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Using the maximum entropy production principle to constrain the value of the cosmological constant

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Abstract: The universe is dominated by a non-zero energy of the vacuum (ρ_Λ) that is making the expansion of the universe accelerate. This acceleration produces a cosmic event horizon with an associated entropy $S_{CEH} \sim \rho_\Lambda^{-1}$ [1]. Thus, the smaller the value of ρ_Λ, the larger the entropy of the event horizon. When this entropy is included in the entropy budget of the universe, it dominates the entropy of the next largest reservoir, supermassive black holes, by 19 orders of magnitude: $10^{122}\, k >> 10^{1\circ 3}\, k$. Here we address the issue of how one might apply the maximum entropy production principle (MEPP) [2] to a cosmological scenario in which ρ_Λ is treated as a variable. The growth of S_{CEH} is a maximum when the energy density of the vacuum is a minimum, greater than zero. We derive an entropy-based probability for the values of ρ_Λ and we find that low values of ρ_Λ are most probable: $P(\rho_\Lambda) \sim \frac{1}{{\rho_\Lambda}^2}\, exp\left[\frac{1}{\rho_\Lambda}\right]$. This probability distribution is an MEPP-based constraint on ρ_Λ that is independent of anthropic constraints and may help explain why the observed value of ρ_Λ is ~2 orders of magnitude lower than expectations based on a combination of anthropic constraints and quantum physics [3].

Keywords: entropy of the universe; maximum entropy production principle; cosmological constant

1. Introduction

The universe is far from equilibrium and is producing entropy. However, it cannot export this entropy to any external universe. Thus, the entropy of the universe is going up (Figure 1). But is the entropy of the universe going up at a rate that one could call a maximum rate? Does it obey the maximum entropy production principle (MEPP)? Is there a range of configurations available to the entropy-producing processes in the universe, among which the actual configuration is one that produces the maximum amount of entropy? Was there a range of ρ_Λ values available from which a most probable value emerged that maximized entropy production?

When the entropy of the cosmic event horizon [1] is included in the entropy budget of the universe (second inventory described in [4]), it dominates all other contributions. Our universe has an event horizon because the expansion of the universe is accelerating due to the vacuum energy density $\rho_\Lambda > 0$ (Eq. 2). The entropy of the supermassive black holes within our event horizon is $1.2^{+1.1}_{-0.7} \times 10^{103}\,k$ and dominates all other sources of entropy except the entropy of the cosmic event horizon which is $2.6 \pm 0.3 \times 10^{122}\,k$ [4]. Thus, the entropy of the cosmic event horizon dominates the entropy of all other sources by 19 orders of magnitude: $10^{122}\,k >> 10^{103}\,k$. The main entropy production mechanism is the growth of the entropy of the cosmic event horizon. Our goal in this paper is to summarize what is known about cosmic event horizon entropy and try to more precisely formulate the question: Is cosmic event horizon entropy production maximal?

First we describe the entropy of event horizons, then we discuss the rate of change of the entropy of the cosmic event horizon. Finally, we compute an entropy-based constraint that (in combination with anthropic constraints and quantum physics) may resolve some tension between the observed and predicted values of ρ_Λ.

