

Evaluating Dynamic Hedging Strategies for American Options in Markets with Jumps

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INTRODUCTION & AIM

- American put options are difficult to hedge in markets with jumps because sudden price movements can generate large hedging errors.
- Closed-form continuous-time hedging solutions are generally unavailable under jump-diffusion dynamics.
- This study uses a discrete-time lattice framework and backward dynamic programming to examine dynamic hedging strategies.
- The main aim is to compare global quadratic hedging, local risk minimization, and Delta-Gamma hedging.
- Particular attention is given to hedging performance under extreme market movements.
- Standard externally imposed exercise rules are compared with an internal exercise rule linked to the current hedging portfolio value.

METHOD

- Continuous-time jump-diffusion models are approximated by a discrete-time lattice framework.
- A finite set of possible outcomes is considered at each time step.
- American put option pricing and hedging decisions are solved by backward dynamic programming.
- Three hedging strategies are compared: Delta-Gamma hedging, local risk minimization, and global quadratic hedging.
- The analysis is implemented under both Merton and Kou jump-diffusion models.
- Model parameters are calibrated using S&P 500 and Bitcoin data.
- Hedging performance is evaluated using the distribution of terminal hedging errors.
- Main performance measures include mean error, standard deviation, VaR, CVaR, and tail losses.
- Two exercise rules are examined: an external exercise rule and an internal rule linked to the hedging portfolio value.

RESULTS & DISCUSSION

Hedging Performance

- Higher strike prices require larger initial hedging portfolios
- Delta hedging performs well on average but has weaker tail-risk protection.
- Asymmetric local hedging reduces VaR and CVaR across strikes, showing stronger downside protection.
- The results confirm a cost–risk trade-off: better tail protection requires higher initial capital.

Table 1. Hedging error statistics for the S&P 500 under the MJD model.

Strategy	K	Init.	Exp.	Std.	VaR _{95%}	CVaR _{95%}	VaR _{99%}	CVaR _{99%}
Delta	95	2.197	1.023	2.097	4.857	7.561	9.213	11.931
	100	3.827	0.938	2.366	5.255	7.767	9.414	11.607
	105	6.266	0.609	2.504	5.273	7.674	9.038	11.369
Global quadratic	95	2.622	1.090	2.033	4.576	6.620	7.832	10.231
	100	2.108	2.477	2.338	7.195	9.777	11.504	13.800
	105	4.340	3.000	2.345	7.450	10.174	11.815	14.315
Asymmetric local	95	3.649	-0.065	2.110	3.269	5.124	6.764	8.833
	100	5.517	-0.264	2.450	3.836	5.656	6.957	8.987
	105	8.025	-0.244	2.434	4.242	6.268	7.214	9.248

Adding Third Asset

Table 2. Impact of adding a third hedging asset under the MJD model.

Strategy	Init.	Exp.	Std.	VaR _{95%}	CVaR _{95%}	VaR _{99%}	CVaR _{99%}
Delta-gamma	3.826	1.013	1.754	3.972	5.915	7.240	9.100
Global quadratic	2.108	2.469	2.157	6.589	8.337	9.242	11.096
Asymmetric local	5.517	-0.7605	2.837	3.765	5.660	6.677	8.740

