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Finite-Time Blow-up Solution to a Stochastic Quasilinear ViscolasticWave Equation withNonlinear and Logarithmic Source

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INTRODUCTION & AIM

Stochastic partial differential equations in separable Hilbert spaces have been extensively studied in recent years, with results on existence, uniqueness, stability, blow-up, and other qualitative properties of solutions. In this work, we consider the stochastic viscoelastic wave equation with logarithmic nonlinearity:

$$\begin{cases} u_{tt} - \mu\Delta u - (\lambda+\mu)\nabla(divu) + \int_0^t h(t-s)\Delta u(s)\,ds \\ +|u_t|^{q-2}u_t = u|u|^{p-2}\ln|u|^k + \epsilon\sigma(x,t)W_t(x,t) & \text{in } \mathcal{D}\times]0,+\infty[, \\ u(x,t) = 0 & \text{on } \partial\mathcal{D}\times]0,+\infty[, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) & \text{in } \overline{\mathcal{D}}, \end{cases} \tag{1}$$

where:
$\mathcal{D}$ is a bounded domain in $\mathbb{R}^n$, $n \in \mathbb{N}^*$, with a smooth boundary $\partial\mathcal{D}$, μ, λ are the Lamé constants which satisfy $\mu > 0$, $\lambda+\mu \geq 0$, h is a positive function, the constant k is a small non-negative real number; and $L^2(\mathcal{D})$ is the set of square integrable function on $\mathcal{D}$ equipped
$W(x,t)$ is an infinite dimensional Wiener process, $\sigma(x,t)$ is $L^2(\mathcal{D})$ valued progressively measurable and ϵ is positive constant which measures the strength of noise. We rewrite (1) as an equivalent Itô's system

$$\begin{cases} du = vdt, \\ dv = \Big[\mu\Delta u + (\lambda+\mu)\nabla(divu) - \int_0^t h(t-s)\Delta u(s)\,ds \\ -|v|^{q-2}v + u|u|^{p-2}\ln|u|^k\Big]dt + \epsilon\sigma(x,t)dW_t(x,t) & \text{in } \mathcal{D}\times]0,+\infty[, \\ u(x,t) = 0 & \text{on } \partial\mathcal{D}\times]0,+\infty[, \\ u(x,0) = u_0(x), \quad v(x,0) = u_1(x) & \text{in } \overline{\mathcal{D}}, \end{cases} \tag{2}$$

which can be written as the integral equation

$$\begin{cases} u(t) = u_0 + \int_0^t v(s)ds, \\ v(t) = v(0) + \int_0^t \Big[\mu\Delta u + (\lambda+\mu)\nabla(divu) - \int_0^s h(s-r)\Delta u(r)\,dr \\ -|v|^{q-2}v + u|u|^{p-2}\ln|u|^k\Big]ds + \int_0^t \epsilon\sigma(x,s)dW_s(x,t) & \text{in } \mathcal{D}\times]0,+\infty[, \\ u(x,t) = 0 & \text{on } \partial\mathcal{D}\times]0,+\infty[, \\ u(x,0) = u_0(x), \quad v(x,0) = u_1(x) & \text{in } \overline{\mathcal{D}}. \end{cases} \tag{3}$$

Energy Functional

$$\begin{aligned} e(t) = {} & \frac{1}{2}\|u_t\|_2^2 + \frac{1}{2}\Big(\mu - \int_0^t h(s)ds\Big)\|\nabla u\|_2^2 + \frac{\lambda+\mu}{2}\|divu\|_2^2 \\ & + \frac{1}{2}(h\,o\,\nabla u)(t) + \frac{k}{p^2}\|u\|_p^p - \frac{1}{p}\int_D |u|^p \ln|u|^k dx \end{aligned} \tag{4}$$

$$(h\,o\,v)(t) = \int_0^t h(t-s)\|v(.,t) - v(.,s)\|^2 ds.$$

METHOD

(A1) Assume that $h : \mathbb{R}^+ \to \mathbb{R}^+$ is a C^1 nonincreasing function satisfying

$$h(0) > 0, \ \mu - \int_0^\infty h(s)ds = l > 0$$

and there exist tow nonnegative constants ς_1 and ς_2 such that

$$-\varsigma_1 h(t) \leq h'(t) \leq -\varsigma_2 h(t), \quad t \geq 0.$$

(A2)

$$\int_0^\infty h(s)ds < \mu\frac{(p-2)p}{(p-1)^2}.$$

(A3) $p > q \geq 2$ and

$$\begin{cases} 2 < p \leq \frac{2(n-1)}{n-2} & \text{if } n \geq 3 \\ 2 < p \leq +\infty & \text{if } n = 1, 2. \end{cases} \tag{5}$$

