IOCMA 2026 Conference

The 2nd International Online Conference on Mathematics and Applications

10–12 June 2026 | Online

MDPI

Existence of global solutions of a nonlinear reaction-fractional diffusion system with anomalous diffusion

Authors: Nabila Barrouk[1], Manel Medkour[2] and Abdelatif Toualbia [3]

Address: [1,2] Faculty of Science and Technology, Department of Mathematics, Mohamed Cherif Messaadia University, B.P. 1553 Souk Ahras 41000, Algeria

[3] Faculty of Exact Sciences And Natural and Life Sciences, Department of Mathematics and Informatics, LAMIS Laboratory, University of Tebessa, Tebessa 12000, Algeria

Email: n.barrouk@univ-soukahras.dz \ m.medkour@univ-soukahras.dz \ abdelatif@univ-tebessa.dz

INTRODUCTION & AIM

In the present work, we study the folowing mathematical model of fractional reaction diffusion systems

$$\begin{cases} \frac{\partial \vartheta_i}{\partial t} - d_i(-\Delta)^{\alpha_i}\vartheta_i = f_1(\vartheta), & \text{in }]0,t^*[\times\Omega, \\ \frac{\partial \vartheta_i}{\partial \eta} = 0 \quad \text{or} \quad \vartheta_i = 0, \quad \text{for all } 1 \le i \le m & \text{on }]0,t^*[\times\partial\Omega, \\ \vartheta_i(0,x) = \vartheta_{i_0}(x), & \text{in } \Omega, \end{cases} \quad (1)$$

where $\vartheta = (\vartheta_1, \ldots, \vartheta_m)$, $m \ge 2$, Ω is a bounded and regular domain of $\mathbb{R}^N$ with boundary $\partial\Omega$, $N \ge 2$, $\vartheta_i = \vartheta_i(t,x)$, $1 \le i \le m$ for $(t,x) \in Q_T = (0,T) \times \Omega$ and f_i are real functions, the presence of the non local operator $(-\Delta)^{\alpha_i}$, $0 < \alpha_i < 1$ for all $1 \le i \le m$, which accounts for the anomalous diffusion, means that the sub-populations face some obstacles that slow their movement, and the constants of diffusion d_i are assumed to be nonnegative, $f_i : \mathbb{R}^m \to \mathbb{R}^m$ are enough regular, ϑ_{i_0} are nonnegative functions in $L^1(\Omega)$, for all $1 \le i \le m$. The local existence in time of the solution ϑ_i is classical. The positivity of the solution stems from the positivity of ϑ_{i_0}, which are assumed to be continuous, for all $1 \le i \le m$.

METHOD

Let A be a m-dissipative operator in the Banach space X its infinitesimal generator of a semigroup $S(t)$, F a function locally Lipschitz, then for all $u_0 \in X$ there exists $T_{\max} = T(u_0)$ such that the system

$$\begin{cases} u \in C([0,T], D(A)) \cap C^1([0,T], X) \\ \frac{du}{dt} - Au = F(s,.,u(s)) \\ u(0) = u_0 \end{cases}$$

admits a unique local solution u verifying

$$u(t) = S(t)u_0 + \int_0^t S(t-s)F(s,.,u(s))\,ds, \ \forall t \in [0, T_{\max}]$$

Theorem 1.

The solution of problem (1) given by

$$\begin{cases} \vartheta_i \in C([0,+\infty[, L^1(\Omega)) \\ f_i(t,x,u,v) \in L^1(]0,t^*[\times\Omega) \\ \vartheta_i(t) = S_i(t)u_0 + \int_0^t S_i(t-s) f_i(\vartheta)\,ds, \ \forall t \in [0,T[\end{cases}$$

where $S_i(t)$ are contraction semigroups in $L^1(\Omega)$ generated, respectively, by $-d_i(-\Delta)^{\alpha_i}$.

RESULTS & DISCUSSION

Proposition 1.

Let $\vartheta_i \in L^1(\Omega)$, then there exists a maximal time $T_{\max} > 0$ and a unique solution $\vartheta_i \in C\left([0, T_{\max}), \left(L^1(\Omega)\right)^n\right)$ of the system (1), with the alternative :

- either $T_{\max} = +\infty$,
- or $T_{\max} < +\infty$ and $\lim\limits_{t \to T_{\max}} \left(\sum_{i=1}^{i=m} \|\vartheta_i(t)\|_\infty \right) = +\infty$.

Lemma 1.

Let u_i be a solution of the system (1) such that

$$\vartheta_{i_0}(x) \ge 0, \qquad \text{in } \Omega$$

then

$$\vartheta_i(t,x) \ge 0, \quad \forall (t,x) \in]0,t^*[\times\Omega.$$

Lemma 2.

Let ϑ_i the solution of the system (1), then there exists $M(t)$ which only depends of t, such that for all $0 \le t \le T_M$, we have

$$\left\| \sum_{i=1}^{m} \vartheta_i \right\|_{L^1(\Omega)} \le M(t)$$

We can conclude from this estimate that the solution ϑ_i given by the Theorem 1 is a global solution.

CONCLUSION

Fractional differential systems have attracted the attention of many researchers due to their enormous use in all scientific fields, especially in biomathematics. In recent years, several theoretical and experimental studies have been devoted to this question. They found that certain thermal, electrochemical, biological systems are governed by fractional differential equations. In the mentioned references we can find other important applications in various scientific fields. The novelty is that the introduction of fractional derivation reduces the number of model parameters.

FUTURE WORK / REFERENCES

N. Barrouk and S. Mesbahi, Existence of global solutions of a reaction-diffusion system with a cross-diffusion matrix and fractional derivatives, Palest. J. Math. 13 (3) (2024) 340-353.

A. Toualbia , and N. Barrouk. Global solutions of ANOMALOUS GLOBAL SOLUTIONS OF DIFFUSION SYSTEMS 3×3. Fractional Differential Calculus 15.1 (2025).