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Spatiotemporal Dynamics in a Fear–Driven Prey–Predator Model with Refuge, Cooperation, and Harvesting

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INTRODUCTION & AIM

- Prey-predator models reveal the factors that determine the existence and persistence of species.
- Traditional prey-predator models focus on temporal changes in populations.
- Diffusive prey-predator models incorporate spatial diffusion to capture the spread of populations in a domain [1,2].

The main objective of this research is to formulate and analyse a diffusive prey-predator model incorporating the following biological/ecological phenomena; to study how spatial diffusion and ecological parameters drive pattern formation and population stability via Turing instability:

1) **Fear Effect (K):** The psychological and behavioral impact of predators on prey (anti-predation adaptations leading to reduced birth rate).
2) **Hunting Cooperation (α):** Social interaction among predators to capture prey more efficiently.
3) **Prey Refuges (m):** Physical or environmental "safe zones" where a portion of the prey population is protected from predation.
4) **Harvesting (h):** The removal of individuals from the system by humans.

METHOD

The following diffusive prey-predator model motivated by [3] is analyzed:

$$\frac{dx}{dt} = d_1\nabla^2 x + \frac{rx}{(1+Ky)} - \mu x - \delta x^{\beta+1} - \frac{(1+\alpha y)(1-m)xy}{1+H(1+\alpha y)x} = f(x,y) + d_1\nabla^2 x$$

$$\frac{dy}{dt} = d_2\nabla^2 y + \frac{(1+\alpha y)(1-m)xy}{1+H(1+\alpha y)x} - y - hy = g(x,y) + d_2\nabla^2 y$$

where x and y are prey and predator populations, $\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}$ is the Laplacian operator, d_1 and d_2 are diffusion coefficients.

- The proposed model has 3 types of equilibrium points: Trivial equilibrium $E_0(0,0)$, predator-free equilibrium $\hat{E}\left(\left(\frac{r-\mu}{\delta}\right)^{1/\beta}, 0\right)$ and coexistence equilibrium $E^*(x^*, y^*)$.
- Local stability of coexistence equilibrium $E^*(x^*, y^*)$ was analysed using the Jacobian matrix given by $J(x^*, y^*) = \begin{bmatrix} f_x & f_y \\ g_x & g_y \end{bmatrix}_{|(x^*,y^*)}$
- **Local stability criterion:**
  $$f_x + g_y < 0, f_x g_y - f_y g_x > 0$$
- Numerical bifurcation analysis was performed in MATLAB.
- **Dispersion equation:**
  $$\lambda^2 + B_1(k^2)\lambda + B_2(k^2) = 0$$
  where, k is the wave number, $B_1(k^2) = f_x + g_y - k^2(d_1 + d_2)$, $B_2(k^2) = d_1 d_2 k^4 - (d_1 g_y + d_2 f_x)^2 k^2 + f_x g_y - f_y g_x$
- **Turing instability criterion:**
  $$d_1 g_y + d_2 f_x > 0, (d_1 g_y + d_2 f_x)^2 - 4 d_1 d_2 (f_x g_y - f_y g_x) > 0$$
- The diffusive system was solved numerically in MATLAB using the 5-point discretized Laplacian operator using the central difference rule on a two-dimensional space under zero-flux boundary condition.

Fixed parameter values

r	K	μ	δ	β	m	H	h	d_2
1	0.0769	0.3	0.1	1.5	0.2	0.01	0.2	0.002

RESULTS & DISCUSSION

Numerical simulations indicate that spatial diffusion destabilizes the homogeneous coexistence equilibrium and generates rich variety of stationary Turing patterns.

