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Optimal Stochastic Control of Pension Asset Sustainability for Ghana's Basic National Social Security Scheme

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Introduction & Aim

Pension systems around the world are increasingly exposed to ageing populations, changing labour-market dynamics and economic uncertainty [1,2]. In Ghana, the Tier 1 (Basic National Social Security Scheme) of the three-tier pension system (Fig. 1), administered by the Social Security and National Insurance Trust (SSNIT), is a defined-benefit scheme that currently faces growing pressure from rising benefit obligations, evolving contributor–retiree ratios and volatile investment returns [2,3,4]. Although successive reforms have strengthened governance and expanded pension coverage [3], actuarial projections continue to raise concerns about the scheme's long-term sustainability. Recent actuarial valuations indicate declining sustainability indicators (Fig. 2) and the potential for reserve depletion [2]. To address these challenges, we develop a stochastic optimal control framework that jointly optimizes investment allocation and pension indexation within a 30-year horizon in balancing benefit adequacy with sustainability of the fund.

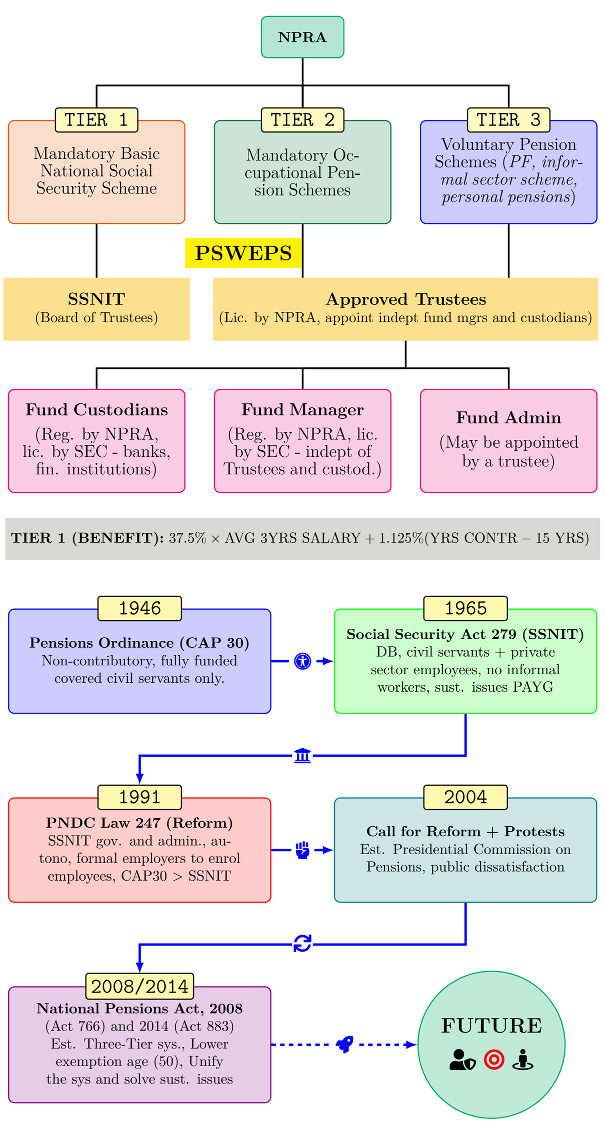

Figure 1: New pension system, reformations and issues over the years.

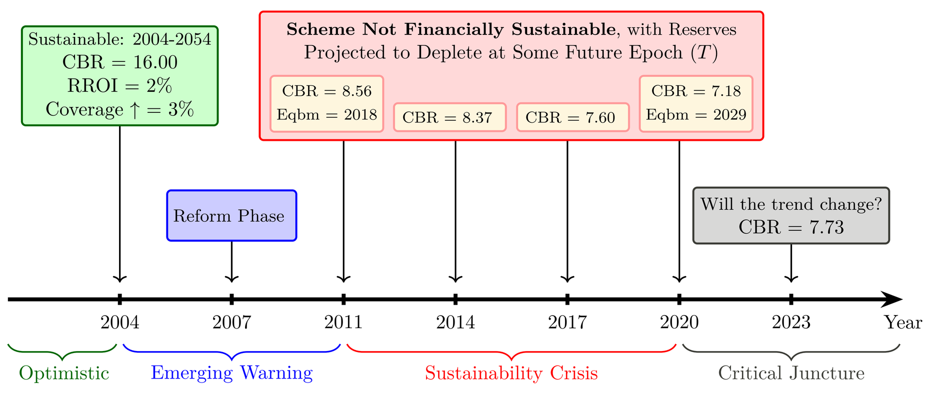

Figure 2: Timeline of Triennial Actuarial Valuation of SSNIT (Tier 1)

Methodology

The pension fund evolves under two trustee-controlled policy variables: the risky asset π_t and the benefit indexation factor β_t. The objective is to maximize retirees' expected discounted welfare and preserve the long-term solvency of the pension fund.

