



Singular Value Decomposition Applied to Non-Cooperative Target Identification

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Abstract: One of the main concerns in the Security and Defense area is to identify quickly and reliably different in-flight targets, especially in hostile scenarios. A non-cooperative classification system could identify targets at long range and under conditions of poor visibility without requiring aircraft collaboration. According to that, to face this problem, an approach to Non-Cooperative Target Identification based on feature extraction techniques applied to High Resolution Range Profiles is presented. The identification methodology is conducted by comparison of a collection of range profiles of a unknown target, namely test set, with a pre-loaded database of known potential signatures, namely training set. In order to evaluate the performance of the presented algorithms, range profiles are obtained through numerical simulation of seven civil aircraft at defined trajectories taken from an actual measurement campaign. The most evident issue of using synthetic signatures instead of measured profiles is that simulated profiles implies an ideal recognition scheme, since datasets have the same high quality. So as to confirm the validity of the approach, additive white Gaussian noise has been considered to the profiles in the test set.

Keywords: HRRP; NCTI; simulated/synthetic database; SVD

1. Introduction

A High Resolution Range Profile is the projection onto the radar line of sight of the radar energy returned by a moving aircraft. HRRP is a one dimensional image which represents the target reflectivity into the range domain. Each profile consists of a number of range bins that contain the distribution of the different reflecting parts, or scattering centers of an illuminated target, supplying information about its structure [1]. According to that, this information, different for each target, is suitable for automatic recognition since it depends on its geometry. Radar recognition has received intensive attention from the NCTI community, since it permits to recognize targets at long distance and under poor visibility conditions without the need of communication with targets and even with them being unaware of it [2]. Non Cooperative Target Identification (NCTI) by radar relies on a comparison between the measured target signatures and a reference database [3–5]. This can be carried out by different techniques, including Jet Engine Modulation (JEM), HRRPs and Inverse Synthetic Aperture Radar (ISAR) images. JEM do not provide an all aspect identification capability, since the aircraft should be in an aspect angle such that the radar energy can be scattered back by the engines. Furthermore, high Signal to Noise Ratio (SNR) is desired, so the range at which aircraft can be recognized is reduced. Even if HRRP and ISAR images give an all aspect classification, the processing to obtain ISAR is very complex and time consuming, so NCTI techniques based on HRRPs are the most suitable option.

In this research, a NCTI system based on the exploitation of feature extraction techniques is presented. The process is carried out using a sequence of consecutive profiles so as to obtain information about the evolution of the position of the scattering centers along each trajectory. Two collections of synthetic HRRPs are considered, the profiles corresponding to the target to be identified, namely test set, and the ones that populate the database, namely training set.

2. Identification Methodology

The objective of the feature extraction techniques is to reduce dimensionality of the problem by keeping the valuable information. In this research, in order to model each aircraft as a subspace and to reduce the unwanted information Singular Value Decomposition is applied to a matrix of profiles, to extract the main features of both sets. SVD is a powerful technique for the factorization of any matrix into orthogonal subspaces [6]. Let be a matrix of aligned and consecutive range profiles, $X \in \mathfrak{R}^{N \times M}$ (assuming N the number of range bins greater than the total number of profiles, M), there exists orthogonal matrices such that $X = U\Sigma V^T$. The columns of $U \in \mathfrak{R}^{N \times N}$ and $V \in \mathfrak{R}^{M \times M}$ are the i th left and i th right singular vectors respectively while σ_i are the singular values of X contained in descending order in the diagonal matrix $\Sigma \in \mathfrak{R}^{N \times M}$. The left singular vectors in U , u_i , span the orthogonal basis space in the range domain while right singular vectors in V , v_i , span the angle domain. Since HRRP represents the radar energy into the range domain, only u_i will be used. The higher the value of σ_i , the higher the contribution of its corresponding u_i in forming the target signal [6]. Therefore, SVD permits to define two subspaces: a *signal subspace*, which contains the valuable information and is spanned by the prominent singular vectors; and a *noise subspace*, which include the negligible data and is spanned by the rest of the singular vectors. The division into signal and noise subspace is defined by an energy threshold, $\eta = \frac{\sum_{i=1}^K \sigma_i}{\sum_{i=1}^M \sigma_i}$, in order to determine the most significant u_i and to discard the noise subspace. This way, denoting the *signal subspace* as X^R with the K first u_i of the test set and the u_i^s as the i th left

singular vectors of each aircraft s in the training set, the accumulated weighted angle between X^R and u_i^s is given by (1) [4]. The angle is weighted such that the singular value, σ_i^s , associated with the left singular vectors sets the significance of this angle in the final solution. Conclusively, the aircraft, s , of the training set that minimizes the angle between *signal subspaces* will be the aircraft that the radar is illuminating.

