IOCMA 2026 Conference

The 2nd International Online Conference on Mathematics and Applications

10–12 June 2026 | Online

MDPI

Stochastic Dynamics of a Harvested Two-Species Fishery Model with Random Birth and Death Effects

Pamodya Teshini Thalawage (teshini-ps19282@stu.kln.ac.lk)

Mihiri Madushani De Silva (mihiris@kln.ac.lk)

[1] Department of Mathematics, Faculty of Science, University of Kelaniya, Kelaniya, Sri Lanka

INTRODUCTION & AIM

Background: Deterministic models assume homogeneous, continuous environments, often oversimplifying natural biological systems. In reality, random births, random deaths, and external interactions introduce non-trivial variability, known as demographic stochasticity.

Problem Statement: The random fluctuations of population can drastically warp steady states, amplify extinction thresholds, or skew population viability away from classical deterministic trajectories.

Aim of the Study:

1. To construct and mathematically validate a deterministic predator-prey or competing species system under constant harvesting effort (E).
2. To transition the system into an Itô Stochastic Differential Equation (SDE) framework using a Continuous-Time Markov Chain (CTMC) approach.
3. To evaluate and contrast a Standard SDE Formulation against a Variance-Amplified SDE Formulation to show how structural noise refinement alters long-term ecological predictions.

METHODOLOGY

Deterministic Baseline (ODE)

The system tracks the population densities of two interacting species, x and y, subjected to constant harvesting effort E:

$$\frac{dx}{dt} = (r - q_1 E)x - \frac{r}{K}x^2 - \alpha xy$$

$$\frac{dy}{dt} = (s - q_2 E)y - \frac{s}{L}y^2 - \beta xy$$

Where r, s are intrinsic growth rates; K, L represent carrying capacities; q_1, q_2 are catchability coefficients; and α, β dictate the interspecies pressure.

Stochastic Derivation (CTMC to SDE)

By establishing small-interval transition probabilities (Δt) matching discrete birth-death events, the system is mapped into a coupled Itô SDE matrix:

$$dX(t) = f\big(X(t)\big)dt + G\big(X(t)\big)\, dW(t)$$

1. Standard SDE Model: Employs standard aggregate noise parameters where diffusion coefficients scale fundamentally as a function of basic population densities:

$$\text{Diffusion Coefficient} \quad g_{11} = \sqrt{\left(r_1 x + \frac{r_1}{K}x^2 + \alpha xy\right)}$$

2. Amplified SDE Model: Decouples the net parameters into precise, separate birth a_{ij} and death b_{ij} coefficients satisfying the precise bounds ($a_{11} - b_{11} = r_1,\ etc$). This structural split reveals hidden variance in deterministic traditional models.

Numerical Simulation

1. **Sample Paths:** Solved via an adaptive Euler-Maruyama scheme with a structural path resolution optimized ($N = 180$) to isolate macroscopic environmental waves rather than high-frequency microscopic fuzz.
2. **Statistical Ensemble:** Conducted over 10,000 parallel Monte Carlo path trajectories evaluated via an ultra-fine grid resolution ($N = 3000,\ T = 10$) to compute stable probability density functions.

RESULTS & DISCUSSION

Trajectory Patterns: Standard SDE model paths tightly trace the deterministic (ODE) lines. Amplified SDE model paths display much wider, winding macro-level oscillations and path crossings due to decoupled birth-death variance.

