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MDPI

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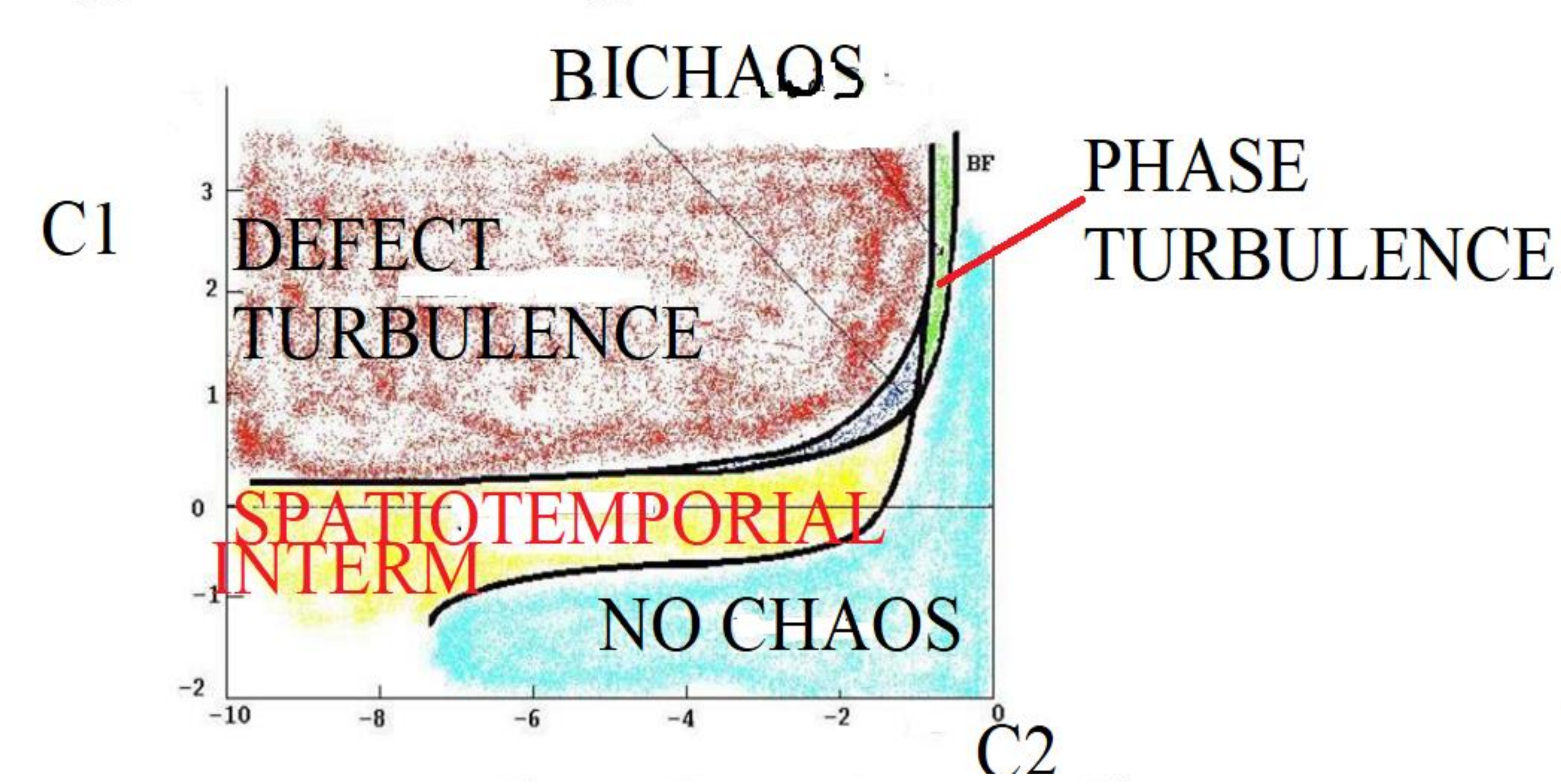
•Stability Estimates for Discontinuous Galerkin in space Methods for the Nonlinear Ginzburg-Landau System

INTRODUCTION & AIM

Complex Ginzburg-Landau Equation

general CGLE

$$\frac{\partial}{\partial T}A = A + (1 + jc_1)\frac{\partial^2}{\partial t^2}A - (1 + jc_2)|A|^2 A$$