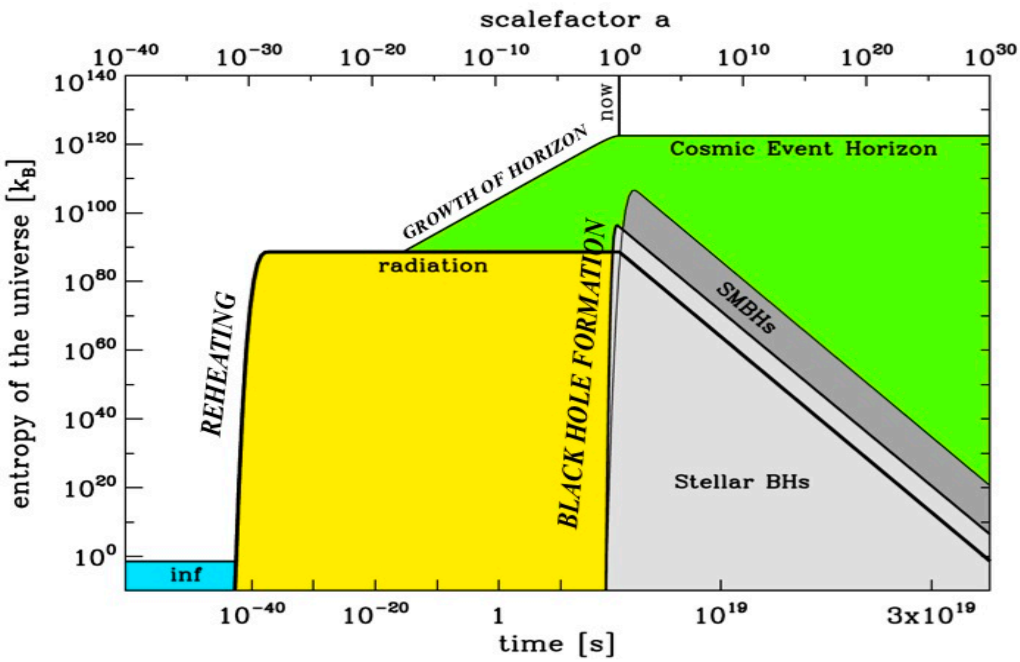

Figure 1. Entropy as a function of time, and of scalefactor a(t), in the various components of the universe within the cosmic event horizon. Reheating at the end of inflation before ~ 10^{-34} s increased the entropy to ~ 10^{90} k. The entropy of the cosmic event horizon began to dominate the entropy in radiation at about 10^{-15} s after the big bang, and has increased steadily until recently as it approaches its maximum value of ~10^{122} k in a cosmological constant dominated universe. The entropy in black holes (both stellar BHs and supermassive black holes, SMBHs) increased rapidly as a result of structure formation during the epoch of

matter domination and is still increasing. Black hole entropy will reach a peak sometime within the next few billion years and will then decrease as the comoving size of the cosmic horizon shrinks (Figure 2, top panel) and black holes cross over the cosmic event horizon carrying their entropy with them [5]. These three major processes (reheating, the growth of the cosmic event horizon and black hole formation) are summarized in Table 1. Figure modified from [4].

Table 1. Entropy production during cosmological epochs when entropy increased the most.

	Reheating and Baryogenesis	**Growth of the cosmic Event horizon**	**Black hole formation**
Entropy ΔS produced [k]	10^{90}	10^{122}	10^{112}
Time frame Δt [s]	10^{-34}	10^{17}	10^{18}
Entropy production $\Delta S/\Delta t$ [k/s]	10^{124}	10^{105}	10^{94}

$R_{CEH}(t)$ is the distance that light will be able to travel between now and the infinite future, normalized to the scalefactor at any time t'. Notice that for the future time interval $t < t' < \infty$, we have $a(t)/a(t') < 1$. The integral in Eq. 2 can be thought of as a converging infinite series, where a(t)/a(t') is the factor responsible for the convergence of an otherwise diverging integral.

2. Methods: Cosmic Event Horizons

In 1956 Rindler [6] showed that a universe with a scalefactor $a(t)$ possessed a particle horizon $R_{obs}(t)$ that defines the boundary of the observable universe (the distance that light has travelled from the big bang until now):

$$R_{obs}(t) = a(t)\, c \int_0^t \frac{dt\prime}{a(t\prime)}, \quad (1)$$

where $c\, dt'$ is the infinitesimal distance that light travels at time t'. This distance is scaled up to its current size using the ratio of the scalefactor today a(t), divided by the scalefactor at earlier times, a(t'). Since the universe is expanding, this ratio is greater than one: $a(t)/a(t') > 1$. Rindler also showed that if the condition

$$R_{CEH}(t) = a(t) \int_t^\infty \frac{c\, dt\prime}{a(t\prime)} < \infty \quad (2)$$

were satisfied, the universe possessed a cosmic event horizon at distance $R_{CEH}(t)$ from all observers.