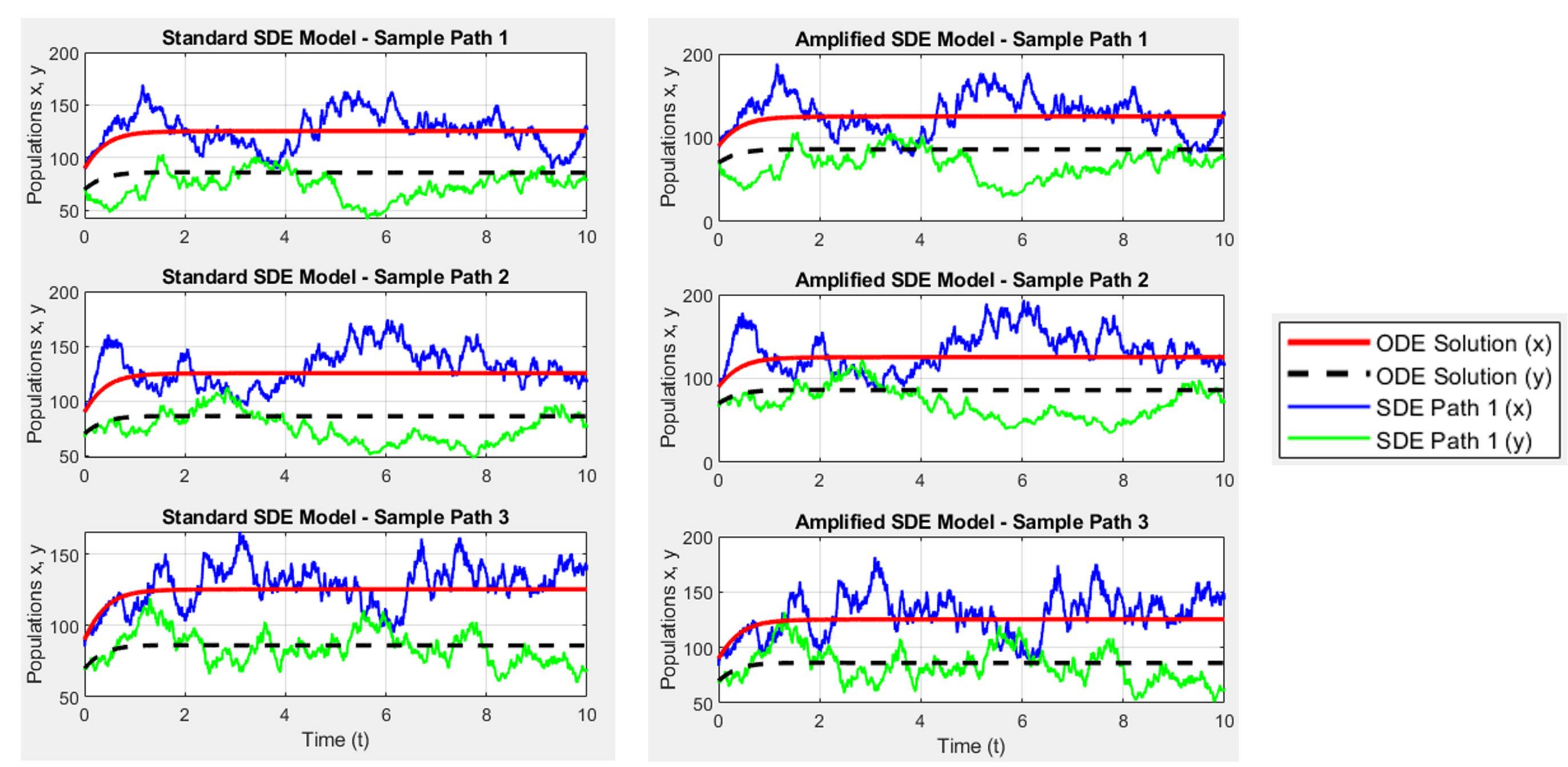

Figure 01

Statistical Consistency: Both SDE systems preserve long-term deterministic mean states, confirming structural integration accuracy.

Variance Discrepancy: The Coefficient of Variation is significantly higher in Amplified SDE model than the standard SDE model.

