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Analytical Modeling of a Dynamic Probability Mixture via Sigmoidal Weighting for Heterogeneous System Degradation

Halyna Tuluchenko
National Technical University "Kharkiv Polytechnic Institute"
Kharkiv, Ukraine

INTRODUCTION & AIM

Traditional mixture models with constant weights are well-defined but fail to capture sharp phase transitions in aging heterogeneous systems.

We propose a dynamic mixture

$$p(x)=\frac{1}{C}\cdot\left(\omega(x)\,f_{Exp}(x)+\left(1-\omega(x)\right)f_{Erlang}(x)\right)$$

using a sigmoidal weighting function (3) to model non-linear degradation.

The aim of the research is to develop a model for a dynamic mixture of Erlang and exponential distributions with sigmoidal weighting functions to describe phase transitions in heterogeneous systems, as well as to establish methods for identifying its parameters.

MATHEMATICAL MODEL

Components of the distribution mixture:

Exponential distribution:

$$f_{Exp}(x)=\lambda\exp\left(-\lambda x\right), \qquad (1)$$

Erlang distribution:

$$f_{Erlang}(x)=\frac{\mu^{k}}{(k-1)!}x^{k-1}\exp\left(-\mu x\right), \qquad (2)$$

for $x\in[0,+\infty)$, where $\lambda>0,\ \mu>0,\ k\in\mathrm{N}$.

The sigmoid weighting function is

$$\omega(x)=\frac{1}{1+\exp\left(-r\left(x-a\right)\right)}, \qquad (3)$$

where a is phase transition point (threshold),
r is transition intensity parameter.

RESULTS

Representation of the normalizing constant using the Lerch function:

$$C=\frac{\lambda}{r}\Phi\left(-\exp(ra),1,\frac{\lambda}{r}\right)+1-\frac{\mu^{k}}{r^{k}}\Phi\left(-\exp(ra),k,\frac{\mu}{r}\right).$$

Approximate calculation of the normalizing constant using the Sommerfeld method:

$$C\approx 1+\exp\left(-\lambda a\right)-\sum_{n=0}^{k-1}\frac{\left(\mu a\right)^{n}}{n!}\exp\left(-\mu a\right)+$$

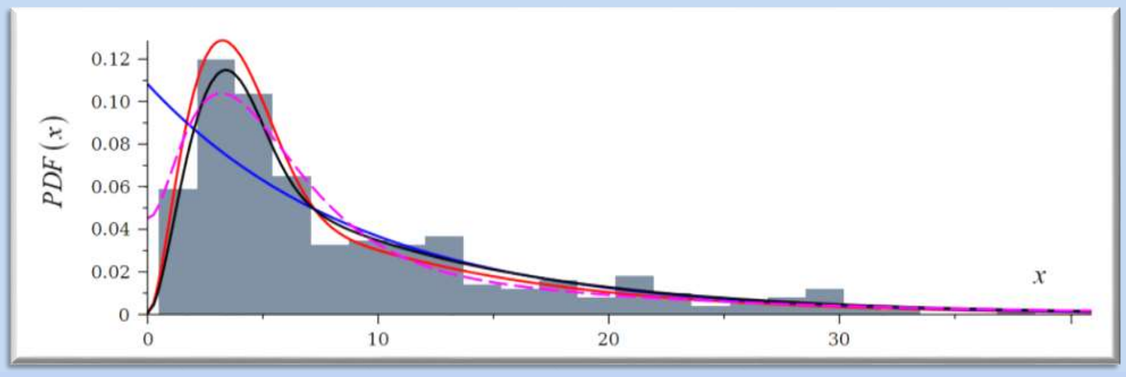
$$+\frac{\pi^{2}}{6r^{2}}\left(f^{(1)}_{Erlang}(a)-f^{(1)}_{Exp}(a)\right)+\frac{7\pi^{4}}{360r^{4}}\left(f^{(3)}_{Erlang}(a)-f^{(3)}_{Exp}(a)\right).$$

The proposed model was tested on synthetic data with known parameters and on real degradation trajectories from the NASA C-MAPSS dataset.

MODEL VALIDATION

Model parameters are identified via MLE with gradient-based optimization.

Application Example (Synthetic Data)



Models	Maximum log-likelihood	AICc	BIC
● Mixture with true parameters	–941.1	1894.5	1916.4
● Proposed Dynamic Mixture	–938.8	1889.8	1911.8
● Mixture with fixed weights	–951.3	1910.8	1925.5
● Erlang Distribution	–966.9	1937.9	1945.3