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Exact Lump and Breather Dynamics of a (1+1)-Dimensional Gilson–Pickering Equation

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INTRODUCTION & AIM

Introduction:

The Gilson–Pickering (GP) equation

$$u_t + 2ku_x - \xi uu_x - \eta u_x u_{xx} - \nu u_{xxt} - \theta uu_{xxx} = 0, \longrightarrow \quad (1)$$

is a higher-order nonlinear evolution equation that arises in the study of complex wave propagation phenomena, where nonlinear steepening, higher-order dispersion, and mixed-derivative effects coexist. The intricate balance among these mechanisms gives rise to a rich variety of nonlinear wave structures and makes the equation an important mathematical model in areas such as fluid dynamics, plasma physics, and nonlinear optics. Due to its strong nonlinear nature, obtaining exact analytical solutions of the GP equation is essential for understanding the qualitative and quantitative features of wave evolution. In the present work, Hirota's bilinear method is employed to transform the governing equation into an equivalent bilinear form, providing a systematic framework for the construction of exact solutions. Through appropriate ansatz functions, rational lump solutions and breather solutions are derived. The obtained lump waves exhibit strong spatial localization and algebraic decay, whereas the breather solutions reveal oscillatory and time-periodic dynamics. The interaction behavior, localization characteristics, and propagation properties of these nonlinear structures are further illustrated through graphical analysis. These results contribute to a deeper understanding of energy localization, nonlinear wave interactions, and the underlying dynamics of higher-order dispersive systems governed by the GP equation.

Aim:

- Derive the Hirota bilinear form of the Gilson–Pickering equation.
- Construct exact rational lump and breather solutions.
- Investigate the localization and interaction dynamics of nonlinear wave structures.
- Explore the influence of higher-order nonlinear and dispersive effects on wave propagation.

METHOD

Hirota Bilinear Form:

To derive the bilinear representation of Eq. (1), we employ Hirota's direct method and introduce the dependent variable transformation

$$u = 2(\ln f)_x,$$

under this transformation, equation 1 is converted into the Hirota bilinear form

$$(D_x D_t + 2kD_x^2 - \nu D_x^3 D_t)f.f = 0,$$

where D_{x} and D_{t} denote Hirota's bilinear differential operators. Expanding the bilinear operators yields the corresponding equation for f:

$$f_{xt}f - f_x f_t + 2k(f_{xx}f - f_x^2) - \nu(f_{xxxt}f - 3f_{xxt}f_x + 3f_{xx}f_{xt} - f_{xxx}f_t) = 0.$$

Lump Solutions:

To construct lump solutions of Eq. (1), the following positive quadratic ansatz is introduced for the bilinear equation:

$$f = g^2 + h^2 + a_7,$$

where

$$g = a_1 x + a_2 t + a_3,$$
$$h = a_4 x + a_5 t + a_6.$$

Here, a_{i} (i=1,2,…,7) are arbitrary constants to be determined. Substituting the above ansatz into the bilinear form and equating the coefficients of like powers of the independent variables x and t yields a system of nonlinear algebraic equations. Solving this system with the aid of MAPLE provides the required parameter constraints, from which the lump solutions of Eq. (1) are obtained.

Breather Solutions:

To construct breather solutions of Eq. (1), the following ansatz is introduced for the bilinear equation:

$$f = e^{\xi_1} + \gamma_1 \cos \xi_2 + \gamma_2 e^{\xi_3},$$

where

$$\xi_1 = -v_1(\epsilon_0 t + x),$$
$$\xi_2 = v_0(\omega_0 t + x),$$
$$\xi_3 = v_1(\epsilon_0 t + x).$$

Here, $\epsilon_0, \omega_0, \mu_0, \gamma_1$ and γ_2 are arbitrary constants. Substituting the above ansatz into the bilinear form and equating the coefficients of the resulting terms yields a system of nonlinear algebraic equations. The system is solved using MAPLE, leading to the parameter relations required for the existence of breather solutions of Eq. (1).