When $\dot{a}(t)$ increases with time (when the expansion of the universe is accelerating), Eq. (2) converges. This convergence is what gives our accelerating universe a cosmic event horizon. Events beyond the cosmic event horizon will never be observed. The faster $\dot{a}(t)$ increases with t, the smaller $R_{CEH}(t)$ will be. In a purely radiation-dominated universe, $a \sim t^{1/2}$, and therefore $\dot{a}(t)$ decreases with time. Thus, the integral in Eq. (2) diverges, and there is no cosmic event horizon. In a purely matter-dominated universe,

$a \sim t^{2/3}$ and again there is no cosmic event horizon. In a purely ρ_Λ -dominated universe, $a \sim e^{Ht} \sim e^{\sqrt{\frac{8\pi G\rho_\Lambda}{3}}} \sim e^{\sqrt{\frac{\Lambda}{3}}t}$. Thus $\dot{a}(t)$ increases with time and Eq. (2) is satisfied and yields

$$R_{CEH}(t) = \sqrt{\frac{3c^2}{8\pi G\rho_\Lambda}} = \sqrt{\frac{3c^2}{\Lambda}}, \qquad (3)$$

where we have used $\Lambda = 8\pi G\rho_\Lambda$ [e.g. 7]. The smaller the cosmological constant, the greater the distance to the cosmic event horizon, and the larger the entropy of the cosmic event horizon.

The universe is not purely dominated by only one component. It has gone through three epochs and is now entering the fourth. The sequence of dominant components is: 1) the false-vacuum-energy inflation epoch dominated by $\Omega_{\Lambda_{inf}}$, 2) the radiation epoch dominated by Ω_r , 3) the matter epoch dominated by Ω_m , 4) the vacuum-energy epoch dominated by the Ω_Λ of the current universe [8,9]. Using Eq. (2) to compute the distance to the cosmic event horizon in our more complicated universe (or any hypothetical homogeneous, isotropic universe described by general relativity) with a mixture of components, we need to know the functional dependence of the scalefactor $a(t)$ on the contents of the universe. That is given by the Friedmann equation which, in a spatially flat universe is [8,9],

$$H^2 = H_o^2\left[\Omega_{\Lambda_{inf}}\, a^0 + \Omega_r\, a^{-4} + \Omega_m\, a^{-3} + \Omega_\Lambda\, a^0\right] \qquad (4)$$

where $H = \frac{\dot{a}}{a}$ is Hubble's constant and H_o is its current value. Eqs. (2) & (4) were used to plot the event horizon for our universe in Figure 2. Notice that in the lower panel of Figure 2, at early times, R_{CEH} starts out small and thus, the entropy of the cosmic event horizon was small.

In 1998, it was discovered that the expansion of the universe is accelerating [10,11]. The consensus view to explain this acceleration is that the universe is filled with a vacuum energy or cosmological constant, which can be represented either as a mass density ρ_Λ (dimensions mass/volume) or by the Greek letter Λ (dimensions time^{-2}). These are related by $\Lambda = 8\pi G\rho_\Lambda$ e.g. [7]. Some authors use $\Lambda c^2 = 8\pi G\rho_\Lambda$, e.g. [8, p 79, eq. 3.49] in which case the dimensions of Λ are length^{-2}.

Most of the growth of $R_{CEH}(t)$ occurs for scalefactors $a < 0.4$ which corresponds to a time $t \lesssim 4$ billion years after the big bang -- before the expansion of the universe started to accelerate. This is seen most easily in the bottom of the lower panel of Figure 2. This growth happens during the radiation and matter dominated epochs. However, the final value that $R_{CEH}(t)$ can grow to, is set by the constant ρ_Λ (Eq. 3). When there are no other energy components in the universe, ρ_Λ also sets the initial value of R_{CEH} which is equal to the final value (Eq. 3). Thus, the entropy cannot increase since it starts and finishes with the same horizon entropy. When there are other components (radiation and matter, as there is in our universe), the decrease in $\dot{a}(t)$ during the radiation and matter dominated epochs makes $R_{CEH}(t)$ initially small, allowing it to grow and asymptotically approach the final value set by ρ_Λ alone (Eq. 3).