METHOD

$$V_h = \{w \in L^2(\Omega) : w|_K \in P_r(T_h) \forall K \in T_h\}$$

$$\|u_h\|_{DG}^2 = \sum_{K\in T_h} \|\nabla u_h\|_{L^2(K)}^2 + \sum_{e\in E_h} \left(\frac{\sigma}{h_e}\|[u_h]\|_{L^2(e)}^2\right)$$

$$a_h(u_h, v_h) = \sum_{\{K\in Th\}} \int_K \nabla u_h \nabla v_h - \sum_{\{e\in\varepsilon h\}} \int_e \{\nabla uh\}[vh]ds + \theta \sum_{\{e\in\varepsilon h\}} \int_e \{\nabla vh\}[uh]ds + \sum_{\{e\in\varepsilon_h\}} \int \frac{e_\sigma}{h_e}[uh][vh]ds$$

- If $\theta = -1$ (SIPG) Sym. Int. Pen. Gal.
- If $\theta = 1$ (NIPG) Nonsym, Int Pen Gal
- If $\theta = 0$ (IIPG) Incomp. Int. Pen Gal

RESULTS & DISCUSSION

$$\int_{t^{n-1}}^{t^n}\int_\Omega \frac{\partial u_{h1}}{\partial t} v_{h1}dxdt = \int_{t^{n-1}}^{t^n} -a_h(u_{h1}, v_{h1}) + \alpha a_h(u_{h2}, v_{h1})dt + \int_{t^{n-1}}^{t^n}\int_\Omega u_{h1}v_{h1}dxdt - \int_{t^{n-1}}^{t^n}\int_\Omega (u_{h1}^2 + u_{h2}^2)u_{h1}v_{h1}dxdt + b\int_{t^{n-1}}^{t^n}\int_\Omega (u_{h1}^2 + u_{h2}^2)u_{h2}v_{h1}dxdt$$

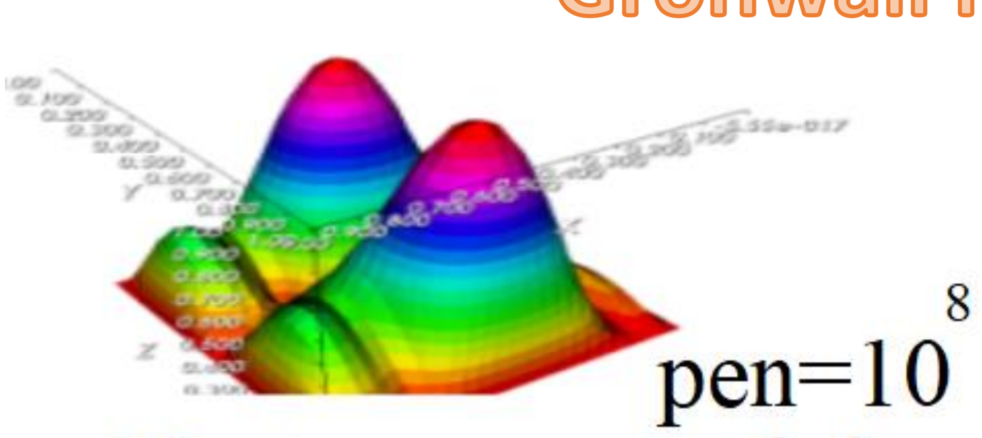
$$\int_{t^{n-1}}^{t^n}\int_\Omega \frac{\partial u_{h2}}{\partial t} v_{h2}dxdt = \int_{t^{n-1}}^{t^n} -\alpha a_h(u_{h1}, v_{h2}) - a_h(u_{h2}, v_{h2})dt + \int_{t^{n-1}}^{t^n}\int_\Omega u_{h2}v_{h2}dxdt - \int_{t^{n-1}}^{t^n}\int_\Omega (u_{h1}^2 + u_{h2}^2)u_{h2}v_{h2}dxdt - b\int_{t^{n-1}}^{t^n}\int_\Omega (u_{h1}^2 + u_{h2}^2)u_{h1}v_{h2}dxdt$$

$$\frac{d}{dt}\left(\|u_{h1}\|_{L^2(\Omega)}^2 + \|u_{h2}\|_{L^2(\Omega)}^2\right) \le 2\left(\|u_{h1}\|_{L^2(\Omega)}^2 + \|u_{h2}\|_{L^2(\Omega)}^2\right).$$

