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Exact Traveling Wave Solutions and 3D Visualization of the (2+1)-Dimensional Date–Jimbo–Kashiwara–Miwa Equation

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INTRODUCTION & AIM

The Date–Jimbo–Kashiwara–Miwa (DJKM) equation is an integrable nonlinear evolution equation in (2+1)-dimensions that arises in the study of multidimensional wave propagation. It serves as an important model for understanding nonlinear phenomena in physical contexts such as plasma physics, fluid dynamics, and other wave-bearing media. Due to its rich mathematical structure, the DJKM equation admits a variety of exact solutions that describe localized and propagating wave patterns.

AIM: The present work aims to construct exact traveling wave solutions of the (2+1)-dimensional DJKM equation using the extended modified auxiliary equation mapping (EMAEM) method and to illustrate the main qualitative features of the solutions through selected 3D visualizations.

METHOD

- The governing nonlinear evolution equation considered in this study is given by,

$$\phi_{xxxxy} + 4\phi_{xxy}\phi_x + 2\phi_{xxx}\phi_y + 6\phi_{xy}\phi_{xx} + \phi_{yyy} - 2\phi_{xxt} = 0. \quad (1)$$

- By applying the traveling-wave transformation

$$\phi(x, y, t) = U(\xi), \xi = kx + ly - mt, \quad (2)$$

the original partial differential equation is reduced to an ordinary differential equation written as,

$$k^4 l\, U''' + 3k^3 l (U')^2 + (2k^2 m + l^3) U' = 0. \quad (3)$$

- According to EMAEM method with, the solution is assumed as:

$$U(\xi) = a_0 + a_1\psi(\xi) + b_1\psi^{-1}(\xi) + d_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^1, \quad (4)$$

where a_0, a_1, b_1, d_1 are real constants to be determined.

- The functions $\psi(\xi)$ and $\psi'(\xi)$ satisfy the auxiliary equation system:

$$\begin{cases} \psi'(\xi) = \alpha_1(\psi(\xi))^2 + \alpha_2(\psi(\xi))^3 + \alpha_3(\psi(\xi))^4 \\ \psi''(\xi) = \alpha_1\psi(\xi) + \frac{3}{2}\alpha_2(\psi(\xi))^2 + 2\alpha_3(\psi(\xi))^3 \\ \psi'''(\xi) = \alpha_1\psi'(\xi) + 3\alpha_2\psi(\xi)\psi'(\xi) + 6\alpha_3(\psi(\xi))^2\psi'(\xi) \\ \psi''''(\xi) = \begin{cases} \alpha_1{}^2\psi(\xi) + \frac{15}{2}\alpha_1\alpha_2(\psi(\xi))^2 + \frac{15}{2}(\alpha_2)^2(\psi(\xi))^3 \\ +20\alpha_1\alpha_3(\psi(\xi))^3 + 30\alpha_2\alpha_3(\psi(\xi))^4 + 24(\alpha_3)^2(\psi(\xi))^5 \end{cases} \end{cases}$$

where $\alpha_1, \alpha_2, \alpha_3$ are the three key parameters of the method.

- Substituting the solution assumption and auxiliary equations into the ODE (3), collecting terms with same powers of $\psi'^{j}(\xi)(\psi(\xi))^{i}$ and setting each coefficient to zero generates an algebraic system, which is then solved to obtain the solution families.

RESULTS & DISCUSSION

Solving the algebraic system yields the following solution family:

$$a_0 = 0, \alpha_1 = \frac{-2k^3m - l^3}{k^4 l}, \alpha_3 = -\frac{k^4 l \alpha_2{}^2}{8k^2m + 4l^3}, b_1 = 0, d_1 = -2k.$$