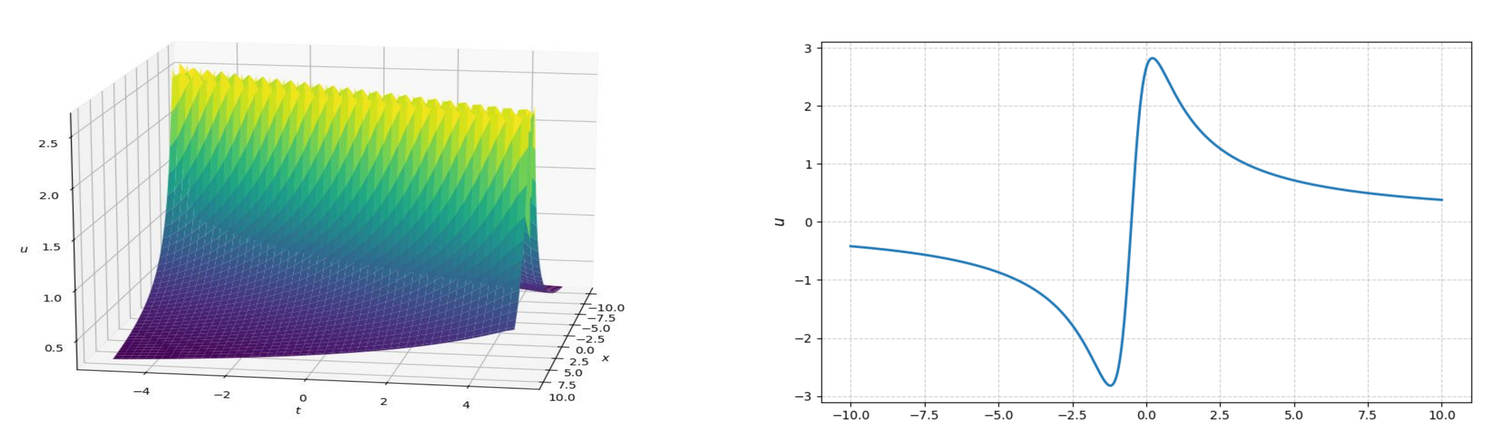
RESULTS & DISCUSSION

Lump solution:

$$k = k,\ \nu = 0,\ a_1 = a_1,\ a_2 = -2k\,a_1,$$
$$a_3 = a_3,\ a_4 = a_4,\ a_5 = -2k\,a_4,\ a_6 = a_6,\ a_7 = a_7.$$

Under these constraints, Eq.1 has a solution

$$u(x,t) = 2(\ln f)_x,$$
$$= 4\left[\frac{a_1(a_1 x - 2ka_1 t + a_3) + a_4(a_4 x - 2ka_4 t + a_6)}{(a_1 x - 2ka_1 t + a_3)^2 + (a_4 x - 2ka_4 t + a_6)^2 + a_7}\right].$$