$$\int_{t^{n-1}}^{t^n}\left(\|u_{h1}(t)\|_{L^2(\Omega)}^2 + \|u_{h2}(t)\|_{L^2(\Omega)}^2\right)dt \le \left(\|u_{h1}(t^{n-1})\|_{L^2(\Omega)}^2 + \|u_{h2}(t^{n-1})\|_{L^2(\Omega)}^2\right)\frac{e^{2\Delta t} - 1}{2}$$

$$\frac{\|u_{h1}^n\|_{L^2(\Omega)}^2}{2} + \frac{\|u_{h2}^n\|_{L^2(\Omega)}^2}{2} \le e^{2\Delta t}\left(\frac{\|u_{h1}^{n-1}\|_{L^2(\Omega)}^2}{2} + \frac{\|u_{h2}^{n-1}\|_{L^2(\Omega)}^2}{2}\right)$$

Gronwall inequality



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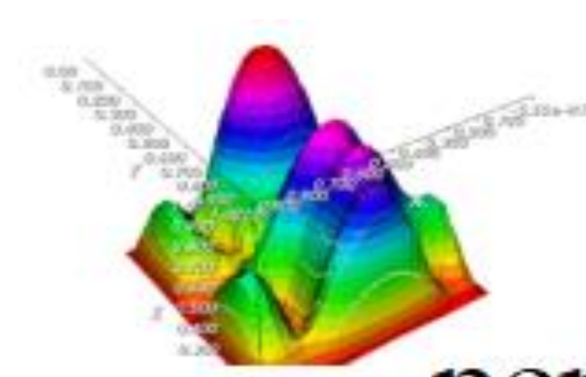
Patern stable
u1 -1.4 to 1.5
u2 -4.5 to 4.5



pen 10^3

Patern Unstable

Mesh	Method	10^3	10^8
	SIPG	2.24 (Stable)	2.10 (Stable)
0.0589	NIPG	10.01 (Unstable)	2.12 (Stable)
	IIPG	2.93 (Weak)	2.11(Stable)

CONCLUSION

All methods for N. G L. are Stable

SIPG>IIPG>NIPG

FUTURE WORK / REFERENCES

Extending our results other systems

Hu, J., Shu, C.-W., & Shu, J. (2013). Discontinuous Galerkin methods for the Fokker-Planck-Landau equation. Journal of Computational Physics, 237, 54-77. https://doi.org/10.1016/j.jcp.2012.12.027