Model & Optimal Control Setup

The asset (A_t), and the salary processes (S_t) are defined on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in[0,T]}, \mathbb{P})$, where $\{\mathcal{F}_t\}$ is the information available to the trustee. (*See Table 1 for all parameter names and values.*)

State Dynamics

$$\begin{cases} dA_t = \left[\pi_t \dfrac{dE_t}{E_t} + (1-\pi_t)\dfrac{dP_t}{P_t}\right] A_t + (C_t^a - B_t^r)dt, \\ dE_t = \mu_t E_t dt + \sigma_t E_t dZ_t^E, \;\; E_0 = e, \\ dP_t = r_t P_t dt, \;\; P_0 = p, \\ C_t^a - B_t^r = \left[\kappa W_t - \bar{b}(1+\beta_t)R_t\right] S_t, \\ dS_t = \alpha_t S_t dt + \nu_t S_t^{\varphi} dZ_t^S, \;\; S_0 = s, \varphi \neq 1, S_t \sim \text{C.E.V.}, \\ \dfrac{dW_t}{dt} = (\eta - \lambda - \phi)W_t, \quad W_0 = w, \\ \dfrac{dR_t}{dt} = \phi W_t - \delta R_t, \quad R_0 = z. \end{cases} \tag{1}$$

where $\mathbb{E}[dZ_t^E dZ_t^S] = \rho\, dt$, $\mathbb{E}[(dZ_t^E)^2] = \mathbb{E}[(dZ_t^S)^2] = dt$. The equation (1) therefore yields

$$dA_t = \left[r_t A_t + (\mu_t - r_t)\pi_t A_t + (\kappa W_t - \beta_t \bar{b} R_t)\, S_t\right] dt + \pi_t \sigma_t A_t\, dZ_t^E \tag{2}$$

with initial and boundary conditions: $A_0 > \xi$, $S_0 > 0$, $W_0 \geq 0$, $R_0 \geq 0$ and solvency floor $A_t \geq \xi$ with ruin time $\tau_\xi = \inf\{t : A_t \leq \xi\}$

Now, we derive the optimal investment policy π_t and indexation β_t that maximize the expected discounted utility of pension benefits and the fund's sustainability. The objective becomes $J^{\pi,\beta}(A,S,t) = \mathbb{E}_{A,S,t}\left[\int_t^T e^{-\rho(u-t)}\, U(B_u^r)\, du \; + \; e^{-\rho(T-t)}\, \tilde{U}(A_T)\right]$ with the value from the objective given as $V(A,S,t) = \sup_{(\pi,\beta)\in\mathcal{U}} J^{\pi,\beta}(A,S,t)$. The value is the *maximal discounted welfare* attainable from state (A,S) at time t under the admissible policies $\mathcal{U} = (\pi_t, \beta_t)_{t\in[0,T]}$. This is given fully in (3)

$$V(A,S,t) = \sup_{(\pi,\beta)\in\mathcal{U}} \mathbb{E}_{A,S,t}\Big[\int_t^T e^{-\rho(u-t)} U(B_u^r)\, du + e^{-\rho(T-t)} \tilde{U}(A_T)\Big] \tag{3}$$

Using the Dynamic Programming Principle on (3) yields the Hamilton–Jacobi–Bellman (HJB) equation

$$0 = \sup_{(\pi,\beta)} \left\{V_t + \mathcal{L}^{\pi,\beta} V + U(B_t^r) - \rho V\right\}, \tag{4}$$

where $\mathcal{L}^{\pi,\beta}$ denotes the infinitesimal generator of the controlled state process. The $\mathcal{L}^{\pi,\beta} f$ where $f = f(A,S,t)$ with $f \in C^{2,1}(\mathbb{R} \times [0,T])$ together with (4) result in the Hamiltonian, $\mathcal{H}(\pi,\beta \mid A,S,t) = (\mu - r)\pi A\, V_A + \frac{1}{2}(\pi\sigma A)^2\, V_{AA}$ $+ \rho_{ES}\, (\pi\sigma A)(\nu S^{\varphi})\, V_{AS} - (\bar{b}[1+\beta]R) S\, V_A$ $+ U(\bar{b}[1+\beta]RS)$ which consequently gives the optimal policies $(\pi^*(A,S,t), \beta^*(A,S,t))$.