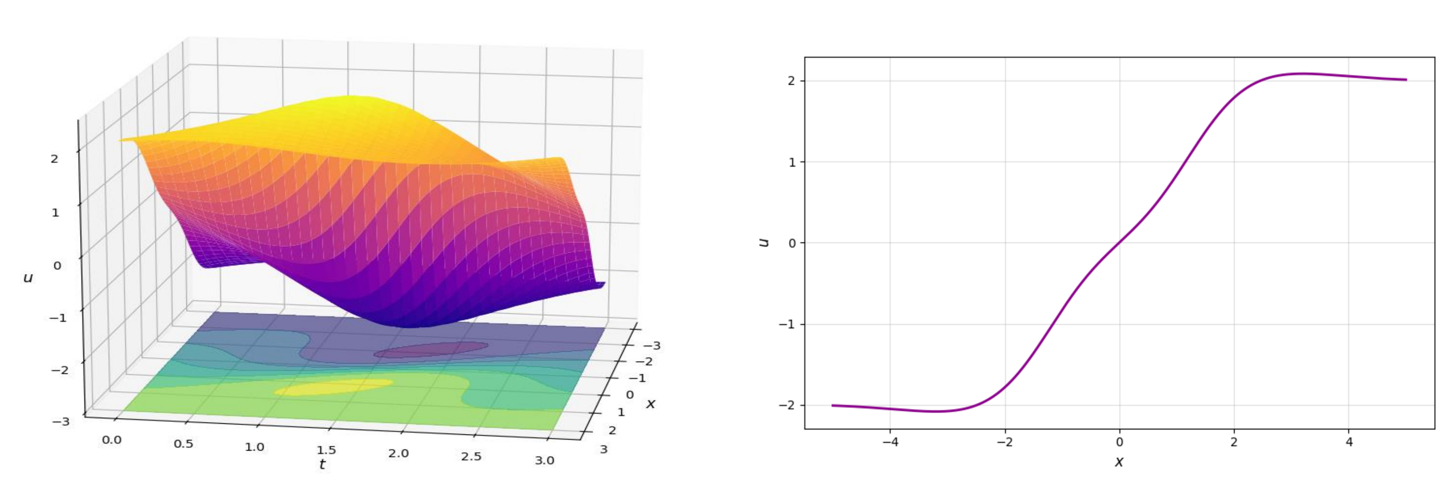
3D and 2D representations of the solution $u(x,t)$ corresponding to $a_1 = 1$, $a_2 = -1$, $a_3 = 0.5$, $a_4 = 1$, $a_5 = -1$, $a_6 = 0.5$, $a_7 = 1$, and $k = 0.5$.

Breather Solution:

$$\gamma_1 = \gamma_1,\ \gamma_2 = \gamma_2,\ k = k, \nu = -\frac{1}{v_0^2 - 3v_1^2}, \omega_0 = \frac{2k\left(\gamma_1^2 v_0^2 + 2\gamma_2 v_0^2 - 6\gamma_2 v_1^2\right)\left(v_0^2 - 3v_1^2\right)}{3\gamma_1^2 v_0^2\left(v_0^2 + v_1^2\right)},$$
$$v_0 = v_0,\ v_1 = v_1, \varepsilon_0 = -k\,\frac{v_0^4 - 4v_0^2 v_1^2 + 3v_1^4}{v_1^2\left(v_0^2 + v_1^2\right)},$$

where, $3\gamma_1^2 v_0^2\left(v_0^2 + v_1^2\right) \neq 0$ and $v_1^2\left(v_0^2 + v_1^2\right) \neq 0$. Under these constraints, Eq.1 has a solution

$$u(x,t) = 2(\ln f)_x$$
$$= 2\left[\frac{-v_1 e^{-v_1(\epsilon_0 t + x)} - \gamma_1 v_0 \sin v_0(\omega_0 t + x) + \gamma_2 v_1 e^{v_1(\epsilon_0 t + x)}}{e^{-v_1(\epsilon_0 t + x)} + \gamma_1 \cos v_0(\omega_0 t + x) + \gamma_2 e^{v_1(\epsilon_0 t + x)}}\right].$$



3D and 2D representations of the solution $u(x,t)$ corresponding to $\gamma_1 = 1$, $\gamma_2 = 1$, $v_0 = 1$, $v_1 = 1$, $k = 1$ and $t = 0$.

CONCLUSION

Exact lump and breather solutions of the Gilson–Pickering equation were derived via Hirota's bilinear method. The obtained wave structures exhibit pronounced localization and oscillatory dynamics, providing valuable insight into nonlinear wave evolution and energy localization in higher-order dispersive systems.

FUTURE WORK / REFERENCES

Future Works:

- Future work includes the construction of lump–kink and stripe-soliton solutions, along with a stability analysis of the associated wave structures.

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