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A Two-Split, Three-Peak Fourier Extension of the Lee–Carter Mortality Model

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INTRODUCTION & AIM

Addressing the Limitations of the Lee–Carter (LC) Model
The traditional LC model assumes mortality parameters are constant over time, which often leads to significant errors in forecasting mortality for older age groups.

Understanding Longevity Risk
Longevity risk is the potential for individuals to live longer than expected, which can destabilise pension systems and insurance pricing if not accurately modeled.

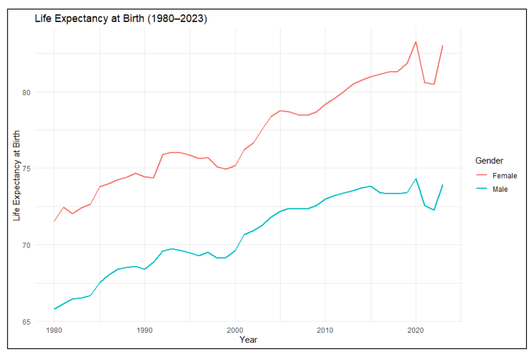

Figure 1: Improving trend in life expectancy at birth and the sharp dip during the COVID-19 era

Goal: A Flexible Fourier Extension
The aim is to implement a Fourier series extension that captures complex, non-linear mortality dynamics across different age groups and time periods.

Equation (1): The Baseline Lee–Carter (LC) Model

$$\ln m_{x,t} = a_x + b_x k_t + \varepsilon_{x,t}$$

METHOD

The Two-Split, Three-Peak Fourier Extension

1980 — 2023

Data and Cohort Analysis
The model utilizes high quality Malaysian abridged mortality rates (ages 0–75) from 1980 to 2023, analysed via 24-year overlapping sub-cohorts

Equation (2): The Proposed Three-Segment Fourier Series

$$b_{x,t} = \begin{cases} a_{1,t} + A_{1,t}\sin\left(2\pi f_{1,t}x + P_{1,t}\right) & x \le x_1 \\ +a_{2,t} + A_{2,t}\sin\left(2\pi f_{2,t}x + P_{2,t}\right) & x_1 < x \le x_2 \\ +a_{3,t} + A_{3,t}\sin\left(2\pi f_{3,t}x + P_{3,t}\right) & x > x_2 \end{cases}$$

Visualizing Age Sensitivity (b_x)

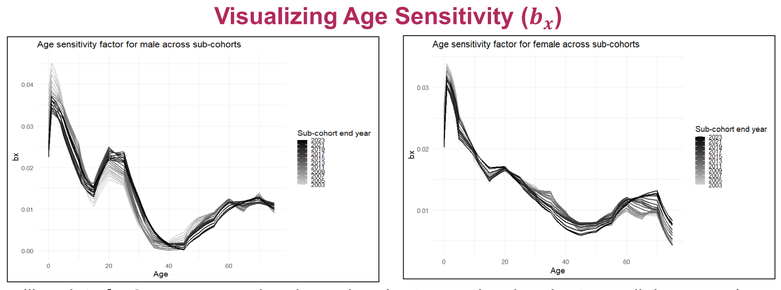

Figure 2: The wave-like plots for b_x across overlapping sub-cohorts are showing the two splitting ages (around 15 and 42/45) and the multi-peak dynamics

Length (year)	Male a_1	Male A_1	Male A_2	Male A_3	Female a_1	Female A_1	Female A_2	Female A_3
10	0.535	0.229	0.295	0.171	0.334	0.314	0.277	0.629
20	0.341	0.171	0.131	0.111	0.127	0.103	0.121	0.420
24	**0.277**	0.218	**0.118**	**0.096**	**0.123**	0.122	**0.091**	**0.248**

Table 1: Selection of a 24-year sub-cohort length showing an overall low coefficient of variation

RESULTS & DISCUSSION

Superior Forecasting Accuracy
The Fourier extension consistently yields lower MAPE than the baseline Lee–Carter model, with the same best in-sample and out-of-sample performance for both males and females. It also produces lower mortality estimation errors, particularly at older ages.

Enhanced Fit for Age-specific Patterns
The model successfully captures multi-peak dynamics, offering a more refined understanding of longevity risk for both male and female populations.

Future Longevity Projections (2024-2043)
Projections indicate higher life expectancy estimates, suggesting the model captures smoother long-term mortality improvements missed by period estimates.