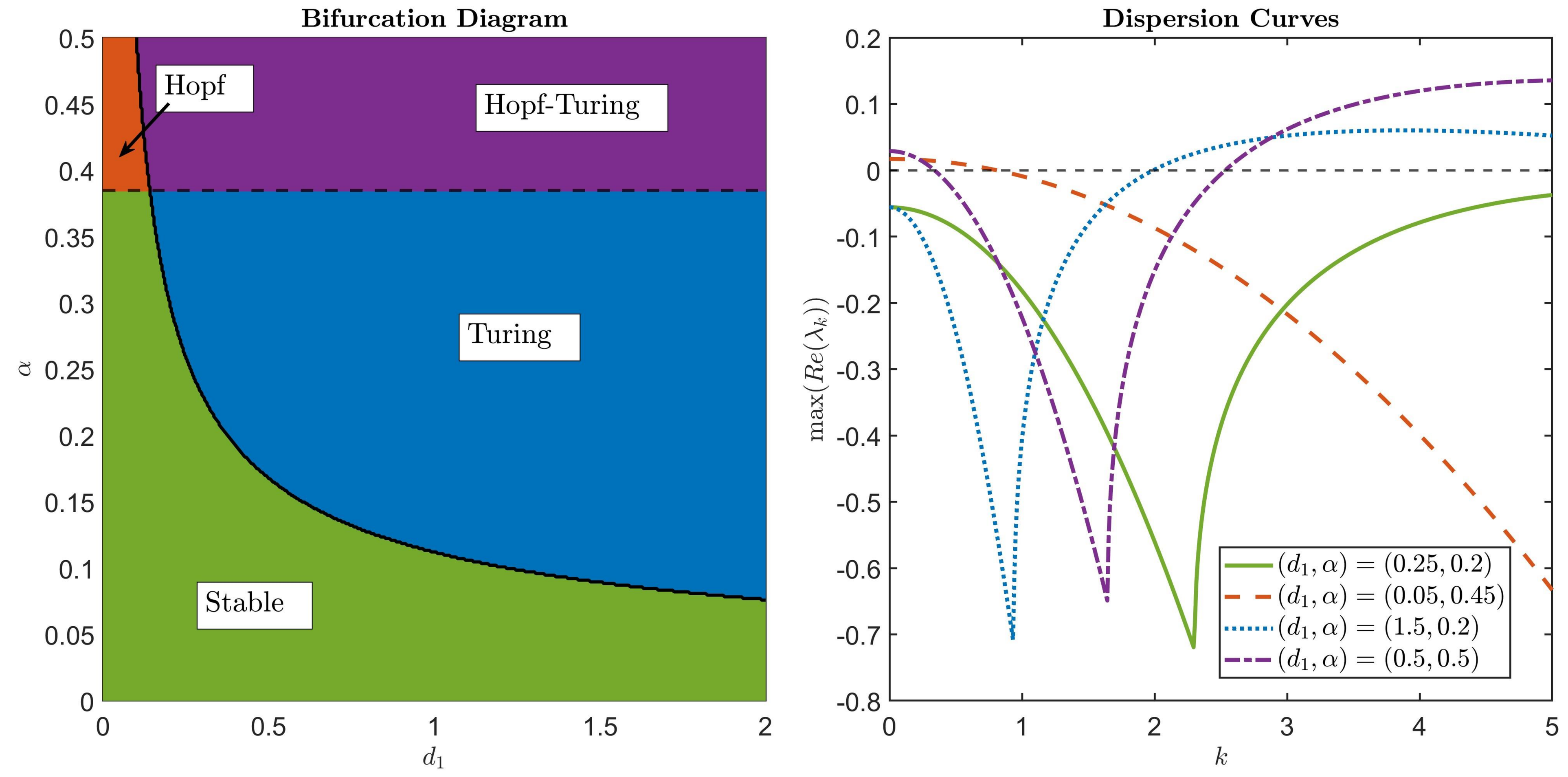

Fig. 1: Bifurcation diagram and dispersion curves for prey diffusion coefficient (d_1) and hunting cooperation (α)

