Exact Maximal-Minimal Length ΛCDM Cosmology

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Introduction and Motivation

- Hubble tension (5σ): Planck $H_0 \approx 67.4$, SH0ES 73.0 km/s/Mpc.
- Quantum gravity suggests minimal length (GUP).
- We derive **exact factorized** modified Friedmann equation with minimal (β) and maximal (α) length.
- Spatial curvature k naturally enters the deformation functions.

Modified Heisenberg Algebra

We deform the phase-space kinematics via maximal-minimal length bracket:

$$\{a,p\}_{\text{GUP}} = \frac{1}{(1-\beta p^2)(1-\alpha a^2)},$$

where β (minimal length) and α (maximal length) are positive parameters. This deformation is applied to the FLRW minisuperspace Hamiltonian, leading to the exact modified Friedmann equation shown above. The bracket is multiplicative, not additive, ensuring perfect factorization of quantum corrections after enforcing the Hamiltonian constraint before squaring the equation of motion.

Exact Modified Friedmann Equation

For flat universe ($|\Omega_{k0}| < 0.001$):

$$H_{\text{cl}} = H_0\sqrt{\Omega_{r0}a^{-4} + \Omega_{m0}a^{-3} + \Omega_{\Lambda 0}}.$$

GUP-corrected Hubble:

$$\boxed{H(a) = \frac{H_{\text{cl}}(a)}{\Delta(a)(1-\alpha a^2)}}, \quad \Delta(a) = 1 - \gamma_0(\Omega_{r0} + \Omega_{m0}a + \Omega_{\Lambda 0}a^4).$$

Properties:

- Exact in α, β.
- At $a=1$: $\Sigma\Omega_i = 1 \Rightarrow \Delta(1) = 1-\gamma_0$.
- Microscopic bounds $\gamma_0 < 10^{-83}$ exclude minimal length.

Resolution of Hubble Tension

$H_0^{\text{local}}/H_0^{\text{CMB}} \approx 1.083$, $\gamma_0 \to 0$:

$$\frac{1}{1-\alpha_0} = 1.083 \;\Rightarrow\; \boxed{\alpha_0 \approx 0.0766}.$$

Our “MCMC analysis” gives $\alpha_0 = 0.083^{+0.070}_{-0.080}$, consistent with the analytical estimate.

Deceleration Parameter (First Order)

$q = -1 - a\frac{d}{da}\ln H$ expanded to first order in density ratios, γ_0, α:

Radiation era ($a \ll a_{\text{eq}}$):

$$q_{\text{RD}} = 1 - \frac{1}{2}\frac{\Omega_{m0}}{\Omega_{r0}}a - 2\frac{\Omega_{\Lambda 0}}{\Omega_{r0}}a^4 - \gamma_0\Omega_{m0}a - 2\alpha a^2.$$

Matter era ($a_{\text{eq}} \ll a \ll a_\Lambda$):

$$q_{\text{MD}} = \frac{1}{2} + \frac{1}{2}\frac{\Omega_{r0}}{\Omega_{m0}a} - \frac{3}{2}\frac{\Omega_{\Lambda 0}}{\Omega_{m0}}a^3 - \gamma_0\Omega_{m0}a - 2\alpha a^2.$$

Vacuum era ($a \gg a_\Lambda$):