When the entropy of the universe starts out low, the vacuum energy density that will maximize entropy production is one that has a minimal $\rho_\Lambda \gtrsim 0$. This ensures a maximum final value for S_{CEH} (Eq. 6). However, if there were no other radiation or matter in such a minimal-ρ_Λ universe, entropy production would be zero because the universe would have started out at maximal entropy. Thus we need some other components to ensure that the universe does not start with a large event horizon. How much of these other components is necessary to maximize entropy production? If we want the entropy to grow as fast as possible in the shortest amount of time, we need the universe to spend as short a time as possible in the radiation-dominated and matter-dominated phases. Thus we want reheating to produce a minimal value of $(\rho_r + \rho_m) \gtrsim \rho_\Lambda \gtrsim 0$.

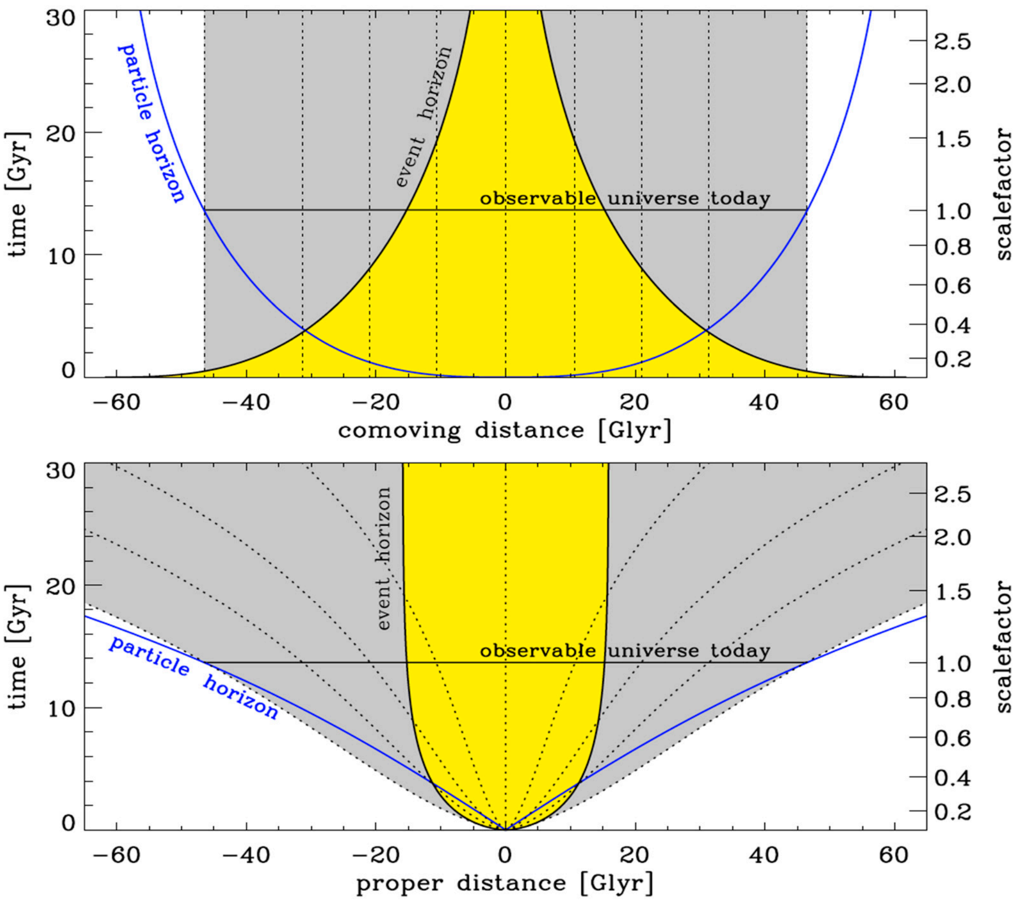

Figure 2. The size of the cosmic event horizon depends on whether one is using comoving distances (top) which do not increase with the expansion of the universe, or proper distances (bottom) which do increase with the expansion of the universe. The dotted lines represent the world lines of galaxies. They remain at constant distances from each other in comoving coordinates and separate from each other in proper coordinates because the universe is expanding. Our galaxy is the vertical line at "zero" in each panel. The age of the universe, 13.8 billion years, is indicated by the horizontal line labelled "observable universe today" denoting the current positions of galaxies that we have been able to see. The width of the yellow area in the lower panel is the diameter of the cosmic event horizon ($= 2\,R_{CEH}$). The cosmic event horizon is shrinking in comoving coordinates and this is responsible for galaxies, black holes and photons passing through the event horizon, never to be seen again. The event horizon is expanding in proper coordinates, but its expansion is slowing down. Most of its growth occurred before a scale factor a ~ 0.4. See Eqs. (1) and (2) for the definitions of the particle horizon and cosmic event horizon respectively. The scale factor "a" is shown on the right hand y-axis of each panel. Figure from [4].

