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Discrete Elliptic Boundary Value Problem Defined by Symbolic Pseudo-differential Operators

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INTRODUCTION & AIM

Discrete elliptic boundary value problems, Discrete analogues of pseudo-differential operators (PDOs) arise in the mathematical modeling of physical phenomena on lattices, networks, discretized domains and are increasingly important in numerical analysis, signal processing.

Pseudo-differential operators (ΨDOs) provide a powerful framework for analyzing elliptic operators through symbolic calculus. They facilitating the study of ellipticity, parametrices, and boundary conditions.

Extending this symbolic calculus to the discrete setting is essential for understanding the behavior of discretized problems.

The purpose of this paper is to develop a systematic theory of discrete elliptic boundary value problems defined via symbolic pseudo-differential operators.

METHOD

The analysis is carried out on discrete domain $\Omega_h \subset Z^n$ discrete pseudo differential operators are defined using the discrete Fourier transform and suitable symbol classes, boundary value problems are formulated by coupling interior difference operator with boundary operators acting on the discrete boundary. Ellipticity is defined through principle symbol estimate, allowing for the construction of parametrices within the discrete symbolic calculus.

RESULTS & DISCUSSION

The main result shows that elliptic difference operators admit parametrices obtained via wave factorization of the symbols. This approach yields a discrete analogues of the Lopatinski-shapiro condition and implies Fredholm properties, as well as existence, uniqueness and regularity of solutions, even for irregular lattice boundaries.

CONCLUSION

This work establishes a rigorous symbolic theory for discrete elliptic boundary value problems. By adapting Vasil'ev's wave factorization and Eskin's boundary problem theory to the discrete setting the paper bridges continuous pseudo-differential theory and discrete model, providing a solid analytical foundation for stability and numerical methods for elliptic problems on lattice.

FUTURE WORK / REFERENCES

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