Solvency Constrained Probability

The survival probability (under the HJB feedback policy (π^*, β^*)) is defined as $p(A,S,t) := \mathbb{P}^{\pi^*,\beta^*}_{A,S,t}(\tau_\xi > T)$, $\tau_\xi := \inf\{u \geq t : A_u \leq \xi\}$. Using the Backward Kolmogorov PDE (Dynkin's formula), we obtain $p_t + \mathcal{L}^{\pi^*,\beta^*} p = 0$ on $(\xi, A_{\max}) \times (S_{\min}, S_{\max}) \times [0,T)$, where $\mathcal{L}^{\pi^*,\beta^*}$ is the generator with π, β frozen at the feedback policy.

Numerical Results

Table 1: Real annualized values of the calibrated model parameters in (1) and (2).

Parameter	Symbol	Value
Investment horizon	T	30
Discount rate	ρ	1.24%
Risk-free rate	r	1.24%
Equity return	μ	5.39%
Equity volatility	σ	26.43%
Salary drift (CEV)	μ_S	−3.54%
Salary volatility	ν	17.32%
Volatility elasticity	φ	0.80
Correlation	ρ_{ES}	0.10
Contribution rate	κ	11.00%
Indexation corridor	$[\beta_{\min}, \beta_{\max}]$	$[0.00, 0.30]$

Parameter	Symbol	Value
Replacement factor	$\bar{b}$	0.64
Solvency floor	ξ	2.75
Initial assets	A_0	6.88
Initial salary	S_0	28,019.51
Worker inflow rate	η	0.08
Non-retirement attrition	λ	0.02
Retirement incidence	ϕ	0.015
Retiree exit rate	δ	0.04
Terminal utility floor	ε	10^{-8}
Initial retirees	R_0	244,810
Initial contributors	W_0	1,951,494