The distance to the cosmic event horizon, $R_{CEH}(t)$ is generally time-dependent, increasing when the universe is dominated by an energy component with an equation of state $w > -1$ (radiation and matter) and remaining constant when the universe is ρ_Λ-dominated (assuming a cosmological constant, $w = -1$). Since our universe is presently entering ρ_Λ-domination, the growth of the event horizon has slowed, and it is almost as large now as it will ever become (bottom panel of Figure 2).

The cosmic event horizon is the source of de Sitter radiation, also characterized by a specific temperature [1,6,7,8]. It is the minimum possible temperature of the universe and is known as the de Sitter temperature T_{deS}. We can express the entropy of the cosmic event horizon as a function of mass, area, size, temperature or density:

$$entropy = f(mass) = f(area) = f(size) = f(temperature) = f(density) \qquad (5)$$

$$S_{CEH} = 4\pi \frac{kG}{c\hbar} m_{CEH}^2 = \frac{1}{4}\frac{kc^3}{G\hbar} A_{CEH} = \frac{kc^3}{G\hbar}\pi R_{CEH}^2 = \frac{1}{16\pi}\frac{\hbar c^5}{Gk}\frac{1}{\left(\frac{T_{deS}}{2}\right)^2} = \frac{3}{8}\frac{kc^5}{G^2\hbar}\frac{1}{\rho_\Lambda}. \qquad (6)$$

These parameters are not independent of each other and are related by:

$$m_{CEH} = \frac{1}{4\sqrt{\pi}}\frac{c^2}{G}\sqrt{A_{CEH}}\,, A_{CEH} = 4\pi R_{CEH}^2\,, R_{CEH} = \frac{1}{4\pi}\frac{\hbar c}{k}\frac{1}{\left({T_{deS}}/{2}\right)}, \frac{T_{deS}}{2} = \frac{1}{\sqrt{6\pi}}\frac{\hbar\sqrt{G}}{k}\sqrt{\rho_\Lambda} \qquad (7)$$

The energy density of the vacuum has been measured from cosmological observations of Type Ia supernovae and of the cosmic microwave background radiation. They yield $\rho_\Lambda = 10^{-29}\ g/cm^3$ (or since $\Lambda = 8\pi G\rho_\Lambda$, $\Lambda = 1.2 \times 10^{-35}\ s^{-2}$). Inserting this value into Eq. (6) yields the entropy of the cosmic event horizon $S_{CEH} \sim 2.6 \pm 0.3 \times 10^{122}\ k$. Inserting this value for ρ_Λ into Eq. (7) yields: $T_{deS} = 2.4 \times 10^{-30}$ K.

