IOCMA 2026 Conference

The 2nd International Online Conference on Mathematics and Applications

10–12 June 2026 | Online

MDPI

Bifurcation, Spectral Entropy, and Chaos Control in a Cournot Triopoly Game with Relative Profit Maximization

Dr. Mourad Azioune 1, Prof. Mohammed-Salah Abdelouahab 1, Prof. René Lozi 2

Abdelhafid Boussouf University, Mila, Algeria 1. Côte d'Azur University, France 2

INTRODUCTION & AIM

This study investigates the complex dynamical behavior of a discrete-time Cournot triopoly game by shifting from a standard absolute-profit framework to relative profit objectives under bounded rationality. This shift introduces strategic aggressiveness and stronger competitive interactions among firms, raising the core problem of how these behaviors introduce highly complex and destabilizing market dynamics. To address this, the aim of the research is to analytically derive local stability thresholds for flip and Neimark–Sacker bifurcations, numerically map global chaotic transitions using spectral entropy as a robust complexity measure, and implement an effective parameter feedback control strategy to stabilize unstable dynamics and restore market equilibrium.

TRIOPOLY MODEL FORMULATION

Model Formulation

Let q_i(t) be the output of firm i ∈ {1,2,3}. The total output is:

Q(t) = q1(t) + q2(t) + q3(t)

Inverse demand function:

P(Q) = a – bQ. Profit of firm i: Π_i = [a - b(q1 + q2 + q3) - c_i] q_i

Relative Profit Maximization

Π_i^R = Π_i - (1/2) Σ_{j≠i} Π_j

Discrete-time bounded rationality adjustment model:

q1(t+1) = q1(t) + α1 q1(t)[a - c1 - 2bq1(t) - (b/2)(q2(t)+q3(t))]

q2(t+1) = q2(t) + α2 q2(t)[a - c2 - 2bq2(t) - (b/2)(q1(t)+q3(t))]

q3(t+1) = q3(t) + α3 q3(t)[a - c3 - 2bq3(t) - (b/2)(q1(t)+q2(t))]

NASH EQUILIBRIUM & STABILITY

Symmetric Nash Equilibrium (E*)

For identical marginal costs (ci = c), the unique steady-state output for all three firms is: q* = (a - c) / (3b)

Characteristic Equation: The linearized 3D system stability is governed by:

P(λ) = λ^3 + A1 λ^2 + A2 λ + A3 = 0

Where the coefficients are:

A1 = -3 + 2α1(a - c)

A2 = 3 - 3α1(a - c) + (1/2)α1^2(a - c)^2

A3 = -1 + 2α1(a - c) - (5/4)α1^2(a - c)^2 + (1/4)α1^3(a - c)^3

Jury Stability Conditions

Cond1: 1 + A1 + A2 + A3 > 0

Cond 2: -1 + A1 - A2 + A3 < 0

Cond 3: A3^2 < 1

Cond 4: |1 - A3^2| > |A2 - A1 A3|

Stability Boundary

The system is stable if: α1 < 2 / (a - c)

The numerical simulations shown in Figure 1 illustrate two different routes to chaos following the loss of stability of the Nash equilibrium. Figure 1(a) exhibits a Flip bifurcation, where successive period-doubling bifurcations lead to chaotic dynamics.