$$F_W(k) = \frac{1}{\sum_{j=1}^p \sigma_j^s} \sum_{i=1}^k \sigma_i^s \cdot \angle(X^R, u_i^s); k = 1, \dots, K \quad (1)$$

Since an energy threshold, η , is needed to define the *signal subspace*, the proposed algorithm F_W involves the issue of properly define the dimensionality of the subspace. It is difficult to determine this dimensionality, so in order to avoid it, the Generalized Mutual Subspace Method (gMSM) [5] is presented. This method introduces a *softweighting* on the basis vectors composing the subspace. The basis are combined via this *softweighting* to measure the subspace similarities without definitely setting their dimensionality. Afresh, since the objective is to keep the valuable information, first, a feature extraction technique is applied to reduce the dimensionality of the problem, in this case, the eigendecomposition. The matrix X can be decomposed by $X = V\Lambda V^T$, where $V \in \mathfrak{R}^{N \times N}$ is a matrix containing the eigenvectors and $\Lambda \in \mathfrak{R}^{N \times N}$ is a diagonal matrix containing the eigenvalues, λ_i . SVD and eigendecomposition are highly related; the nonzero singular values of X are the square roots of the nonzero eigenvalues of $X^T X$ or $X X^T$. Both techniques allow the extraction of the main features by reducing the dimensionality. Nevertheless, the fundamental difference between them is that SVD uses two different bases, U and V ; while eigendecomposition uses just one, the eigenvectors in V . Once again, the higher the eigenvalue, the higher the amount of information of the target contained in its associated eigenvector. Accordingly, eigenvectors with high eigenvalues associated will belong to the *signal subspace* and those with low eigenvalues will form *noise subspace*.

While in F_W the subspace dimensionality is defined by setting an energy threshold η , in this case the concept of *softweighting* tries to avoid this parameter. *Softweighting* gives importance to all the eigenvectors, in a way that all the eigenvalues will be weighted by a transformed value of the corresponding eigenvalues, by $\Omega = \min \left[\frac{\lambda}{\lambda_{mm}}, 1 \right]$. The m first values of the diagonal matrix Ω will be the unity and the rest will be proportionally decreasing with the m th eigenvalue. By adding the *softweighting*, the importance of each eigenvector as a basis of the subspace is set. After defining the value of parameter m and taking into account the diagonal matrix Ω of both sets, SVD is applied to $V_R^T \cdot V_S$ as in (2). Where Q'_R, Q'^T_S are the matrices of singular vectors and Θ'_{RS} is the singular value diagonal matrix whose elements represents the canonical angles' cosines.

$$\Omega_R \cdot V_R^T \cdot V_S \cdot \Omega_S \xrightarrow{SVD} Q'_R \cdot \Theta'_{RS} \cdot Q'^T_S; \quad (2)$$

Here, the similarity measure will determined by the squared trace of Θ'_{RS} , that is $s_{RS} = \text{tr}(\Theta'_{RS})^2$. The higher the s_{RS} , the higher the similarity between sets. Thus, the algorithm identifies the aircraft of the database, s , as the one with the highest similarity.

2.1. Data sets

Two sets from 7 civil aircraft in different flightpaths (Table 1) are used to the identification system. Test and Training samples are synthetically obtained with a computer software for all the orientations (aspect angles) contained in the flightpaths. In this research, HRRPs are generated by FASCRO [7], which applies high frequency techniques (Physical Theory of Diffraction, PTD, and Physical Optics, PO), to find the Radar Cross Section (RCS) of a target. The estimated trajectories come from a civil measurement campaign called ORFEO [8]. A total number of 22 trajectories are divided into frames for classification. Each frame (sequence of simulated profiles ordered in time) cover approximately 2.5° in azimuth of the aircraft aspect angle. After the split of the trajectories, the test and the training sets are defined as matrices of HRRPs of size $N \times M$ with $N = 324$ number of range bins and M number of profiles, depends on the trajectory and frame chosen. The principal hesitation of using synthetic profiles instead of measured HRRPs is that simulated profiles imply an ideal recognition scheme, since datasets have the same high quality. In order to assess the robustness, additive white Gaussian noise (AWGN) has been added to the profiles in the test set.

Table 1. Dimensions of aircraft for the RCS prediction.

	B747-400	B767-300	A310-300	MD88	Fokker 100	B737-500	Fokker 28
Length (m)	70.66	54.22	46.66	45.10	35.53	31.10	29.61
Wingspan (m)	64.44	47.52	43.90	32.80	28.08	28.90	25.07
Height (m)	16.79	14.77	12.74	7.43	6.58	9.73	6.62

3. Experimental Results

Experiments are carried out by comparison of each aircraft and frame (test set) with the database for the same frame (training set). The results are assessed and discussed here in order to define the system as accurately as possible. The main information between the 7 aircraft are contained in their respective HRRPs; normalization $L2-Norm$ is applied to both sets before the classification takes place to ensure all profiles have comparable magnitudes.