We are interested in trying to apply MEPP to the universe, so we are interested in the rate at which the entropy of the cosmic event horizon changes. The value of ρ_Λ determines the final and largest value of the entropy of the cosmic event horizon. So the rate of entropy production would be different depending on how large a value the entropy asymptotically approaches. In our universe, Figure 1 and the third row of Table 1 summarize the rates of entropy production: $\Delta S/\Delta t$. The universe had its highest entropy production rate ($\sim 10^{124}\ k/s$) for a very short time during reheating and produced an entropy of $\sim 10^{90}\ k$. The universe produced the largest amount of entropy ($\sim 10^{122}\ k$) after reheating, due to a large, sustained entropy production, ($\sim 10^{105}\ k/s$) as the cosmic horizon grew.

From Eq. (6) $S_{CEH}(t) \propto R(t)_{CEH}^2$. Entropy production is the time derivative,

$$\frac{dS_{CEH}}{dt} \propto R_{CEH}\frac{dR_{CEH}}{dt}. \qquad (8)$$

In a purely ρ_Λ - dominated universe with $\rho_\Lambda = constant$, the cosmic event horizon has a constant distance (see Eq. 3). Therefore, $\frac{dR_{CEH}}{dt} = 0$, entropy is constant, and in Eq. (8), entropy production is zero. Our universe is not a purely ρ_Λ - dominated universe. This allows for the early high values of $\frac{dR_{CEH}}{dt}$ seen in the lower panel of Figure 2 for $a \lesssim 0.4$.

Most cosmologists assume that some kind of symmetry breaking in the early universe allows us to treat ρ_Λ, ρ_r and ρ_m as variables that could have taken on values different from the ones they took on in our universe [e.g. 3, 12, 13, 14,15]. If ρ_Λ could have taken on a value from some range -- if ρ_Λ could have been different -- then in some sense the universe was able to explore a range of values for the cosmological constant ρ_Λ to maximize entropy production (Eq. 8).

3. Results and Discussion

There are various constraints on the possible values of ρ_Λ within an assumed ensemble of universes (the multiverse). Using a quantum cosmological approach, Hawking [16] described a distribution of values for ρ_Λ that peaked at $\rho_\Lambda = 0$. In 1989, Weinberg [3] recognized anthropic constraints on ρ_Λ:

"…if it is only anthropic constraints that keep the effective cosmological constant within empirical limits, then this constant should be rather large, large enough to show up before long in astronomical observations."

Starting with Eq. (6) $S_{CEH} = \frac{3}{8}\frac{kc^5}{G^2\hbar}\frac{1}{\rho_\Lambda}$ we obtain the derivative

$$\frac{dS_{CEH}}{d\rho_\Lambda} \propto -\frac{1}{{\rho_\Lambda}^2}\,. \qquad (9)$$

Any system unconstrained by initial conditions or evolutionary entrenched structure, is more likely to be in a high entropy state than a low entropy state because there are more microstates W available in high entropy states. Thus, the probability $\frac{dP}{dS_{CEH}}$ that S_{CEH} will take on a particular value, is proportional to the number of microstates for that value, and since $S = k\ ln\ W$, we have,

$$\frac{dP}{dS_{CEH}} \propto W(S_{CEH}) \propto exp\left[\frac{S_{CEH}}{k}\right] \propto exp\left[\frac{1}{\rho_\Lambda}\right], \qquad (10)$$

where we have used Eq. (6) in the last step. Using the substitution rule of integration applied to probability densities [17] we can write the probability $\frac{dP}{d\rho_\Lambda}$ that ρ_Λ will take on a particular value:

$$\frac{dP}{d\rho_\Lambda} = \frac{dP}{dS_{CEH}}\left|\frac{dS_{CEH}}{d\rho_\Lambda}\right|, \qquad (11)$$

which, with Eqs. (9) and (10) can be written,

$$\frac{dP}{d\rho_\Lambda} \propto exp\left[\frac{1}{\rho_\Lambda}\right]\frac{1}{\rho_\Lambda^2} \qquad (12)$$

4. Conclusions

Equation (12) is represented in Figure 3 by the green curve labelled "entropics".

