

A Robust Numerical Framework for Ultrasonic Wave Characterization in Complex Petrophysical Environments

Ujjal Mandal

Department of Applied Physics, University of Gour Banga, Malda, WB, 732124

INTRODUCTION & AIM

Ultrasonic wave propagation serves as a critical diagnostic tool in petrophysics for characterizing the internal micro-geometry and fluid saturation states of reservoir rocks. Recent advancements in numerical modeling have shifted towards k-space pseudospectral methods, such as the k-Wave framework, due to their superior ability to handle high-frequency wave propagation in complex, heterogeneous media with minimal numerical dispersion and computational cost. In a petrophysical context, the interaction between ultrasonic waves and rock heterogeneity—primarily pores, micro-fractures, and mineral inclusions—triggers significant scattering and attenuation. Research indicates that attenuation coefficients and velocity dispersion are strongly frequency-dependent, governed by mechanisms such as wave-induced fluid flow (squirt-flow) and matrix an elasticity at the core scale. Furthermore, the relationship between ultrasonic velocity and petrophysical parameters like porosity and inclusion density remains a cornerstone of rock physics. Studies on shale and carbonate rocks demonstrate that velocity is inversely proportional to porosity and highly sensitive to the aspect ratio of micro-cracks, necessitating the use of sophisticated simulation models to invert these properties accurately. However, a significant research gap exists in the seamless integration of high-resolution k-Wave simulations with multi-scale experimental laboratory data. Current literature emphasizes that numerical results, such as contrast ratios and effective attenuation coefficients, must be rigorously validated against semi-analytical methods like the modified Born approach or the Convergent Born Series (CBS) to ensure the reliability of petrophysical interpretations.

METHOD

A heterogeneous rock model was developed using the k-Wave pseudospectral framework to simulate 1 MHz ultrasonic wave propagation. The computational domain consisted of a 128×128 grid with a fluid-saturated host matrix (2500 m/s) containing five low-velocity circular inclusions (350 m/s) representing pores and vugs. A 51-element linear source array was used for excitation, and the steady-state acoustic field was analyzed using a 16384-point FFT. To account for scattering and frequency-dependent attenuation, the Convergent Born Series (CBS), Modified Born Approach (MBA), and fractional viscoacoustic modeling were incorporated. The numerical results were validated through comparison with MBA predictions. The propagation of ultrasonic waves in a heterogeneous rock medium was modeled using the k-Wave k-space pseudospectral framework. Under the assumptions of a lossless, homogeneous, and isotropic medium, acoustic wave propagation is governed by the classical wave equation:

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -S(\mathbf{r}, t)$$

where p is the acoustic pressure, c is the sound speed, and $S(\mathbf{r}, t)$ represents the ultrasonic source term.

The k-Wave solver employs coupled first-order conservation equations for momentum and mass transport: A heterogeneous rock model was constructed on a 128×128 computational grid, consisting of a fluid-saturated host matrix ($v_f = 2500 \frac{m}{s}$) and five circular inclusions ($v_s = 350 \frac{m}{s}$) representing pores and vugs. Wave excitation was generated using a 51-element linear ultrasonic source operating at 1 MHz. The steady-state acoustic field was extracted through a 16384-point FFT analysis. To account for scattering effects, the Modified Born Approach (MBA) was incorporated through the scattering potential:

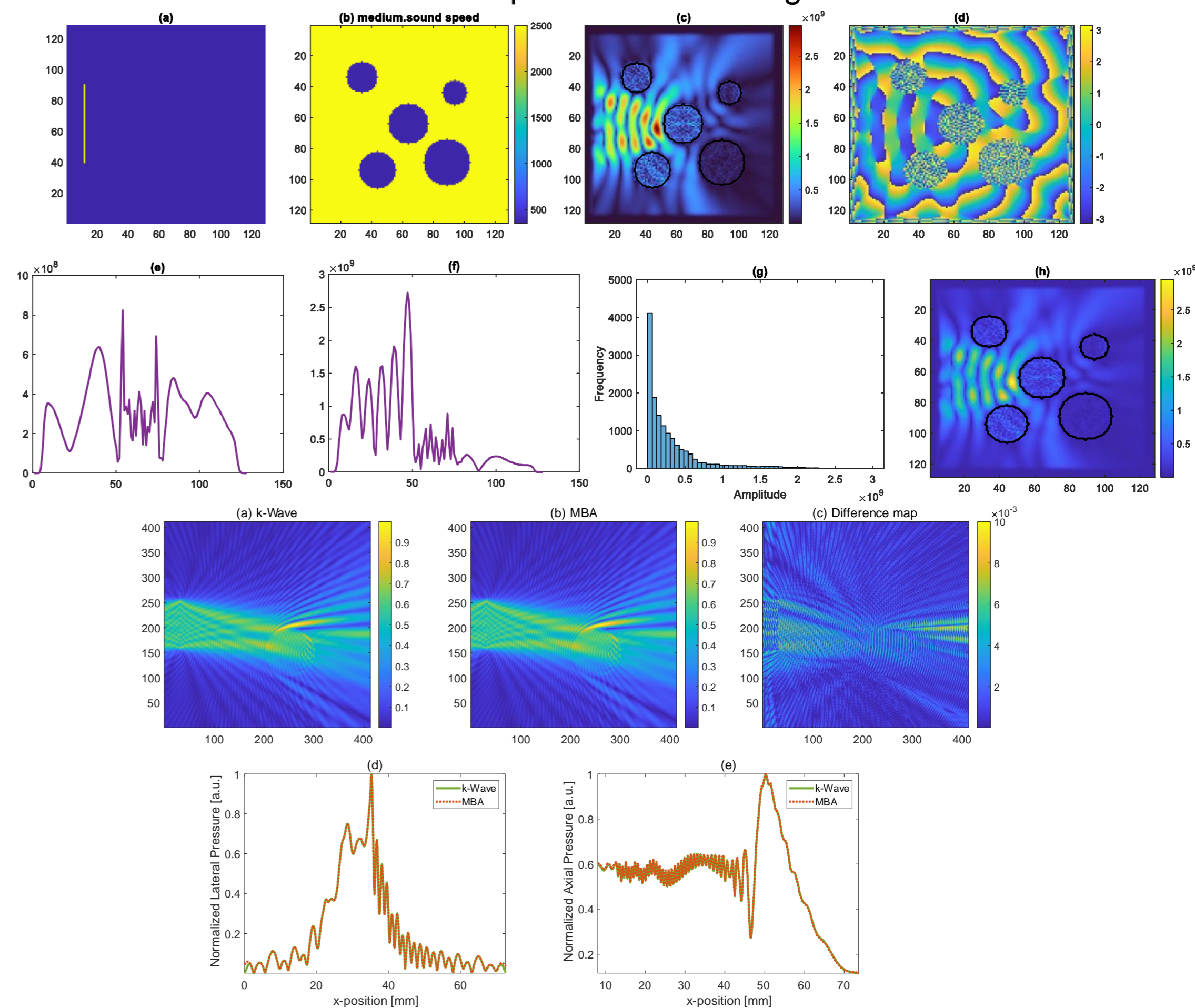
$$V(\mathbf{r}) = k_s^2 - k_f^2 - i\epsilon$$

RESULTS & DISCUSSION

The simulated ultrasonic wave field revealed strong interactions between the propagating wavefront and the embedded rock heterogeneities. Significant scattering, diffraction, and phase distortion were observed as the incident wave encountered the low-velocity inclusions. The steady-state amplitude maps showed the formation of pronounced acoustic shadow zones behind the scatterer cluster, indicating substantial energy redistribution within the medium.

Quantitative analysis demonstrated a **Contrast Ratio (CR) of 3.51 dB**, confirming the capability of the framework to distinguish inclusions from the surrounding matrix. The **effective attenuation coefficient was found to be 421.95 dB/m**, reflecting the combined effects of intrinsic absorption and multiple scattering. Furthermore, a **Transmission Coefficient of 0.3391 (-9.39 dB)** indicated that nearly 66% of the incident ultrasonic energy was either attenuated or scattered by the inclusions.

The accuracy of the proposed model was evaluated through comparison with the Modified Born Approach (MBA). An excellent agreement was obtained, with a numerical discrepancy of **less than 1%**, validating both the stability and reliability of the k-Wave-based framework. These results demonstrate the effectiveness of the proposed methodology for non-destructive characterization of porous and heterogeneous rock formations.



CONCLUSION

A robust k-Wave-based numerical framework was developed to investigate ultrasonic wave propagation in heterogeneous rock media. The model successfully captured scattering, attenuation, and shadow-zone formation caused by rock heterogeneities. Validation against the Modified Born Approach (MBA) showed excellent agreement with an error below **1%**, demonstrating the accuracy and reliability of the proposed method for non-destructive petrophysical characterization.

FUTURE WORK / REFERENCES

Future work will focus on extending the framework to 3D simulations and integrating it with experimental ultrasonic measurements for real rock-core characterization.

[1] Ghimire, S.; Sabri, F. K-wave modelling of ultrasound wave propagation in aerogels and the effect of physical parameters on attenuation and loss. Appl. Phys. A 2023.