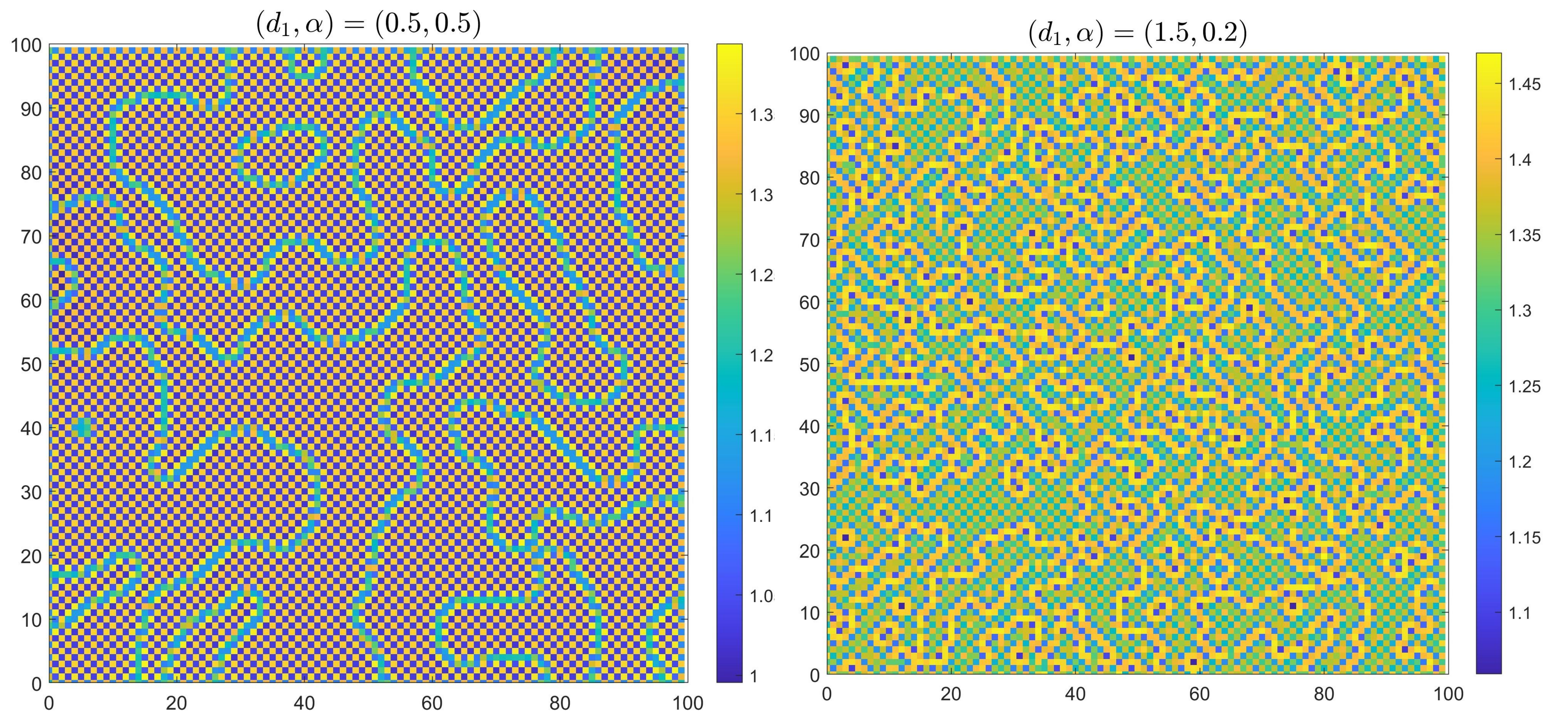

Fig. 2: Labyrinthine patterns for prey population in Hopf-Turing ($d_1 = 0.5, \alpha = 0.5$) and Turing ($d_1 = 1.5, \alpha = 0.2$) regions.

