(x - Vt),\ \tau = \varepsilon^{3/2} Vt \qquad (7)$$

The flow variables are taken in terms of ε as follows:

$$\left.\begin{aligned} n_j &= n_{j0} + \varepsilon n_{j1} + \varepsilon^2 n_{j2} + \cdots \\ u_j &= u_{j0} + \varepsilon u_{j1} + \varepsilon^2 u_{j2} + \cdots \\ \phi &= \varepsilon\phi_1 + \varepsilon^2\phi_2 + \varepsilon^3\phi_3 + \cdots \end{aligned}\right\} \qquad (8)$$

Employing RPT with the use of (7) and (8) in the set of Eqs. (1)-(6) and using the BCs: $u_{d1} = 0, n_{d1} = 0, u_{i1} = 0, n_{i1} = 0$ and $\phi_1 = 0$ at $|\xi| \to \infty$, to the coefficient of ϵ equations, we get first order quantities $n_{d1}, u_{d1}, n_{i1}, u_{i1}$ and n_{e1}. Putting theses values in Poisson's ε-order equation, we obtain the Phase-velocity equation as

$$(1 - \sigma Z_d) - \frac{1}{Q(u_{i0} - V)^2} - \frac{\sigma Z_d^2}{(u_{d0} - V)^2} = 0 \qquad (9)$$

Again, equating the coefficient of ϵ^2 and solving the values of $\frac{\partial n_{d2}}{\partial \xi}$, $\frac{\partial n_{i2}}{\partial \xi}$ and $\frac{\partial n_{e2}}{\partial \xi}$, then substituting these into $\frac{\partial^2 \phi_1}{\partial \xi^2} = Z_d n_{d2} + n_{e2} - n_{i2}$, we get the **KdV equation** as

$$\frac{\partial \phi_1}{\partial \tau} + p\phi_1 \frac{\partial \phi_1}{\partial \xi} + q\frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \qquad (10)$$

where $p = x/y$, $q = -1/y$ with

$$x = 1 + \frac{3\sigma Z_d^3}{(1 - \sigma Z_d)(V - u_{d0})^4} - \frac{3}{Q^2(1 - \sigma Z_d)(V - u_{i0})^4}$$

$$y = -\frac{2V\sigma Z_d^2}{(1 - \sigma Z_d)(V - u_{d0})^3} - \frac{2V}{Q(1 - \sigma Z_d)(V - u_{i0})^3}$$

When $p > (<)\, 0$, there exists a compressive (rarefactive) KdV-DIA solitons. However, when $p = 0$, the KdV equation fails to describe DIA solitons. For higher order nonlinearity, we use new stretched variables

$$\xi = \varepsilon(x - Vt), \qquad \tau = \varepsilon^3 Vt$$

By applying same procedure as above (but up to cubic terms of ε), we get **mKdV equation** as

$$\frac{\partial \phi_1'}{\partial \tau} + p_1(\phi_1')^2 \frac{\partial \phi_1'}{\partial \xi} + q_1 \frac{\partial^3 \phi_1'}{\partial \xi^3} = 0 \qquad (11)$$

where $p_1 = x_1/y,\ q_1 = -1/y$ with

$$x_1 = \frac{1 - k^2}{2} - \frac{15\sigma Z_d^4}{2(1 - \sigma Z_d)(V - u_{d0})^6} - \frac{15}{2Q^3(1 - \sigma Z_d)(V - u_{i0})^6}$$

SOLUTION OF KDV AND MKDV EQS.

To solve KdV eq. (10), we use the transformation $\eta = \xi - C_1\tau$ and BCs: $\phi_1 = \frac{\partial \phi_1}{\partial \eta} = \frac{\partial^2 \phi_1}{\partial \eta} = 0$ at $|\eta| \to \infty$, we get

$$\phi_1 = \phi_0\, \mathrm{sech}^2\left(\frac{\eta}{\Delta}\right)$$

where $\phi_0 = \frac{3C_1}{p}$ and $\Delta = \sqrt{\frac{4q}{C_1}}$ are the amplitude and width of the solitary waves represented by KdV equation.

Similarly, applying same transformation and BCs in mKdV equation (11), we obtained the mKdV solution as

$$\phi_1' = \phi_0'\, \mathrm{sech}\left(\frac{\eta}{\Delta'}\right)$$

Amplitude and width of the solitary waves described by the mKdV equation are $\phi_0' = \sqrt{\frac{6C_1}{p_1}}$ and $\Delta' = \sqrt{\frac{q_1}{C_1}}$.

RESULTS AND DISCUSSION

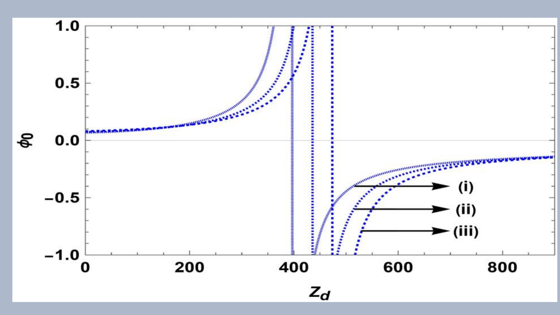

Fig. 1: Amplitude of KdV-DIA solitons versus Z_d for fixed $\sigma = 0.001, u_{d0} = 30$ and $u_{i0} = 5(i), 10(ii), 15(iii)$.

