ICNDT 2026

A simplest Approach for Plotting the Dispersion Curves in a Layered Composite Material Using the Floquet-Bloch Boundary Conditions

Sarah TALEB (sarah.taleb@g.enp.edu.dz) * 1, 2 Algeria
Rafik HALIMI (r.halimi@crti.dz) 3 Algeria
Wahiba Djerir (w.djerir@crti.dz) 3 Algeria
[1] Laboratory of Fundamental and Applied Sciences (LSFA), National Polytechnical School of Algiers, Algeria.
[2] Laboratory of Material physics (LPM), Physics department, USTHB, Algeria
[3] Research Center in Industrial Technologies (CRTI), P.O. Box 64, Algiers, Cheraga, DZ-16014 Algeria

Abstract:

Structural health monitoring via guided waves has been a major player in material inspection and damage identification, as they travel long distances with relatively low attenuation, holding valuable insight about the structural integrity of the material. Nevertheless, this method needs a pre-understanding of the dispersive nature of the waves in order for a prompt identification of any eventual damage in the structure, thus, the importance of solving the dispersion relation.
In the case of anisotropic materials, for example composites, many methods have been implemented for solving the dispersion curves and which are robust and require expertise and mathematical background to solve analytically.
In this regard, this study proposes a simplest approach using the Floquet-Bloch periodic condition (FB), which reduces the propagation problem to a unit cell, that is the smallest repeated cell possible in the structure. In other words, it considers the propagation in the first Brillouin zone only as it captures the eigenfrequencies that correspond to specific wave number values and which represent the modes propagating in the structure. As a result, Lamb waves dispersion curves in a layered CFRP composite laminate will be plotted using the Floquet-Bloch periodicity, as well as an analysis of the displacement in each layer of the CFRP.
Keywords
Composite, Dispersion, Lamb waves, Floquet-Bloch

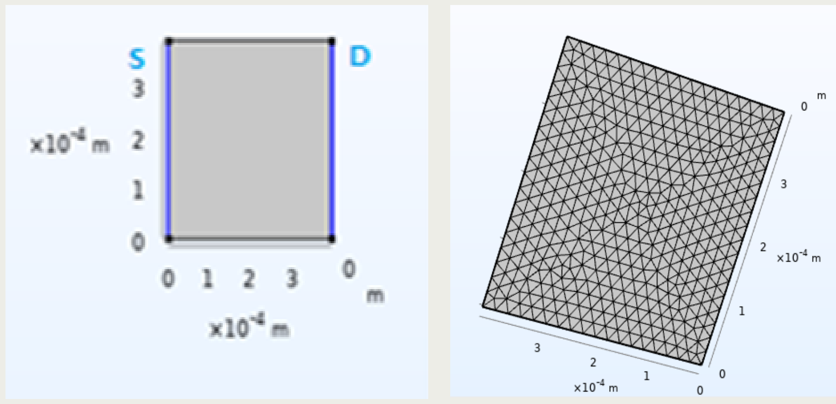
1. Solving the wave dispersion problem via Floquet Bloch periodic condition:

The classical way of plotting dispersion curves for isotropic waveguides has always been an analytical method, by finding the roots of the equation of propagation, or in the other case of anisotropic waveguides, such as composite materials by using the Transfer Matrix Method (TMM), Global Matrix Method (GMM), Stiffness Matrix Method (SMM), and Semi-Analytical Finite Element method (SAFE).
These methods still require a strong mathematical background to handle root finding of huge matrices especially in the case materials composed of many layers, each layer adds more constraints to the problem, and therefore it is often a struggle for researchers to obtain dispersion curves of composite materials.
The core idea of this work is to simplify this issue so that for any given waveguide with any geometry, dispersion curves can be easily obtained. For this reason, a Finite Element based Method was used to simulate the propagation problem using the Floquet BC wave:

$$U = U_s \;\; e^{-i\,k_f (u_d - u_s)} \qquad (1)$$

The idea is that a unit cell is chosen to imitate the behavior of all the entire waveguide [1], in other terms, instead of studying the wave propagation problem in 1 meter plate, we rather study it in the smallest repeatable unit cell possible which is the first Brillouin zone and which exhibit the same behavior if repeated through the structure, which is brilliant in term of time saving and the applicability of it for different frequency intervals .
The unit cell vcan have different geometrical shapes depending on the waveguide it self, so, if the waveguide is a plate it is chosen to be rectangular, if it is a pipe then the unit cell could be a cylinder and so on. The dimensions and the boundaries of the unit cell are the most critical in this problem as they will define the frequency range we are looking for as well as the number of propagating modes (eigen frequencies) we want to capture. By defining the boundaries of the unit cell, we confine the FB wavenumber k_f to propagate between its source (S) and to arrive at the other parallel boundary which is the destination (D), so, the displacement U_s and U_d are both at the source and destination boundaries.
The periodicity of the obtained curves is at $\frac{2\pi}{s}$ which represents one wavelength (λ), where S is the distance between the source and destination boundaries, the wave vector will consequently travel between $-\frac{\pi}{s}$ and $\frac{\pi}{s}$.
In our case, the unit cell is in itself a layered material identical to the GFRP plate, and it is important to mention that the author has found very few work that applied this method to a layered material [2].