Case 1.1: $U_{(1,1)}(\xi) = -2k\left\{\frac{-2k^3m-l^3}{k^4 l} + \alpha_2\left[\frac{1+\epsilon\, tanh\left(\frac{\sqrt{\alpha_1}}{2}(\xi+\xi_0)\right)}{\alpha_2}\right] + \alpha_3\left[\frac{1+\epsilon\, tanh\left(\frac{\sqrt{\alpha_1}}{2}(\xi+\xi_0)\right)}{\alpha_2}\right]^2\right\}^{\frac{1}{2}}$,

$\alpha_1 > 0, \epsilon = \pm 1, \alpha_2{}^2 - 4\alpha_1\alpha_3 = 0.$

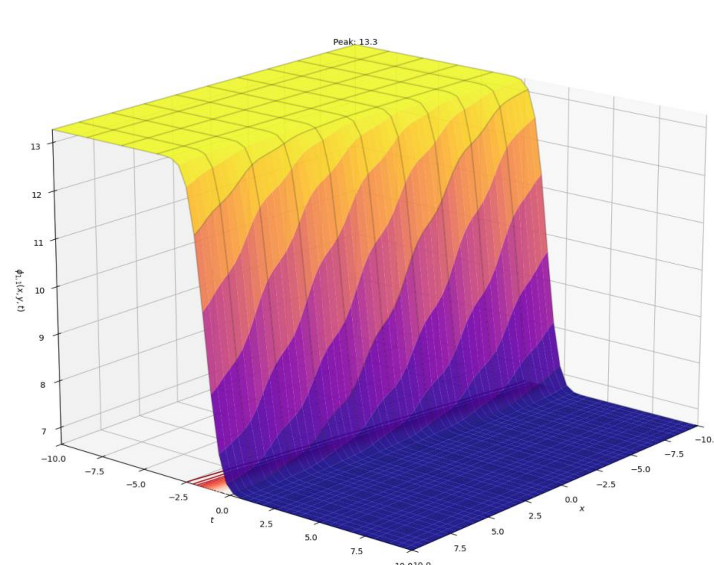
Case 1.2: $U_{(1,2)}(\xi) = -2k\left\{\frac{-2k^3m-l^3}{k^4 l} + \alpha_2\left[\frac{\sqrt{\frac{\alpha_1}{\alpha_3}}\left(1+\frac{\epsilon\, sinh\left(\sqrt{\alpha_1}(\xi+\xi_0)\right)}{\delta+cosh\left(\sqrt{\alpha_1}(\xi+\xi_0)\right)}\right)}{2}\right] + \alpha_3\left[\frac{\sqrt{\frac{\alpha_1}{\alpha_3}}\left(1+\frac{\epsilon\, sinh\left(\sqrt{\alpha_1}(\xi+\xi_0)\right)}{\delta+cosh\left(\sqrt{\alpha_1}(\xi+\xi_0)\right)}\right)}{2}\right]^2\right\}^{\frac{1}{2}}$, $\alpha_1 > 0, \alpha_3 > 0, \epsilon = \pm 1, \delta = \pm 1, \alpha_2 = -\sqrt{4\alpha_1\alpha_3}$.

Case 1.3:

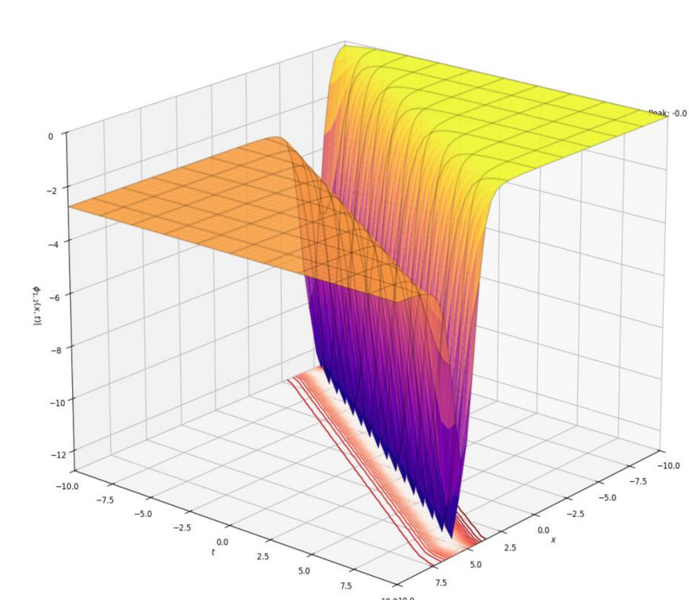
$$U_{(1,3)}(\xi) = -2k\left\{\frac{-2k^3m-l^3}{k^4 l} + \alpha_2\left[\frac{\alpha_1\left(1+\frac{\epsilon\sqrt{1+\mu_0{}^2}\delta+cosh(\sqrt{\alpha_1}(\xi+\xi_0))}{\mu_0+sinh(\sqrt{\alpha_1}(\xi+\xi_0))}\right)}{\alpha_2}\right] + \alpha_3\left[\frac{\alpha_1\left(1+\frac{\epsilon\sqrt{1+\mu_0{}^2}\delta+cosh(\sqrt{\alpha_1}(\xi+\xi_0))}{\mu_0+sinh(\sqrt{\alpha_1}(\xi+\xi_0))}\right)}{\alpha_2}\right]^2\right\}^{\frac{1}{2}},$$

$\alpha_1 > 0, (\epsilon, \delta) = (1,1), (-1,1), (1,-1), (-1,-1).$

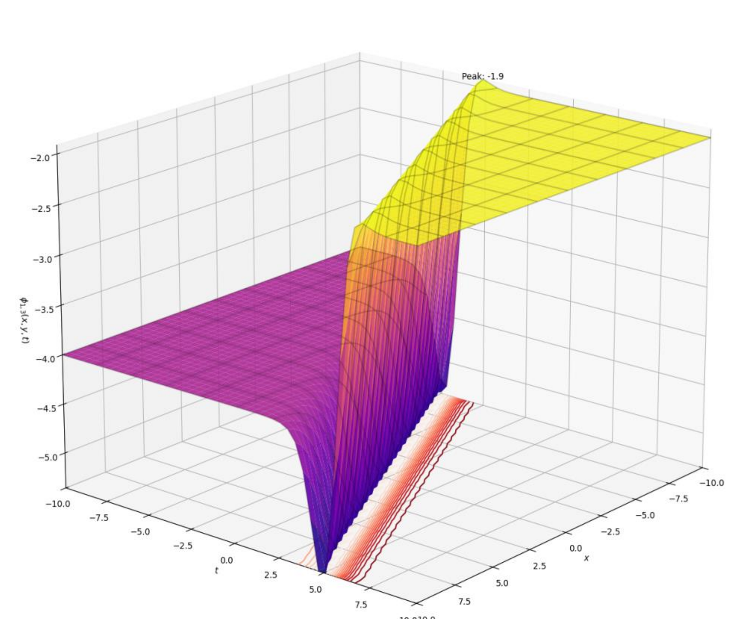
In all above cases $\xi = kx+ly-mt$, μ_0 and ξ_0 are arbitrary constants.



Kink wave: $\phi_{1,1}(x, y, t)$ with parameters k = -1, l = -1, m = 6, $\epsilon = 1, \xi_0 = 0, \alpha_2 = 2$



Dark soliton on a kink background: $\phi_{1,2}(x, y, t)$ with parameters k=1, l=-1, m=1, δ=-1, ϵ=-1, $\xi_0 = 3$.



Kink wave with bright-dark transition: $\phi_{1,3}(x, y, t)$ with parameters k=1, l=-1, m=1, δ=1, ϵ=1, $\xi_0 = 0$

CONCLUSION

Exact traveling wave solutions of the (2+1)-dimensional DJKM equation were successfully derived using the extended modified auxiliary equation mapping technique. The corresponding 3D plots highlight the essential wave characteristics and confirm the effectiveness of the method for studying higher-dimensional nonlinear wave equations.

FUTURE WORK / REFERENCES

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