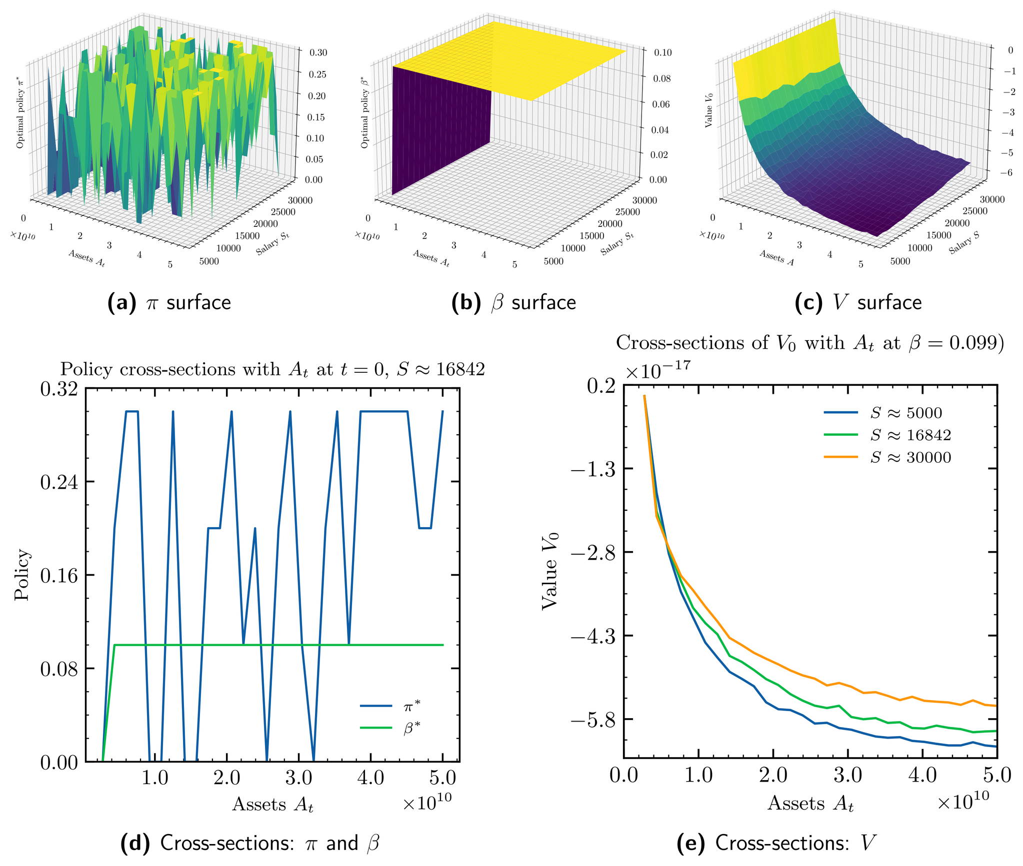

Figure 3: Surfaces and cross-sections of π, β and V. The optimal combination of equity–fixed income investments policies is found to be $\pi = 19.46\%$ and $\beta = 9.90\%$.

We undertook some policy modifications to observe their overall shock on the fund and optimal policies. We raised the contribution rate at 0.5% incrementally and saw at the rate of 11.5%, π^* and β^* were 10% and 15% respectively. Benefits doubled from 26.72B to 52.19B. However, contribution rate beyond 12% saw benefit improvement remain the same as compared to the rate at 11.5% except improved sustainability. Similarly, we allowed contributor growth (worker coverage expansion) at 0.5% and benefits improved from 26.72B to 36.24B with indexation increasing to 12% (*see* Tables 2 and 3).

Table 2: Policy results $\kappa = 11.5\%$ for asset allocations $\pi = 10, 20$ and 30%.

Asset Allocation	10/90			20/80			30/70		
	β								
	10%	12%	15%	10%	12%	15%	10%	12%	15%
Terminal fund	20.39	15.56	8.32	23.34	18.21	10.57	26.70	21.23	13.22
VaR (A_T)	15.49	11.10	4.22	13.25	9.11	2.75	10.97	7.16	2.75
Expected shortfall	14.35	10.10	3.47	11.31	7.46	2.75	8.60	5.20	2.75
Contribution	6.99	6.99	6.99	6.99	6.99	6.99	6.99	6.99	6.98
Benefit	26.79	35.10	52.19	26.79	35.10	51.99	26.79	35.08	51.42
Survival Time	30.00	30.00	30.00	30.00	30.00	29.94	30.00	29.99	29.80
Prob. of success	1.000	1.000	0.990	1.000	1.000	0.949	0.999	0.995	0.908

Note: Monetary values are in billions of local currency unit. Contributions and Benefits $\mu \wedge m = \sup\{x \in \mathbb{R}^+ : x \leq \mu \text{ and } x \leq m\}$.

Table 3: Policy results of 0.5% contributors growth for asset allocations $\pi = 10\%, 20\%$, and 30%.

Asset Allocation	10/90			20/80			30/70		
	10%	12%	15%	10%	12%	15%	10%	12%	15%
Terminal fund	17.65	12.62	5.27	20.29	14.96	7.30	23.30	17.64	9.69
VaR (A_T)	13.29	8.73	2.75	11.30	6.93	2.75	9.26	5.20	2.75
Expected shortfall	12.25	7.76	2.75	9.55	5.50	2.75	7.15	3.76	2.75
Contribution	7.18	7.18	7.18	7.18	7.18	7.17	7.18	7.18	7.15
Benefit	27.67	36.24	53.43	27.67	36.24	52.61	27.67	36.21	51.54
Survival Time	30.00	30.00	29.89	30.00	30.00	29.69	30.00	29.98	29.42
Prob. of success	1.000	1.000	0.845	1.000	0.998	0.813	0.999	0.986	0.778

Note: Monetary values are in billions of local currency units. Contributions and Benefits represent $\mu \wedge m = \sup\{x \in \mathbb{R}^+ : x \leq \mu,\ x \leq m\}$.

Conclusion

The proposed stochastic optimal control identified about 20% equity–80% fixed-income allocation with 10% pension indexation as the optimal strategy for improving the long-term sustainability of Ghana's Basic National Social Security Scheme, while moderate increases in contributions and contributor growth will further strengthen fund resilience.

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