In order to define the *signal subspace* dimensionality, the parameters η for F_W and m for gMSM, have been studied to obtain the optimum values. As stated, if a high η is chosen almost all the u_i will be considered as *signal subspace*, and thus, it may contain data which in fact, is part of the *noise subspace*. In the same way, if m is very high, more u_i are treated with a *softweight* of 1. Singular vectors representing the *noise subspace* will be probably considered as part of the *signal subspace*, thus, recognition performance can be negatively affected. Accordingly, in this research, the chosen threshold for F_W is $\eta = 0.90$ and the parameter m for gMSM is $m = 1$. These values offer a trade-off between the *signal subspace* and the recognition rates.

Considering the synthetic nature of the data sets, two case studies are presented in this paper so as to validate the performance in different identification schemes. Firstly, if comparison of simulated HRRPs is carried out, F_W and gMSM give a great success rate (100%) since both sets have the same nature with a very clean signature. The most evident uncertainty is the use of synthetic profiles in both datasets, since this implies an ideal recognition scheme. In order to study the feasibility of the system, white Gaussian

noise has been added to the profiles in the test set. This way, a more real approach is simulated since profiles in the test set are observably different from those in the database. An illustration of the difference can be seen in Fig. 1 where a HRRP of an A310-300 with ∞dB (no noise) and $-10dB$ of SNR are depicted. Recognition rates for different cases of SNR are studied with the aim of demonstrating the robustness of each algorithm. A lower SNR means that more differences exist between both sets; the dominant scattering centers are reduced and additionally, when the noise is so prominent, it can be even confused with scattering centers.

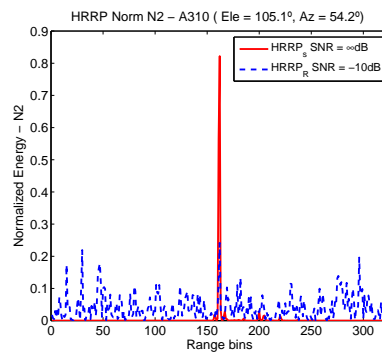


Figure 1. Variation between test and training sets

The recognition rates of 22 trajectories split into frames with a threshold of $\eta = 0.90$ and a value of $m = 1$ applying F_W and gMSM, can be seen in Table 2. As expected, if no noise is added ($SNR = \infty dB$) the methodology gives a success rate of 100%, since datasets have the same SNR. The greater the SNR, the greater similarities between both sets are obtained. This way, better recognition results are obtained (98.88% for gMSM and 98.48% for F_W), since more scatterers are visible. However, a lower SNR means that more unwanted information is present. In this study, the results are not severely influenced by noise; nevertheless when noise is highly corrupting ($SNR = -10dB$), it causes a decrease in the recognition rates (91.63% for gMSM and 87.48% for F_W). This means that, for this classification experiment, better recognition rates are obtained by applying *softweighting*. In fact, in this case, the classification performance has been improved 4% approximately by applying the gMSM algorithm. This means not only a progression in the recognition results, but also encourages us to keep validating the methodology presented in a real possible scenario.

Table 2. Recognition rates (%)

	∞dB	10dB	5dB	3dB	0dB	-3dB	-5dB	-10dB
F_W	100	98.48	96.79	96.23	94.62	92.46	90.69	87.48
gMSM	100	98.88	98.15	98.63	97.11	96.23	93.98	91.63

4. Conclusions

In this research, two metrics for NCTI have been presented and compared, F_W and gMSM. As demonstrated, the main objection of evaluating an algorithm with simulated profiles is the ideal

environment, since synthetic profiles have a very clean signature and real profiles will suffer from unwanted effects such as noise or clutter. In order to validate the feasibility of the recognition approach, additive white Gaussian noise to the test set has been applied. By introducing *softweighting* in the gMSM algorithm, identification is achieved with no need of previously setting the subspace dimension unlike the algorithm F_W , where an energy threshold is needed to define the *signal subspaces*. With the *softweighting*, an increase in the identification rates has been accomplished comparing with F_W . Considering the difference between the samples when additive white Gaussian noise is added to the test set, the identification results presented are promising and encouraging. Nevertheless, in an actual scenario, not only noise is present but also other effects. Thus, future experiments with larger sets and with additional effects will be conducted to assure the precision of the method proposed here using measurements of real targets signatures to perform classification.

Conflicts of Interest

The authors declare no conflict of interest.

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