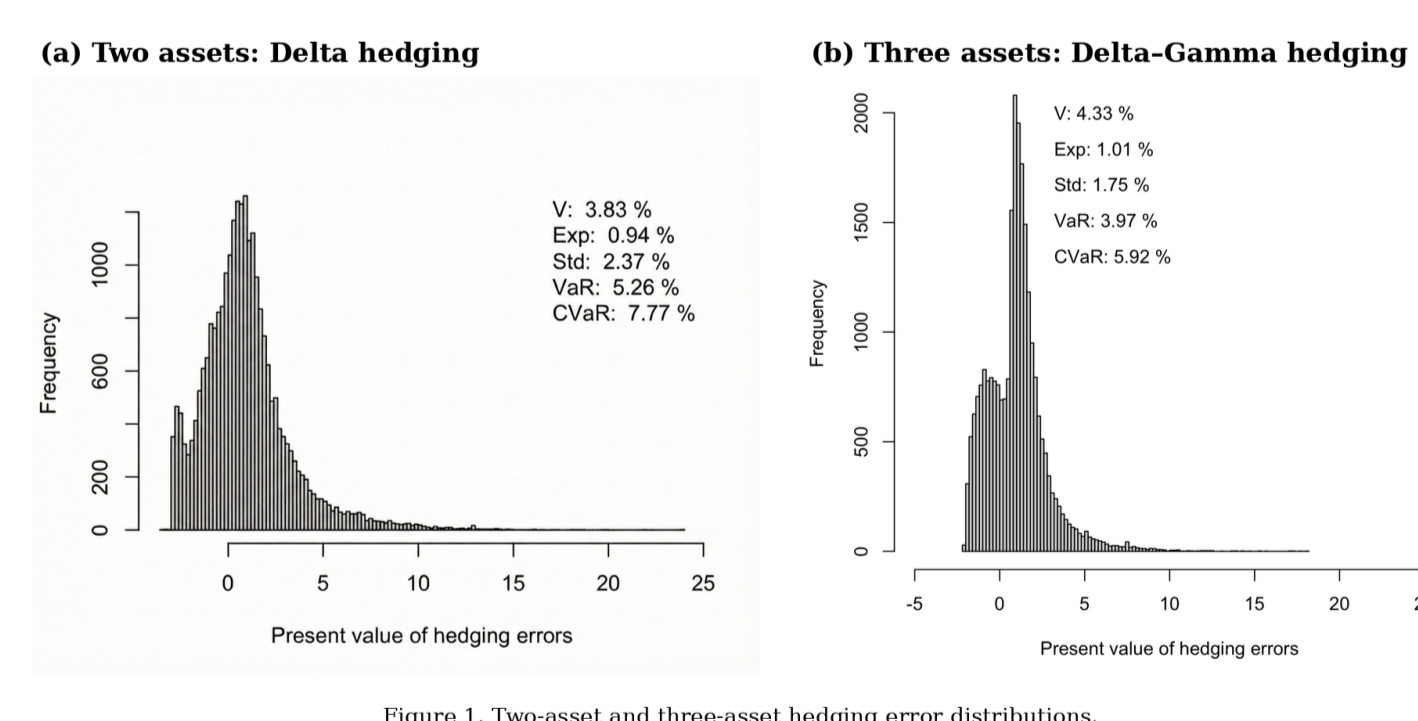


Figure 1. Two-asset and three-asset hedging error distributions.

CONCLUSIONS

- Delta hedging gives smaller average errors but weaker protection against extreme losses.
- Asymmetric local risk minimization provides stronger downside protection, especially under jump risk and early exercise uncertainty.
- Under the internal exercise rule, ζ affects both hedge cost and exercise timing: cheaper hedges lead to earlier exercise and weaker tail-risk protection.
- Model choice matters: Kou often improves downside-risk results, especially for BTC, but no universal ranking is observed.

Hedger's Preference

- The asymmetry parameter ζ controls the hedger's preference between higher initial cost and downside-risk protection.
- A smaller ζ gives a more conservative hedge with higher initial capital but lower VaR and CVaR.
- A larger ζ reduces the initial hedging cost but increases tail-risk exposure.
- The results show a clear cost–risk trade-off: stronger downside protection requires a more expensive hedge.

Table 3. Effect of the hedger's preference parameter under the MJD model.

ζ	Init.	Exp.	Std.	VaR _{95%}	CVaR _{95%}	VaR _{99%}	CVaR _{99%}
0.2	8.990	-3.817	2.383	1.247	2.203	3.558	5.539
0.5	5.517	-0.264	2.450	3.836	5.656	6.957	8.987
0.8	4.281	0.902	2.386	5.090	7.221	8.268	10.676

Internal Decision

- Under the internal exercise rule, the stopping decision is linked directly to the hedging portfolio value.
- A smaller ζ gives a more expensive hedge and higher portfolio value, which delays exercise and improves tail protection.
- A larger ζ lowers the initial hedging cost but makes exercise easier and increases VaR/CVaR.
- The results show a feedback effect: the hedging strategy affects both the hedging error distribution and the exercise mechanism.

Table 4. Internal exercise decision under the MJD model.

ζ	Init.	Exp.	Std.	VaR _{95%}	CVaR _{95%}	VaR _{99%}	CVaR _{99%}
0.2	10.177	-3.413	2.387	0.423	2.065	3.069	4.872
0.5	5.737	0.377	2.239	4.329	6.157	7.225	9.107
0.8	4.303	1.206	2.315	5.336	7.454	8.555	10.839

FUTURE WORK/ REFERENCES/
ACKNOWLEDGMENT

- Incorporate transaction costs to make the hedging framework more realistic.
- Extend the analysis to stochastic volatility or GARCH-type models with jumps.
- Jointly optimize early exercise and hedging decisions in incomplete markets.