Comparative Statistical Performance at t = 10 (10,000 Paths)				
System Model	Variable	Mean (μ)	Std Dev (σ)	Coeff. of Var (CV)
Standard SDE Model	Species X	124.8027	16.0139	0.1283
	Species Y	84.6853	13.7462	0.1623
Amplified SDE Model	Species X	124.3939	21.9643	0.1766
	Species Y	83.3543	18.9238	0.227

Table 01

Risk Profiles: Probability histograms show that the amplified SDE model creates a flatter, wider distribution. This mathematically increases the risk of populations dipping into critical low or near-extinction zones.

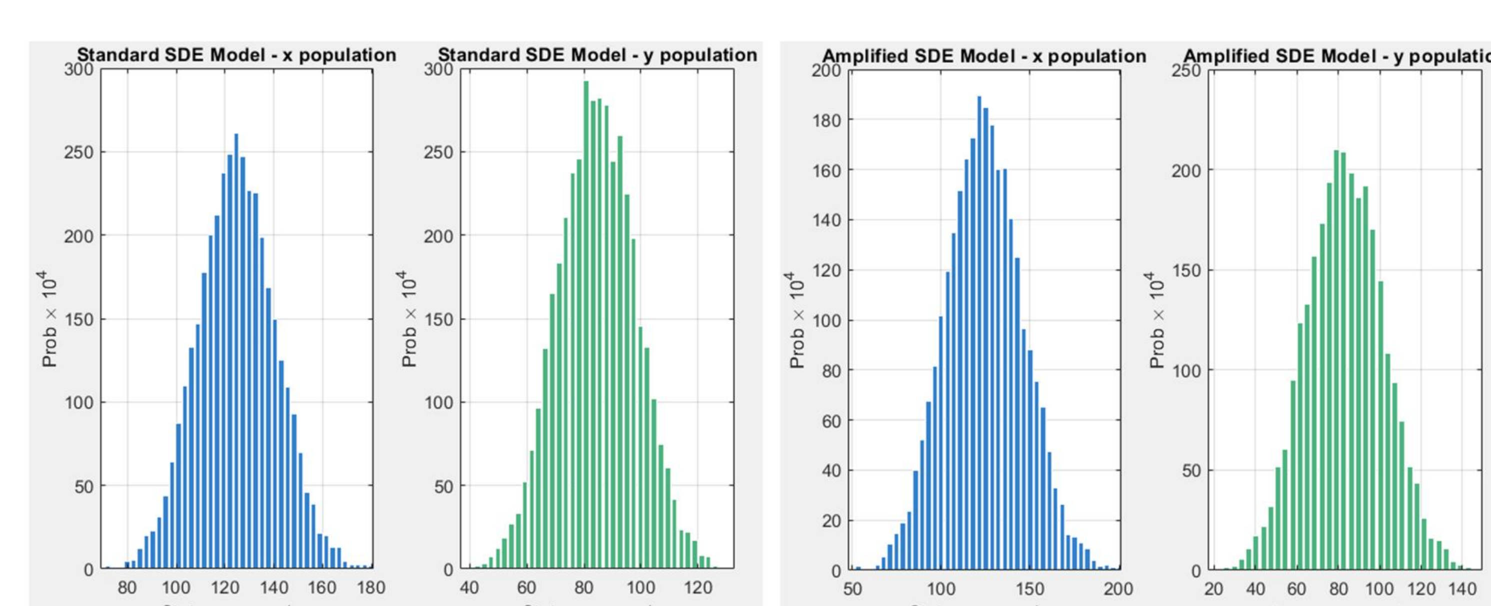

Figure 02

CONCLUSION

Demographic stochasticity highlights ecological risks overlooked by deterministic models. Amplified population variance under harvesting necessitates updated resource management strategies to accurately assess species resilience and avoid overestimations.

REFERENCES

1. De Silva, T. M. M., Jang, S. R. J. (2018). Stochastic modeling of phytoplankton zooplankton interactions with toxin producing phytoplankton. Journal of Biological Systems, 26(01), 87-106.
2. Chaudhuri, K. (1986). A bioeconomic model of harvesting—a multispecies fishery. Ecological Modelling, 32(4), 267–279.