We purpose

$$G(t) = \frac{\epsilon^2}{2}\sum_{j=1}^{\infty}\mathbb{E}\int_0^t\int_D \lambda_j e_j^2(x)\sigma^2(x,s)dxds,$$

$$H(t) = G(t) - \mathbb{E}e(t),$$

$$L(t) := H^{1-\alpha}(t) + \delta\mathbb{E}(u,v),$$

where $0 < \alpha < min\{\frac{p-1}{2(p+1)}, \frac{p+1-q}{(p+1)q}\}$ and δ is a very small constant.

RESULTS & DISCUSSION

Theorem 2.1. *Assume* **(A1) - (A3)** *hold. Let (u,v) be a solution of system (2) with initial data $(u_0, v_0) \in H_0^1(\mathcal{D}) \times L^2(\mathcal{D})$ satisfying*

$$\mathbb{E}e(0) \leq -(1+\beta)E_1, \tag{8}$$

where β is nonnegative constant and E_1 is given (9). If $p > q$, then there exists a positive time $T_0 \in [0,T]$ such that

$$\lim_{t\to T_0^-} \mathbb{E}(e(t)) = +\infty,$$

where

$$T_0 = \frac{1-\alpha}{\alpha K L^{\frac{\alpha}{1-\alpha}}(0)},$$

$$L(0) = H^{1-\alpha}(0) + \delta\mathbb{E}(u_0, u_1) > 0,$$

and K is given later.

proof We purpose

$$G(\infty) \leq \frac{\epsilon^2}{2}Tr(Q)c_0^2\mathbb{E}\int_0^\infty\int_D \sigma^2(x,s)dxds := E_1 < \infty, \tag{9}$$

where

$$Tr(Q) = \sum_{j=1}^{\infty}\lambda_j < \infty \text{ and } c_0 = \sup_{j\geq 1}\|e_j\|_\infty < \infty,$$

where Q is the covariance operator of wiener process W. We showed

$$\begin{aligned} L'(t) \geq {} & \gamma(H(t) + \mathbb{E}\|v\|_2^2 + \mathbb{E}\|divu\|_2^2 + \mathbb{E}(h\,o\,\nabla u)(t) \\ & + \mathbb{E}\|\nabla u(t)\|_2^2 + \mathbb{E}\|u\|_p^p + \mathbb{E}\|u\|_{p+1}^{p+1}) \\ \geq {} & 0, \end{aligned} \tag{10}$$

$$\begin{aligned} (L(t))^{\frac{1}{1-\alpha}} \leq \tilde{C}\Big[& H(t) + \mathbb{E}\|v\|_2^2 + \mathbb{E}\|\nabla u\|_2^2 + \mathbb{E}\|divu\|_2^2 \\ & + \mathbb{E}\|u\|_p^p + \mathbb{E}(h\,o\,\nabla u)(t) + \mathbb{E}\|u\|_{p+1}^{p+1}\Big], \end{aligned} \tag{11}$$

According to (10) and (11), we have

$$(L(t))^{\frac{1}{1-\alpha}} \leq \tilde{K}L'(t). \tag{12}$$

In a direct integration of (12), we get

$$(L(t))^{\frac{\alpha}{1-\alpha}} \geq \frac{1}{(L(0))^{\frac{-\alpha}{1-\alpha}} - \frac{\tilde{K}\alpha t}{1-\alpha}}.$$

Therefore, $L(t)$ blows up in time $T \leq T_0 = \frac{1-\alpha}{\alpha K L^{\frac{\alpha}{1-\alpha}}(0)}$.

CONCLUSION

Under appropriate assumptions on the relaxation function, the damping exponent, and the noise intensity, we construct a suitable energy functional and derive refined energy estimates. By introducing an appropriate Lyapunov functional and exploiting the non-convex structure of the logarithmic potential, we establish sufficient conditions for finite-time blow-up with positive probability. In particular, we prove that if the initial energy is below a critical negative threshold depending explicitly on the noise intensity, then the corresponding solution cannot exist globally in time. Moreover, an explicit upper bound for the blow-up time is obtained in terms of the initial data and the system parameters.

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FUTURE WORK / REFERENCES

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