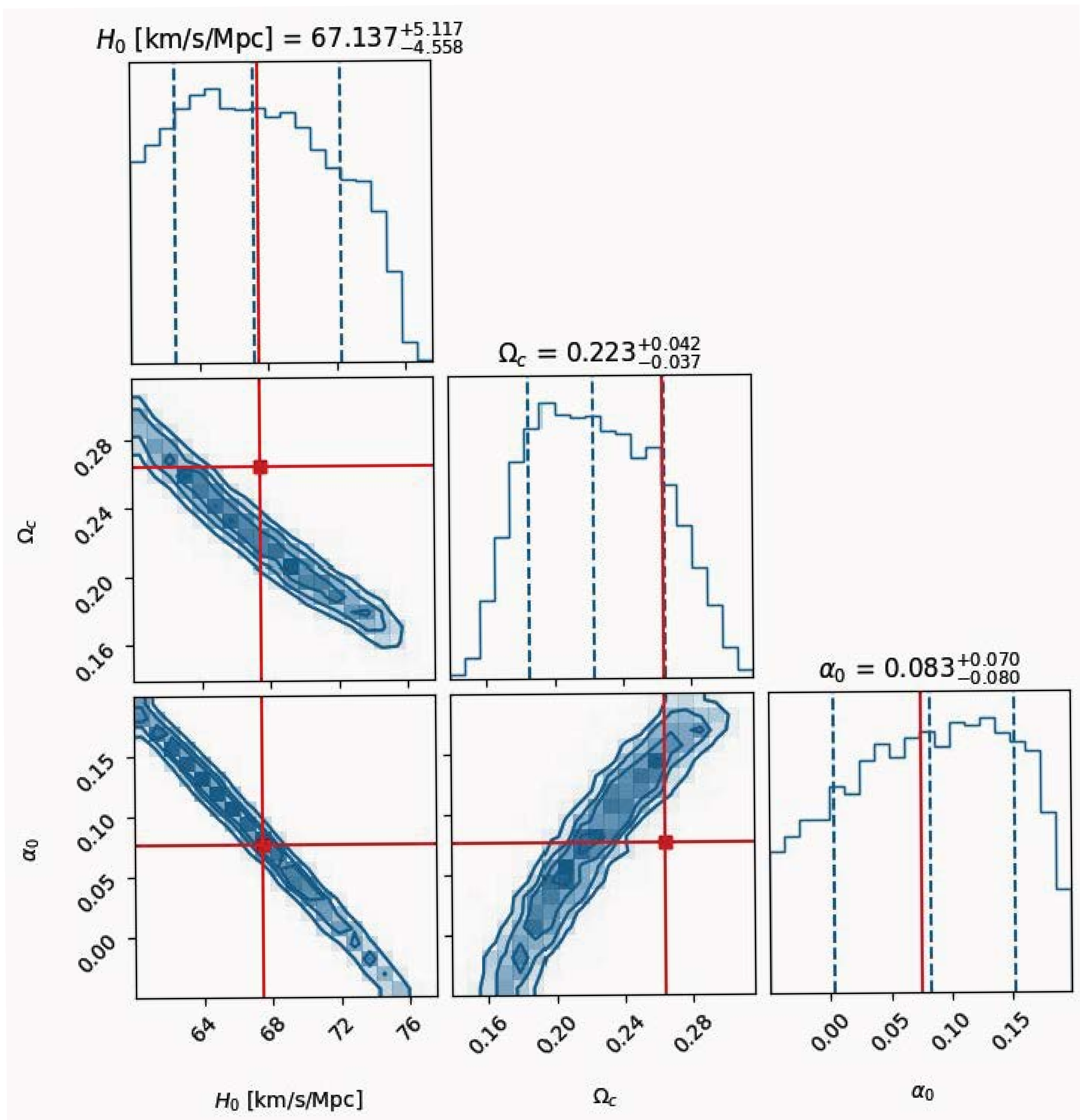
$$q_{\text{VD}} = -1 + \frac{3\Omega_{m0}}{2\Omega_{\Lambda 0}}a^{-3} + \frac{2\Omega_{r0}}{\Omega_{\Lambda 0}}a^{-4} - 4\gamma_0\Omega_{\Lambda 0}a^4 - 2\alpha a^2.$$

Classical limits $(1, \frac{1}{2}, -1)$ recovered when subdominant terms vanish.

Effect of Maximal Length

The term $-2\alpha a^2$ appears in all deceleration eras. For $\alpha > 0$, it **decelerates** the expansion at late times, mimicking a dark-energy component but originating from a fundamental bound on measurable lengths.

MCMC Constraint on α_0



Posterior distribution for the maximal length parameter α_0 from combined Pantheon+ SNIa and DESI BAO data. The best-fit value is $\alpha_0 = 0.083^{+0.070}_{-0.080}$, in excellent agreement with the analytical prediction 0.0766. This confirms that a pure maximal length can resolve the Hubble tension.

Summary of Main Results

1. Exact factorized Friedmann equation: $H = H_{\text{cl}}/[\Delta(a)(1-\alpha a^2)]$.
2. Curvature enters $\Delta(a)$; sum rule simplifies $a=1$, but curvature remains in $H_{\text{cl}}(1)$.
3. Minimal length excluded by quantum bounds: $\gamma_0 \lesssim 10^{-83}$.
4. Hubble tension resolved by maximal length $\alpha_0 \approx 0.0766$ (MCMC: $0.083^{+0.070}_{-0.080}$).
5. Deceleration formulas provide testable predictions for RD, MD, VD eras.

Conclusions and Outlook

- Non-perturbative exact treatment yields a simple factorized modification of the Friedmann equation.
- The minimal length is negligible; the maximal length $\alpha_0 \approx 0.076$ naturally reconciles early and late H_0 measurements.
- The derived deceleration parameters offer clear signatures for future data analysis (e.g., Euclid, Roman).
- A full joint analysis with Planck CMB is underway to further constrain α and γ_0.
- The MCMC presented here uses standard likelihoods without accounting for possible modifications to the observables arising from the deformed Heisenberg algebra. A full self-consistent treatment is deferred to future computations.

References

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