IOCMA 2026 Conference
The 2nd International Online Conference on Mathematics and Applications
10–12 June 2026 | Online
MDPI

A Topological Framework for Detecting Chaos in Erbium Doped Fiber Lasers

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MOTIVATION

Erbium-doped fiber lasers (EDFLs) [1] exhibit complex nonlinear behaviors, including transitions between periodic and chaotic regimes, which are important for the stability and performance of photonic systems. In this work, classical nonlinear dynamical analysis is combined with Topological Data Analysis (TDA) [2] to investigate these regime transitions. Lyapunov exponents are used to identify chaotic behavior, while persistent homology and the Mapper algorithm are employed to capture the global geometric and topological structure of the attractors. The results demonstrate that TDA provides an effective and complementary framework for detecting and understanding dynamical transitions in nonlinear fiber laser systems.

EDFL MATHEMATICAL MODEL

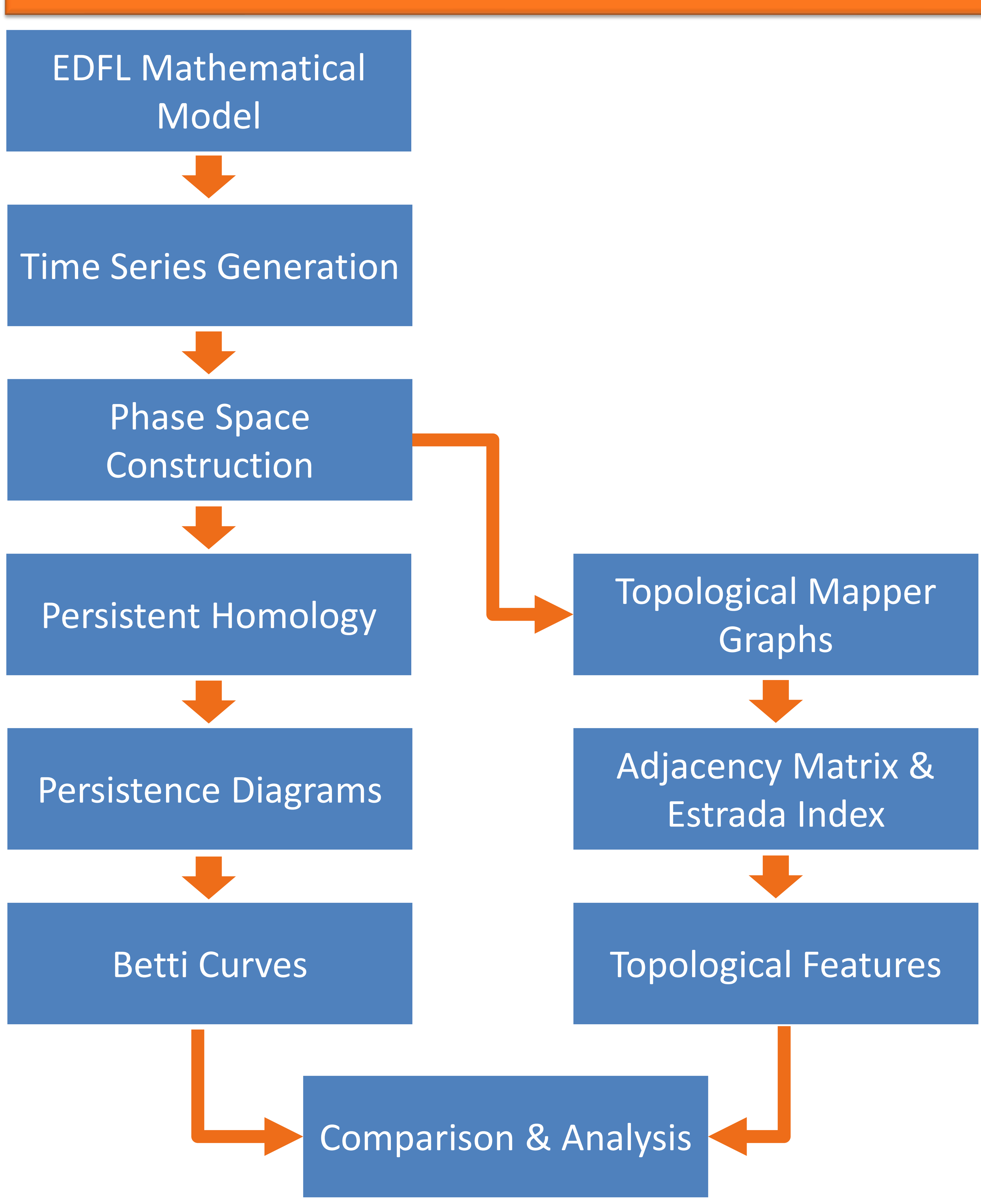
The dynamics of the erbium-doped fiber laser (EDFL) are described by the normalized system:

$$\frac{dI}{d\theta} = aIN - bI + c(N + r_w)$$

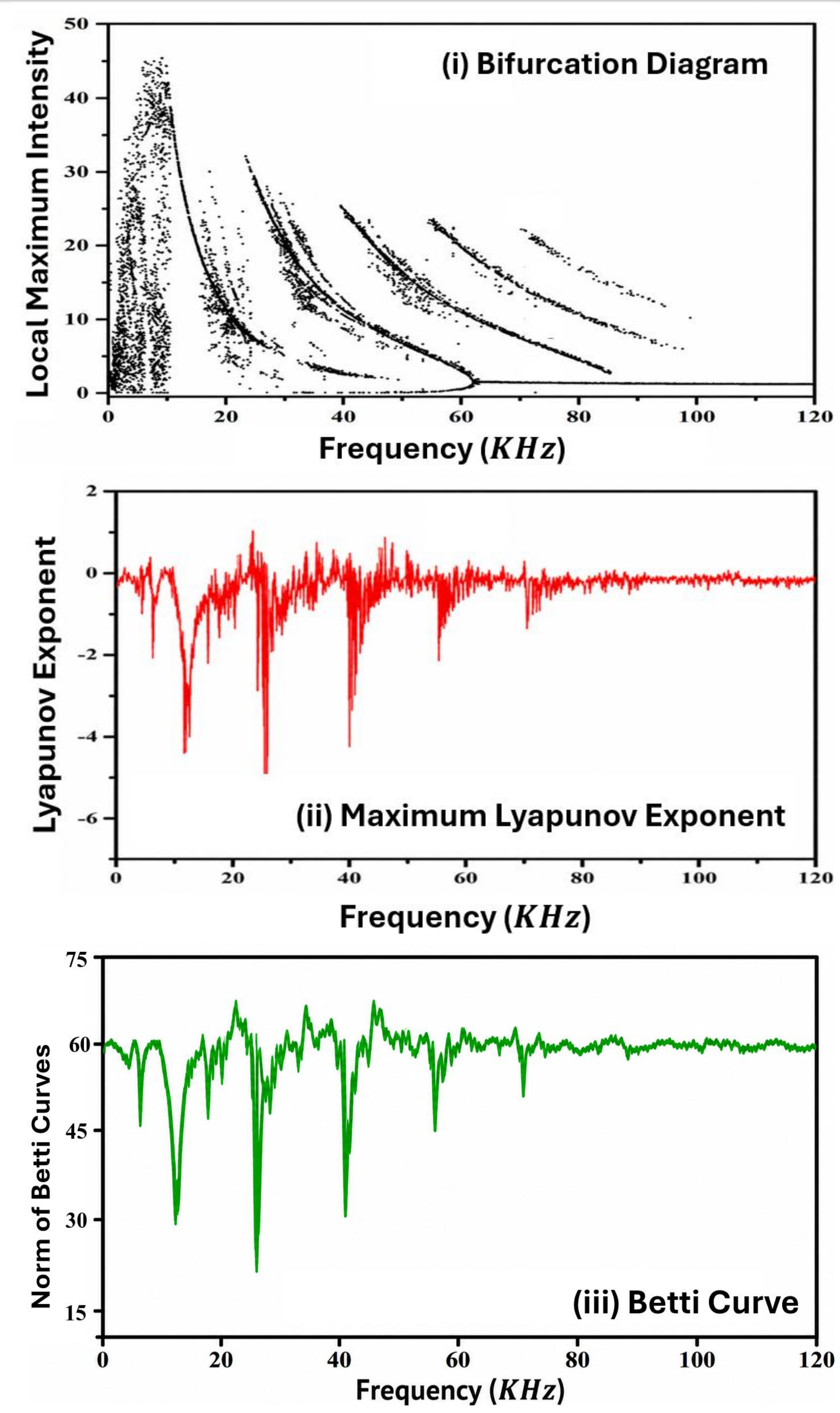
$$\frac{dN}{d\theta} = -dIN - (N + r_w) + e\left[1 - \exp\left(-\beta\alpha_0 L\left(1 - \frac{N + r_w}{\xi_2 r_w}\right)\right)\right]$$

where (I) is the normalized intensity and (N) is the population inversion. Numerical simulations were performed for different control parameters to investigate periodic and chaotic dynamics.