Combining the upper anthropic bound ($\rho_\Lambda \lesssim 10^{-27} g/cm^3$) with the quantum physics prediction (ρ_Λ *~10^{91} g/cm^3*), suggests that ρ_Λ should take on the maximum value consistent with the anthropic bound -- ρ_Λ should be so large that galaxies would barely have had time to form before the acceleration due to ρ_Λ stops structure formation and accelerates everything beyond the cosmic horizon. However, the observed value of ρ_Λ is ~ 100 times smaller than the anthropic bound. If the anthropic bound and the quantum physics prediction were the only constraints, then ρ_Λ should be ~100 times larger than the actual observed value. Thus, there may be another constraint. The entropic constraint in Eq. (12), may be such an additional constraint. Equation (12) is an MEPP-based probability distribution and is a constraint on ρ_Λ that is independent of anthropic constraints and may help explain why the observed value of ρ_Λ is ~2 orders of magnitude lower than expectations based on a combination of anthropic constraints and quantum physics [3].

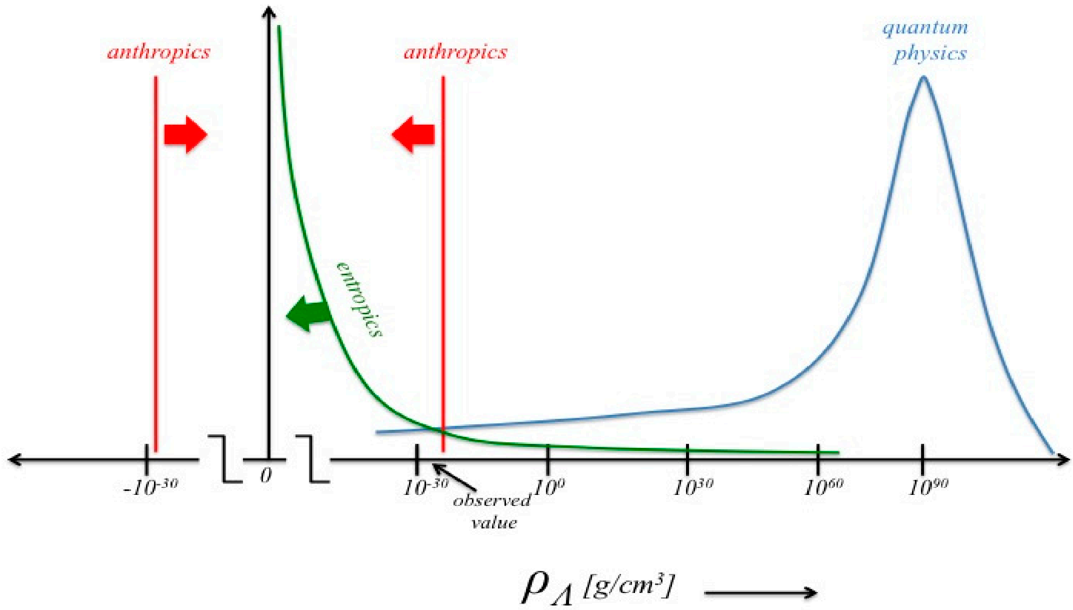

Figure 3. Notional constraints on the value of the cosmological constant. Quantum physics suggests that the energy density of the vacuum ρ_Λ should be ~10^{91} g/cm^3[e.g. 3]. This is $\sim 10^{120}$ larger than the observed value 10^{-29} g/cm^3. The blue curve labelled "quantum physics" represents this constraint. If the value of ρ_Λ is too high ($\rho_\Lambda \gtrsim 10^{-27}$g/cm^3), the acceleration of the universe begins much earlier than it did in our universe -- clouds of gas accelerate away from each other instead of collapsing. Thus there is no time for galaxies and stars to form [3]. If the value is too low, $\rho_\Lambda \lesssim -10^{-27}$ then the universe recollapses without having lasted long enough for biology or observers to evolve [12,13,14,15]. These limits are shown by the vertical red lines labelled "anthropics". The green curve labelled "entropics" represents Eq. (12), which is the result of using a maximum entropy argument to derive the probability distribution of ρ_Λ.

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Conflicts of Interest

The authors declare no conflict of interest.

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