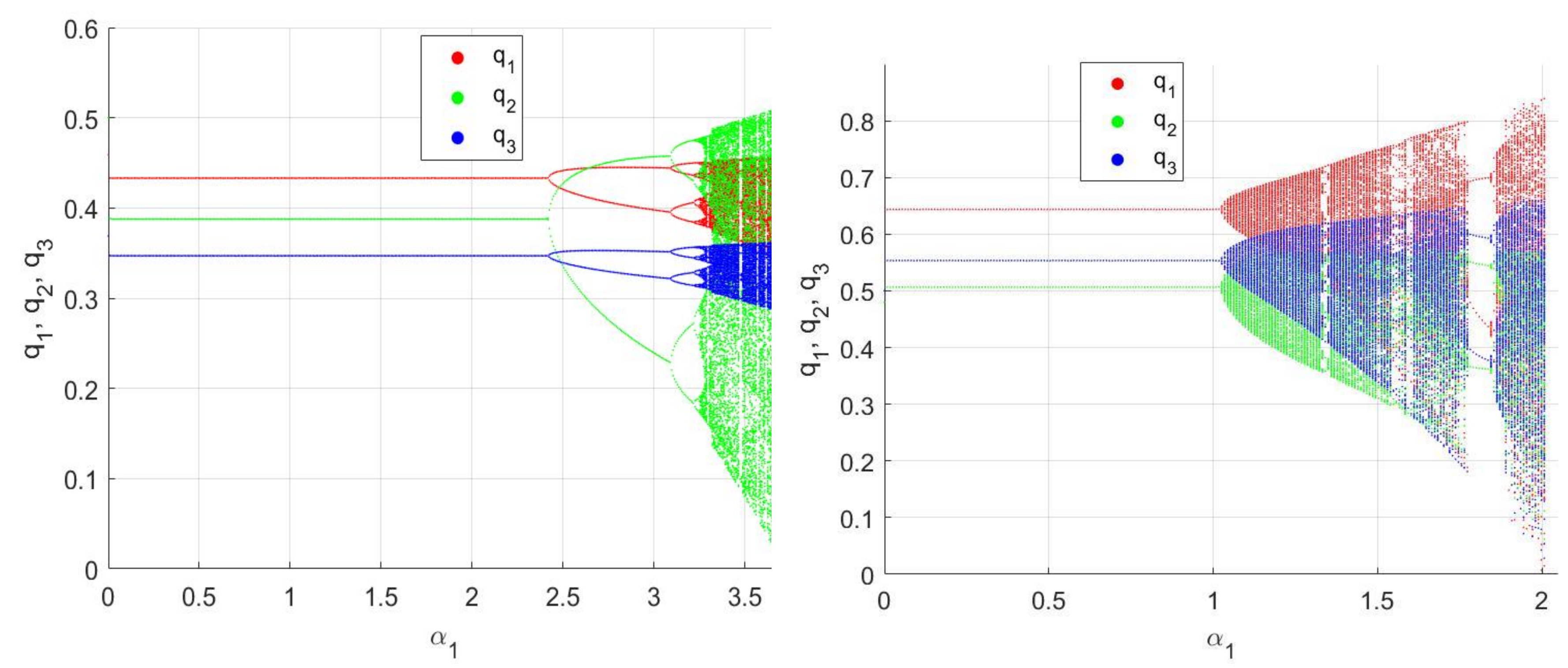

Figure 1. Bifurcation diagrams of the Cournot triopoly game showing the evolution of the outputs of Firms 1, 2, and 3 (q1, q2, and q3) as the adjustment speed parameter α1 varies. (a) The system loses stability through a Flip bifurcation, leading to a sequence of period-doubling bifurcations and eventually chaotic behavior. (b) The system undergoes a Neimark-Sacker bifurcation, generating quasi-periodic oscillations on closed invariant curves before the onset of chaos.

Figure 1(b) shows a Neimark-Sacker bifurcation, characterized by the emergence of quasi-periodic oscillations on a closed invariant curve before the onset of chaos. These results confirm the Jury stability analysis and demonstrate that relative profit maximization can generate complex and unpredictable market dynamics.