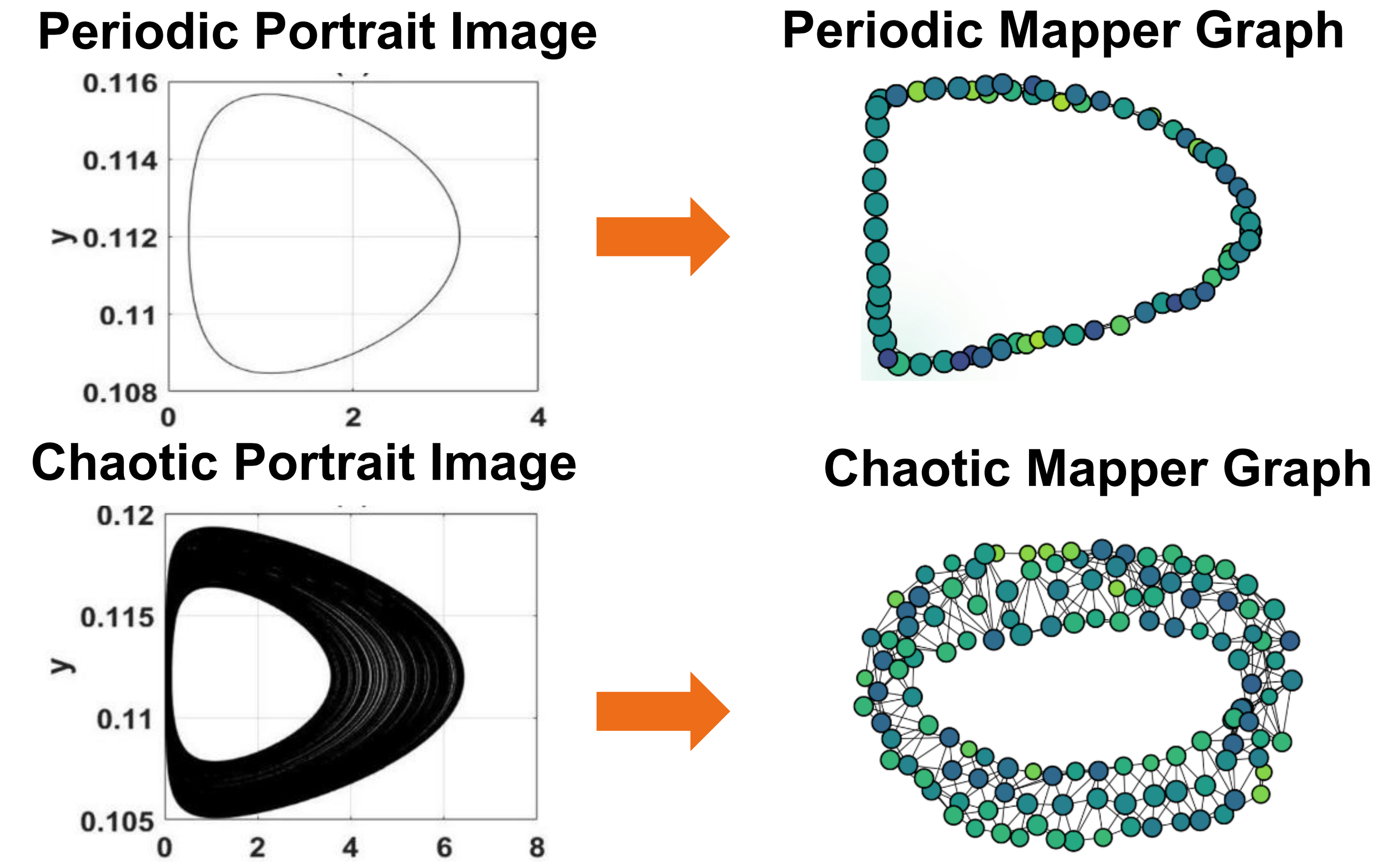
METHODOLOGY



RESULTS & DISCUSSION



TOPOLOGICAL MAPPER GRAPHS



FUTURE WORK / REFERENCES

Topological Data Analysis successfully distinguishes periodic and chaotic regimes in the EDFL system. Persistent homology and Mapper reveal structural changes in attractors that complement Lyapunov exponent analysis. Extend the framework to experimental laser data and real-time regime monitoring. Investigate synchronization and machine learning approaches using topological features.

1: Pisarchik, A. N.; Barmenkov, Y. O.; Kir'yanov, A. V. Experimental characterization of bifurcation structure in an erbium-doped fiber laser. {\em IEEE Journal of Quantum Electronics} {\bf 2003}, {\em 39}, 1567--1571.
2: Otter, N.; Porter, M. A.; Tillmann, U.; Grindrod, P.; Harrington, H. A. A roadmap for the computation of persistent homology. {\em EPJ Data Science} {\bf 2017}, {\em 6}(1), 17.