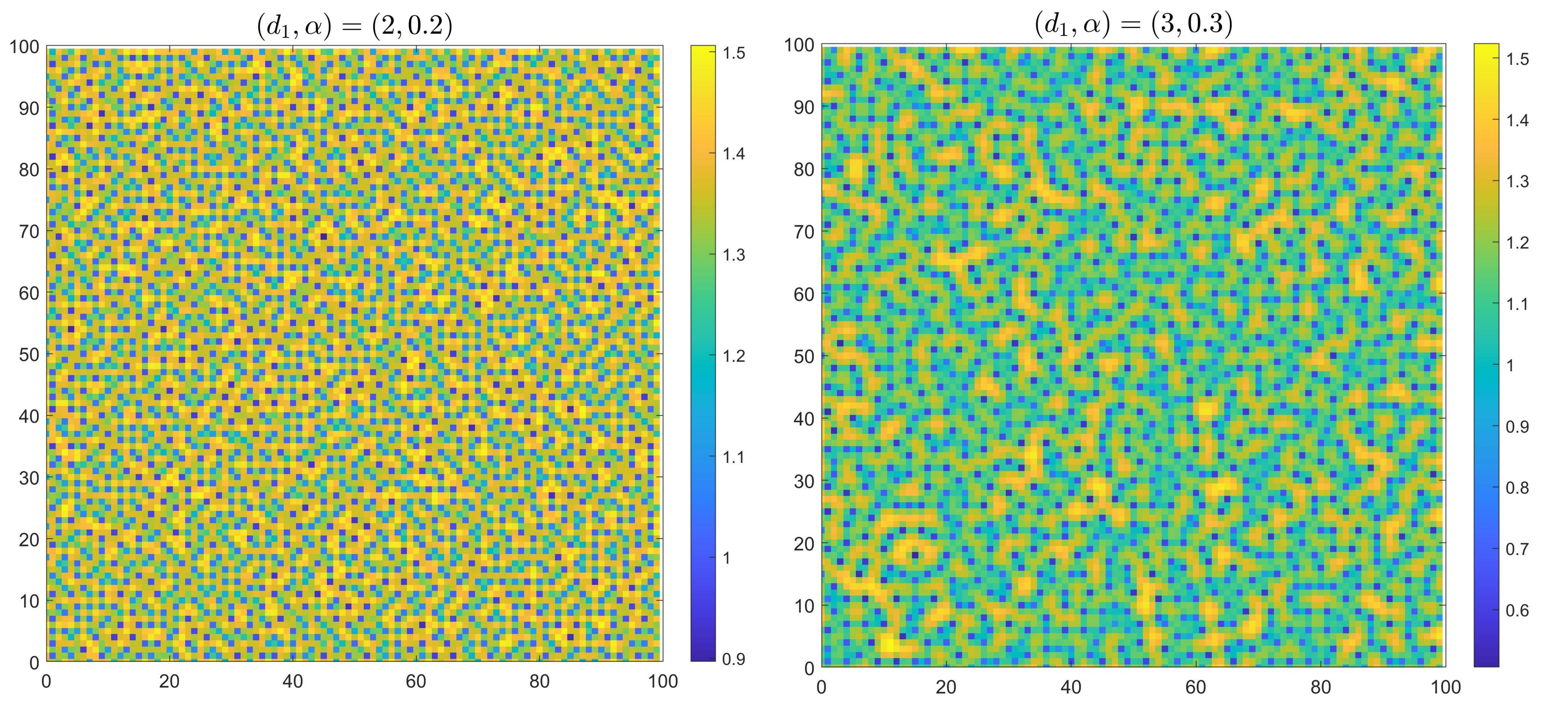

Fig. 3: Spotted patterns for prey population in Turing region (i) ($d_1 = 2, \alpha = 0.2$) (ii) ($d_1 = 3, \alpha = 0.3$)

CONCLUSION

- Hopf bifurcations are noted for fear, refuge, cooperation, and harvesting; leading to oscillatory dynamics and species extinction.
- Turing instability produces labyrinthine and spotted spatial patterns under high prey diffusion.
- These findings highlight that spatial dispersal acts as a critical mechanism for species persistence

FUTURE WORK / REFERENCES

FUTURE WORK: Investigate diffusion-driven spatial patterns in an extension of the proposed model with non-local fear/ disease in prey.

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3) Enatsu, Y., Roy, J. and Banerjee, M., 2024. Hunting cooperation in a prey–predator model with maturation delay. J. Biol. Dyn., 18(1), p.2332279.