The unit cell is chosen to be squared with a 4.10^{-4} m side, and a depth of 1mm, thus, each layer has a thickness of 1.25 10^{-4} m. Layers orientation are 0°/90°/0°/90°/0°/90°/0°/90°/0°.



(a) (b)

Figure1: (a) Source and destination for the FB periodic condition, (b): Meshing of the structurer for the FEM.

A triangular fine mesh was applied to the structure giving 782 elements in an area of 16 e^{-4} mm^2 of the unit cell, meaning that one spatial step (λ) contains 18 elements which is far superior than the minimum required and that is five per wavelength (Nyquist Limit).

2. Floquet wave number sweep and frequency response:

Once the parameters are set, we sweep over the k_f in the rage of the lattice length $\boldsymbol{s}$. The Floquet wave acts as a mechanical perturbation that causes the structure to vibrate (figure 2) on its intrinsic modes which represents the eigen frequencies we are looking for.

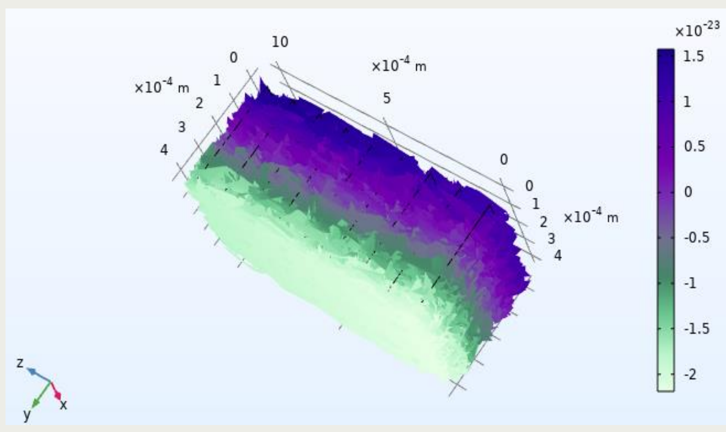


Figure 2 lattice subjected to stresses

Naturally, as the GFRP is composed of eight layers, the Floquet wave will propagate in each layer with different intensities, as it moves forwards inside the structure, some of its energy gets dissipated along the way. From a layer to another, since layers have different orientations, the wave is subjected to multi-reflections and thus the displacements differ in shape and intensity.

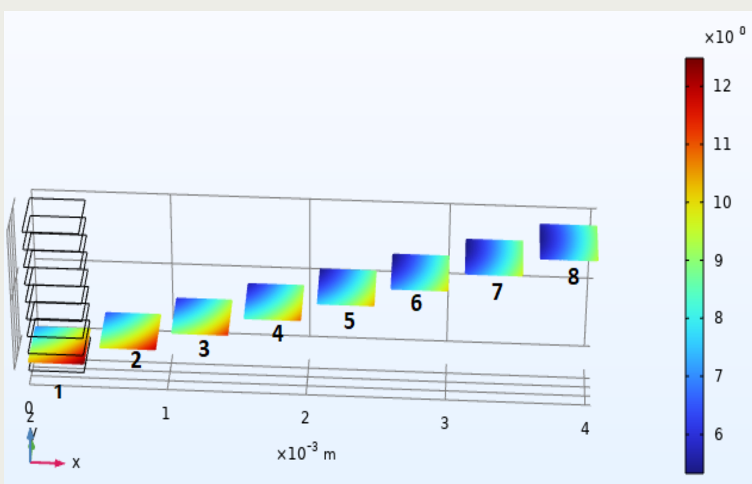


Figure3: Displacement magnitude in each layer of the GFRP.

3. Mode extraction:

By applying a parametric sweep over the floquet wavenumber k_f in the range within the length of the unit cell, it becomes easier to extract all the eigen frequencies propagating in the structure. The search of the latter is performed around a frequency shift where the value can be adjusted based on the frequency range as well as the number of modes desired by the study. In our case, for the GFRP, we've searched around a 200 kHz frequency, and perceived 8 propagating modes in the structure.
The wavenumber values are exactly those of k_f and the eigenfrequencies obtained allow us to plot the corresponding dispersion curves:

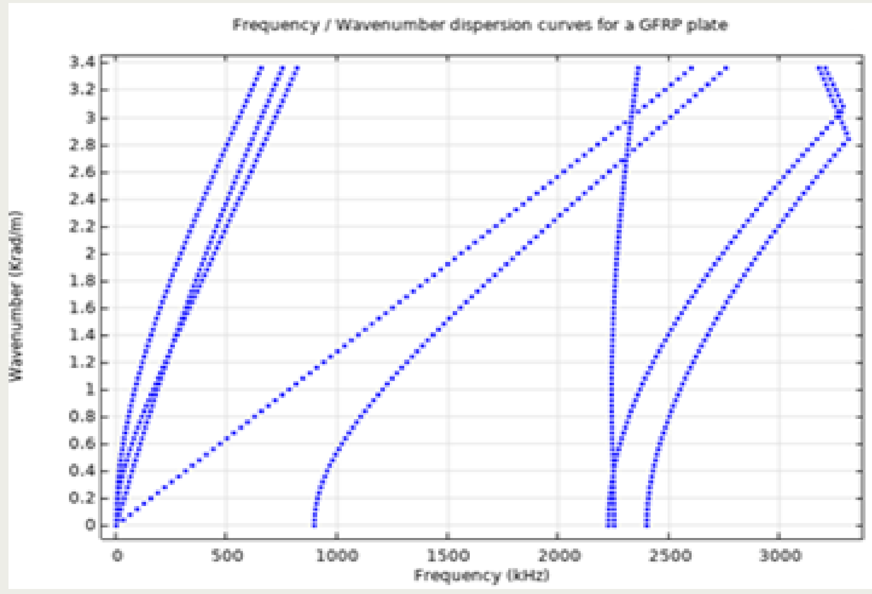


Figure 4: Wavenumber-frequency dispersion curves of a 1mm GFRP plate.

Since the material studied here is a GFRP with 8 layers, the propagation is way more complex than those of simple or lower layers plate. We can see the appearance of SH modes, which can be recognized by their very fast propagation and high wavenumber value, the SH0 fundamental mode is completely non dispersive, then we have the appearance of superior the Symmetric and antisymmetric Lamb modes with higher cut off frequencies.

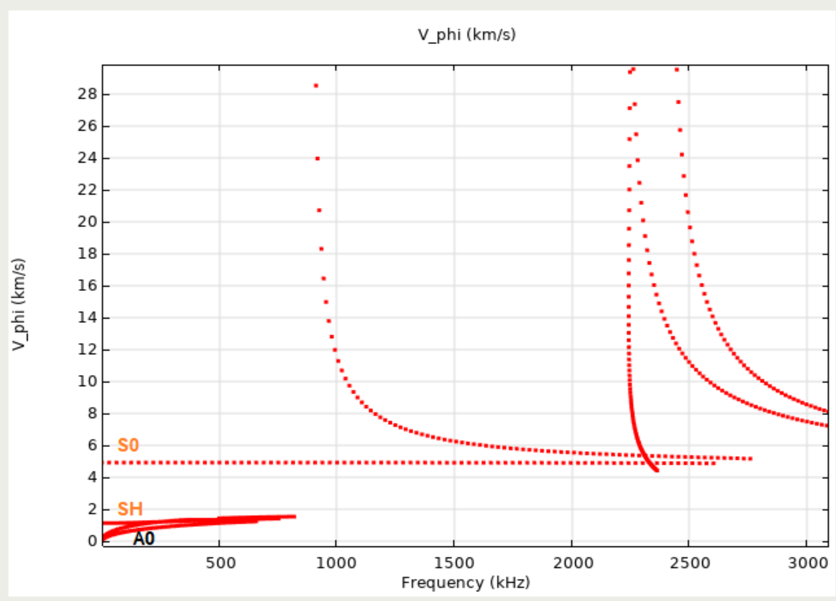


Figure 5: Phase velocity-frequency dispersion curves of a 1mm GFRP plate.

Based on the values of the wavenumber, phase velocity can consequently be driven from the equation:

$$v_{phi} = \frac{2\pi f}{k_f}$$

As show in figure 5, Lamb modes are propagating in the structure at relatively low frequencies and velocities compared to the SH modes, which are in the superior part of the curve.

4. Conclusion:

This works presents a very simple approach of obtaining the dispersion curve of any material and any shape, compared to the analytical complex methods. We have been able to use the FB unit cell periodicity that presents the same solutions as the whole structure to retrieve the eigen frequencies on which the structure vibrate; the dispersion curves were therefore plotted and modes identified.

References:
[1] Hakoda C, Rose J, Shokouhi P, Lissenden C (2018) Using floquet periodicity to easily calculate dispersion curves and wave structures of homogeneous waveguides. In: AIP Conference Proceedings, AIP Publishing LLC, vol 1949, p020016
[2] Gomez Garcia P, Ferna´ndez-A´ lvarez JP (2015) Floquet-bloch theory and its application to the dispersion curves of
nonperiodic layered systems. Mathematical Problems in Engineering 2015(1):475364