SPECTRAL ENTROPY

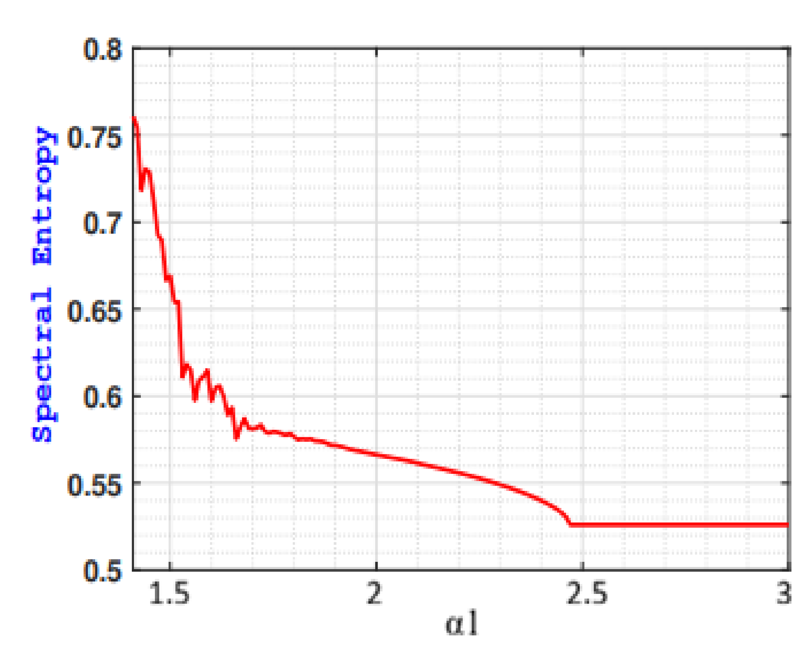

Figure 2 illustrates the evolution of Spectral Entropy (SE) with respect to the adjustment parameter α1 . The SE increases as α1 varies, indicating a growth in the complexity of the system dynamics. Higher SE values correspond to more irregular and chaotic behavior, revealing parameter regions where the system transitions from regular to chaotic regimes.

FROM CHAOS TO STABILITY

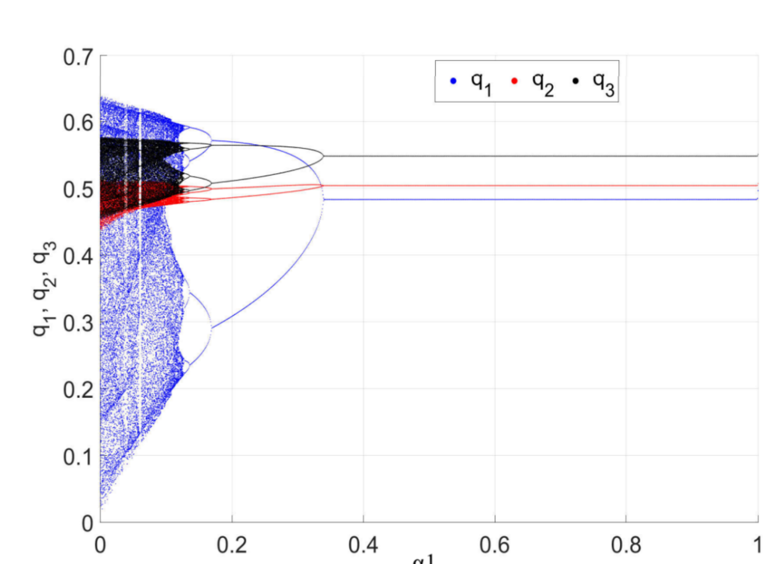

Figure 3 illustrates the control of chaos via feedback. The controlled system is governed by the state equation:

$q(t+1) = (1 - \alpha_1) f(q(t)) + \alpha_1 q(t)$.

As the control parameter α_1 increases, chaotic oscillations are progressively suppressed, and the system converges to a stable equilibrium for $\alpha_1 \gtrsim 0.34$, demonstrating the effectiveness of the feedback control strategy

CONCLUSION

The results demonstrate that the introduction of relative profit maximization significantly enriches the dynamics of the Cournot triopoly game, leading to the emergence of flip and Neimark-Sacker bifurcations, quasi-periodic oscillations, and chaotic behavior. Spectral entropy analysis successfully captures the transitions between regular and complex dynamics, while the proposed feedback control strategy effectively suppresses chaos and restores stability. These findings provide valuable insights into the impact of competitive behavior on market stability and highlight the usefulness of control techniques in managing complex economic systems.

REFERENCES

[1] Azioune, M., & Abdelouahab, M. S. (2025). Controlling chaos and mixed mode oscillations in a Bertrand duopoly game with homogeneous expectations and quadratic cost functions. Mathematics and Computers in Simulation, 233, 553–566.

[2] Azioune, M., Abdelouahab, M. S., & Lozi, R. (2025). Bifurcation analysis of a Cournot triopoly game with bounded rationality and chaos control via the OGY method. International Journal of Bifurcation and Chaos, 35(08),