In-sample	Out-sample	Male MAPE (LC)	Male MAPE (Fourier)	Female MAPE (LC)	Female MAPE (Fourier)
1980-2006	2007-2023	18.27	8.45	16.17	13.83
1980-2007	2008-2023	17.53	8.41	15.99	13.79
1980-2008	2009-2023	16.69	8.22	15.31	13.14
1980-2009	2010-2023	16.23	8.43	15.01	13.07
1980-2010	2011-2023	15.8	8.59	15.44	13.73
1980-2011	2012-2023	15.49	8.86	15.50	14.26
1980-2012	2013-2023	15.20	8.80	15.66	14.86
1980-2013	2014-2023	14.56	8.14	12.98	11.75
1980-2014	2015-2023	14.06	8.26	12.71	11.74
1980-2015	2016-2023	12.60	9.01	12.36	11.80
1980-2016	2017-2023	**12.02**	**7.87**	12.16	**11.40**
1980-2017	2018-2023	12.19	8.25	**12.13**	11.44
1980-2018	2019-2023	12.47	9.10	12.13	11.71

Table 2: Fourier extension consistently outperforms LC in MAPE across all in/out-sample lengths, with identical optimal periods for males and females

Age	Male LC	Male Fourier	Female LC	Female Fourier
65	3.57	3.49	6.64	5.05
66	3.97	3.80	9.36	6.63
67	4.80	4.37	12.25	8.43
68	5.52	5.12	15.05	10.82
69	6.15	5.81	17.41	13.64
70	6.70	6.49	19.44	16.59
71	5.59	5.19	18.53	13.66
72	4.71	4.23	17.89	11.82
73	4.00	3.77	17.43	10.71
74	3.68	3.76	17.08	10.14
75	3.93	4.22	16.81	9.95
Average	4.78	**4.57**	15.26	**10.68**

Table 3: Comparison of MAPE for fitted mortality rates at older ages where Fourier outperforms LC for both males and females

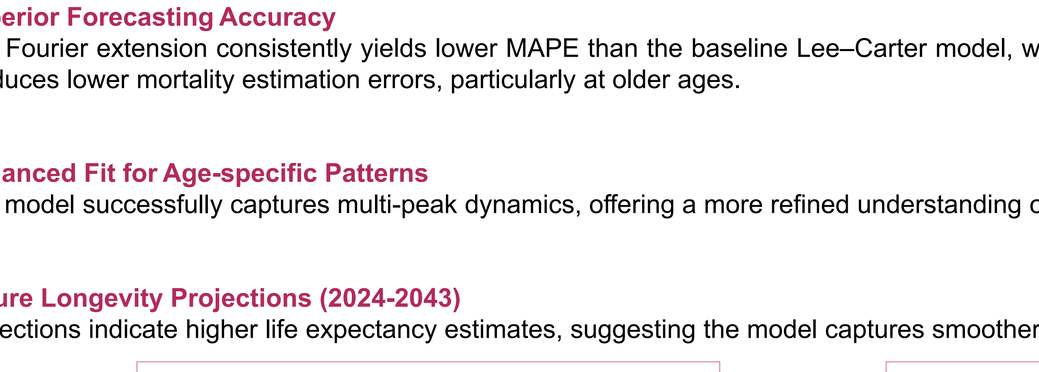

Figure 3: The small-multiple graphs of the LC-fitted vs. Fourier-fitted b_x show how closely the Fourier extension tracks the original model while capturing finer details

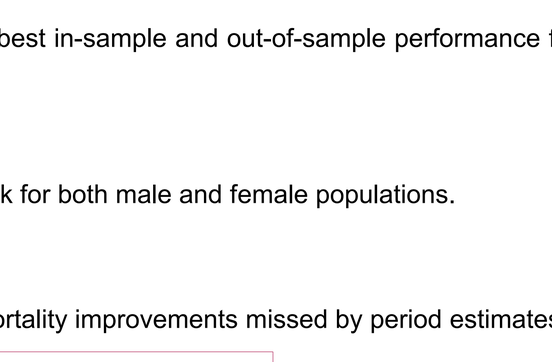

Figure 4: The Fourier-forecasted life expectancy at birth produces higher estimates and suggests that the model captures smoother long-term mortality improvements, particularly at older ages

CONCLUSIONS

Enhanced Fit for Policy Planning
The three-peak approach provides a better representation of mortality dynamics, offering crucial insights for public health planning and social security reforms.

Economic Resilience
Sophisticated mortality forecasting enhances national economic stability by providing reliable estimates for long-term fiscal planning and social welfare sustainability.

FUTURE WORK/ REFERENCES/ACKNOWLEDGMENT

Expansion to Asia-Pacific
Future research aims to apply this two-split, three-peak model to other populations in the Asia-Pacific region to test its robustness across different mortality regimes.

Impact on Annuity Pricing
Investigations will continue into how this improved model affects capital requirements and retirement income adequacy.