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# Algebraic structures of 2D and 3D fields of real vectors

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## INTRODUCTION

A geometric algebra (Clifford algebra) is an extension of elementary algebra to work with geometrical objects such as vectors. It is built out of two fundamental operations: addition and geometric product. The multiplication of vectors alone results in objects called multivectors, among which are bivectors, the name applied to the objects of the bivector field $R^2$, corresponding to the field of vectors $V_2$. The product of two complex numbers (two ordered pairs), in the form of the geometric product $wz = w^* \circ z + w^* \wedge z$, where $w^*$ is conjugate w, is the basis for modifying Grassmann's geometric product of two vectors, which is defined as the sum of the inner (scalar) and outer (bivector) products. By this modification, the geometric product of two vectors becomes commutative, similar to the product of two complex numbers themselves, which supports vector division.

## METHOD

An analogous analysis of the most important results of complex analysis and the results obtained in the vector field $V_2$, confirms that the algebraic structure of the vector field $V_2$ is well defined.

## CONCLUSION

All fundamental integral identities, from Cauchy's integral identity of complex analysis, through the Green's integral identity and the Stokes integral identity, as well as Gauss-Ostrogradsky's integral identity, all the way to the Newton-Leibniz formula, can be expressed by one vector integral identity.

## MAIN RESULTS

**Definition 1.** Let $\mathbf{r}^*1$ and $\mathbf{r}2$ be two vectors in $\mathbf{V}2$. Then, the commutative binary operation

$$\mathbf{r}^*_1\mathbf{r}_2 = (1/2)(\mathbf{r}^*_1\mathbf{r}_2 + \mathbf{r}_1\mathbf{r}^*_2) + (1/2)(\mathbf{r}^*_1\mathbf{r}_2 - \mathbf{r}_1\mathbf{r}^*_2)= \\ = (\mathbf{r}^*_1\cdot\mathbf{r}^*_2)\mathbf{e} - (\mathbf{r}^*_1\times\mathbf{r}^*_2) \times \mathbf{e} = (\mathbf{r}_1\cdot\mathbf{r}_2)e + (\mathbf{r}_1\times\mathbf{r}_2) \times \mathbf{e},$$

is the geometric product of $\mathbf{r}^*_1$ and $\mathbf{r}_2$.

The geometric product of vectors $\mathbf{r}^*_1 = a\mathbf{e} - b\mathbf{e}^\wedge$ and $\mathbf{r}_2 = c\mathbf{e} + d\mathbf{e}^\wedge$ corresponds to the product (a,-b)(c,d) = (ac + bd,ad -bc) of two bivectors (a,-b) and (c,d),

The concepts of geometric product and bivector, introduced above, are closely related to the same concepts in Clifford algebra. However, there is evidently a crucial difference between Clifford algebra and the algebra of the field $V_2$, which is reflected in the fact that the geometric product of the elements of the field $V_2$, on the one hand, is the element of the field $V_2$, corresponding to the bivector, the element of the bivector field $R^2$, and on the other hand, it is also commutative, which means that the field $V_2$, in addition to being an additive Abelian group, is also a multiplicative Abelian group.

The Euclidean space $R^3$ consists of three Euclidean spaces $R^2$, which means that the field of vectors $V_3$, isomorphic to it, consists of three fields of vectors $V_2$.

The commutative geometric product of two vectors in $V_3$ is defined as the sum of the geometric products of the component vectors in the component fields of vectors, of which the 3D field of real vectors consists.

## FUTURE WORK

These results can be applied to the field of complex vectors $V_C$, which corresponds to the *i*vector field C (field of complex numbers C),