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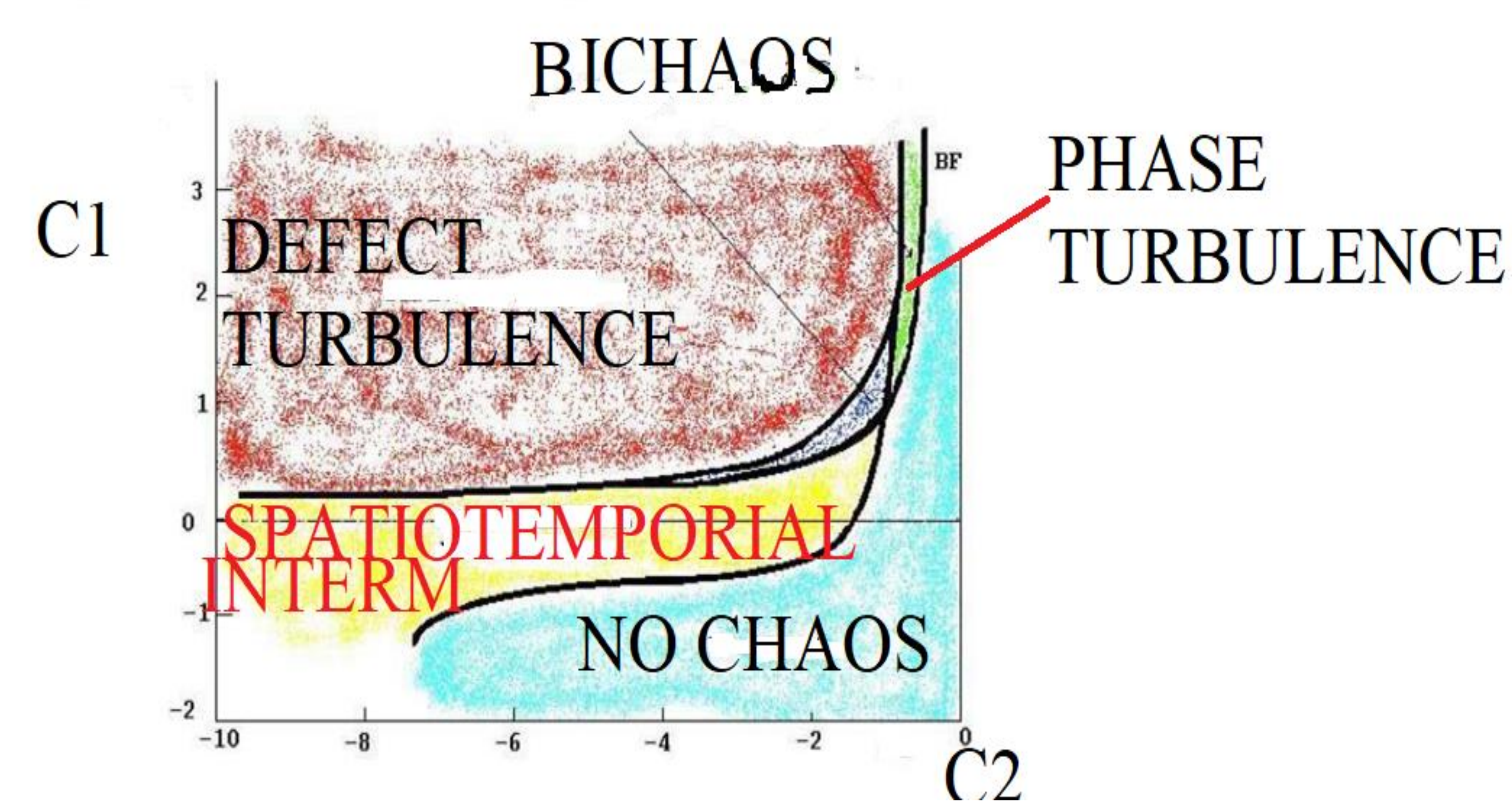

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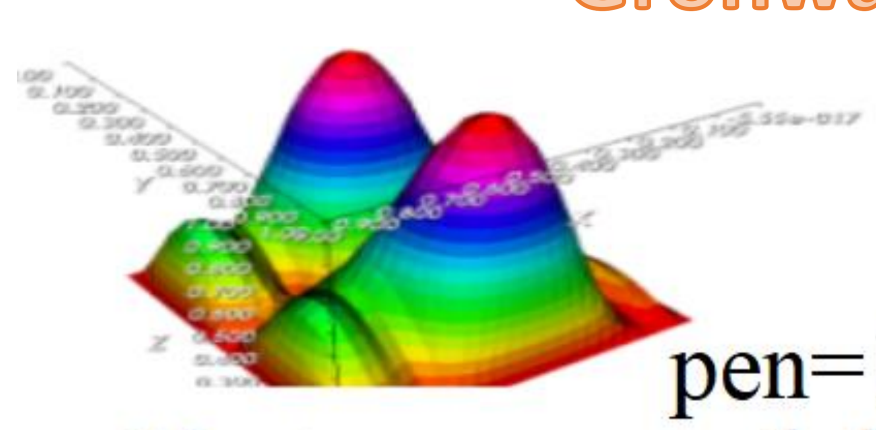

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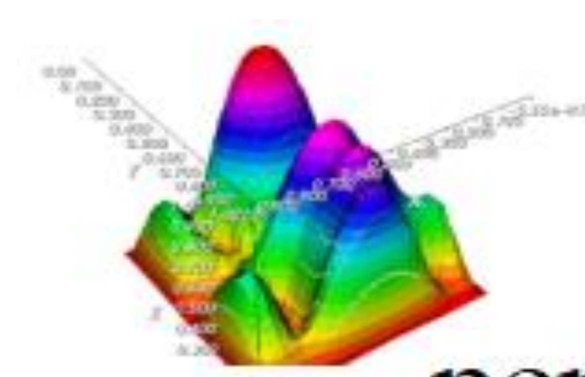

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