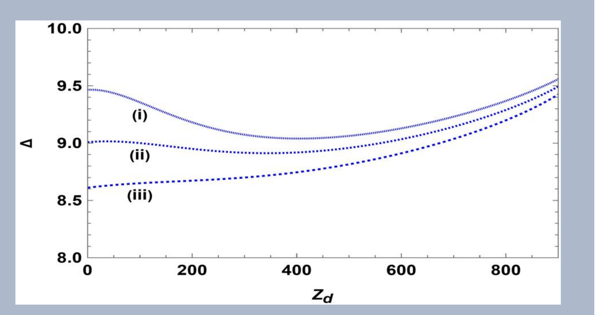

Fig. 2: Width of KdV-DIA solitons versus Z_d for fixed $\sigma = 0.001, u_{d0} = 30$ and $u_{i0} = 5(i), 10(ii), 15(iii)$.

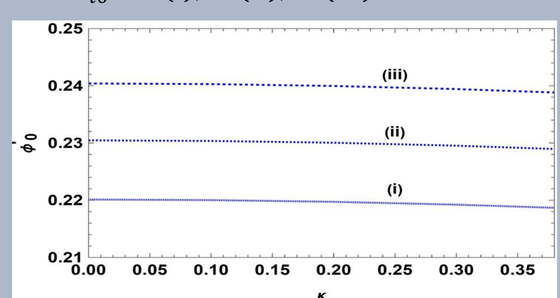

Fig. 3: Amplitude of mKdV solitons versus κ for fixed $u_{d0} = 0.1, Z_d = 100$ and $u_{i0} = 5(i), 10(ii), 15(iii)$.

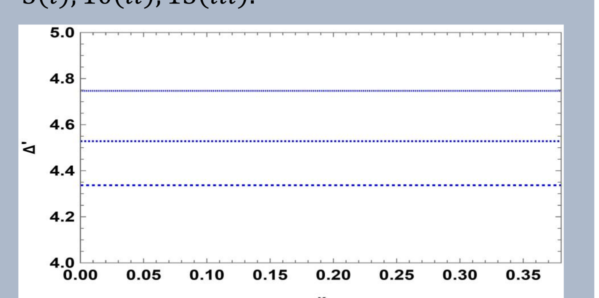

Fig. 4: Width of mKdV solitons versus κ for fixed $\sigma = 0.001, u_{d0} = 0.1, Z_d = 100$ and $u_{i0} = 5(i), 10(ii), 15(iii)$.

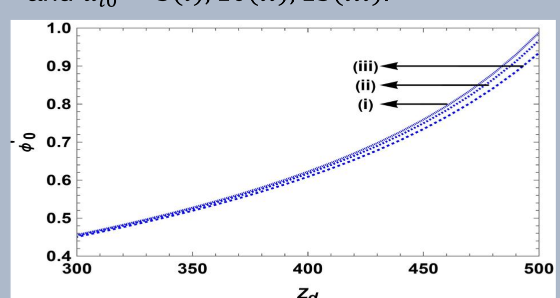

Fig. 5: Amplitude of mKdV solitons versus Z_d for fixed $u_{d0} = 40, u_{i0} = 5$, $\sigma = 0.001$ and $\kappa = 0.1(i), 0.2(ii), 0.3(iii)$.

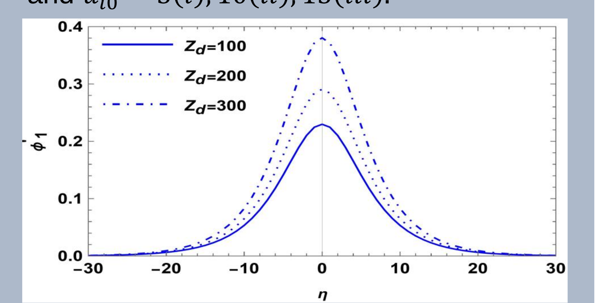

Fig. 6: mKdV solitons profile versus space η for fixed $\sigma = 0.001, u_{d0} = 30, u_{i0} = 5$, $\kappa = 0.34$,

For $u_{d0} = 30$, both compressive and rarefactive KdV-DIA solitons are observed for $\sigma = 0.001$ and $u_{i0} = 5(i), 10(ii)$, and $15(iii)$ (Fig. 1). In the compressive regime, soliton amplitude initially increases linearly with Z_d, followed by a sharp convex rise over a short range of Z_d for all values of u_{i0}.

Width of the KdV-DIA solitons increases with Z_d (Fig. 2). Lower values of ions streaming speed (u_{i0}) produce DIA solitary waves with wider width, whereas higher values of Z_d lead to narrower solitary structures.

The amplitude (Fig. 3) of the solitons is significantly affected by κ, but the soliton width (Fig. 4) remains practically unchanged, which is a salient feature of this investigation.

Figure 5 illustrate the variation of amplitude of mKdV-DIA solitons versus Z_d for different values κ and seen that the amplitude exhibits a concave upward trend as Z_d increases. As κ increases from 0.1 to 0.3, the amplitude of the solitons decreases.

The mKdV-DIA solitons shape is shown in Fig. 6 for varying Z_d, and observed that higher values of Z_d result in higher amplitude of mKdV-DIA solitons, which is also a key feature of this investigation.

CONCLUSION & FUTURE WORK

The Kaniadakis deformation parameter (k) does not influence on KdV solitons, but plays a significant role on mKdV solitons. It is also observed that the solitons width is not affected by the parameter κ. Higher values of Z_d produce higher amplitude of KdV and mKdV solitons. Our present work may provide valuable insights into the nonlinear behavior of space plasmas in Earth's magnetosphere and the pulsar magnetospheres of astrophysical environments.

The effect of relativistic and magnetic fields on the same plasma model keeps open scope for future study. In general, only the perturbative method is found to be successful in the case of multicomponent plasma, whereas the pseudopotential method cannot be applied in many complex situations. Therefore, for the present multicomponent plasmas, validity of the pseudopotential method will be abundant scope for future research.

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