Dynamic EVT-Copula Models with Endogenous linkages between Extreme Losses and Dependence Structures

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INTRODUCTION & AIM

The Global Financial Crisis and the COVID-19 pandemic have led to an increased focus on the need to model extreme losses and the systemic risk in interconnected financial markets with accuracy. Most traditional risk models are inadequate at taking into account the presence of dynamic dependence between assets and the fact that rare catastrophic events tend to exhibit heavy-tailed properties. The models of Extreme Value Theory (EVT) model tail risks well and the models of Copula models describe complex dependence structures. Most of the current EVT–Copula methods, however, consider the extreme losses and dependence structure separately. he proposed model can better reflect contagion effects, crisis amplification and systemic interconnectedness, and offer better assessment of financial market stability and exposure to extreme risk.

- ➢ Model extreme financial losses using dynamic EVT and Generalized Pareto Distributions (GPD).
- ➢ Develop adaptive volatility-based threshold selection for tail-event detection.
- ➢ Capture time-varying and asymmetric dependence using dynamic copulas.
- ➢ Introduce an endogenous mechanism where extreme losses directly influence dependence dynamics.
- ➢ Extend the framework to high-dimensional financial systems using vine copulas.
- ➢ Estimate systemic risk measures such as VaR, ES, CoVaR, and MES.
- ➢ Improve understanding of contagion, crisis propagation, and systemic amplification in financial markets.

METHODS

- ❖ **Data Collection and Preprocessing**
  Collect financial asset return data.
  Transform returns into loss series for risk analysis
- ❖ **Volatility Filtering**
  Apply the GARCH model to capture volatility clustering and obtain standardized residuals.
- ❖ **Dynamic Threshold Selection**
  Determine adaptive thresholds based on changing market volatility conditions.
  Identify extreme observations exceeding the threshold.
- ❖ **Extreme Value Modeling**
  Model exceedances using the Peaks-Over-Threshold (POT) approach.
  Fit a Generalized Pareto Distribution (GPD) with time-varying parameters.
- ❖ **Marginal Distribution Transformation**
  Transform the fitted marginal distributions into uniform variables using the (PIT).
- ❖ **Dynamic Copula Construction**
  Dynamic copula models to capture nonlinear and tail dependence among financial assets.
- ❖ **Endogenous Linkage Mechanism**
  Introduce mechanism where extreme losses directly influence dependence dynamics.
- ❖ **GAS-Based Dependence Updating**
  Update copula parameters through a Generalized Autoregressive Score (GAS) framework.
- ❖ **High-Dimensional Extension**
  Extend the model using Vine Copulas to analyze large interconnected financial systems.
- ❖ **Systemic Risk Estimation**
- ❖ **Model Validation**

RESULTS & DISCUSSION

In this paper we proposed the model for the endogenous linkage between extreme losses and dependence structures. The extreme loss extraction of the continuously compound return is defined as $R_{i,t} = \log\left(\frac{P_{(i,t)}}{P_{i,t-1}}\right)$. The EVT focuses on extreme losses, returns are transformed into $L_{i,t} = -\log\left(\frac{P_{(i,t)}}{P_{i,t-1}}\right)$. The marginal modelling for the sufficiently high threshold of the excess distribution converges to a GPD from Mario (2004) and Basavaraj and A S Talawar, (2025).

$$\mathrm{F}_{\mathrm{U}_{\mathrm{i,t}}} = \begin{cases} 1-\left(1+\frac{\xi_{i,t}x}{\beta_{i,t}}\right)^{-\frac{1}{\xi_{i,t}}} & if\,\xi_{i,t} \neq 0 \\ 1-e^{\left(-\frac{x}{\beta_{i,t}}\right)} & if\,\xi_{i,t} \neq 0 \end{cases}$$

The time varying severity of extreme losses is

$$l_i = \sum_{t=1}^{T}\left\{-\log(\beta_{i,t}) - \left(\frac{1}{\xi_{i,t}}+1\right)\log\left(1+\frac{\xi_{i,t}x}{\beta_{i,t}}\right)\right\}$$

The equation of the copula modelling of dependence structure of the marginal CDF is

$$f_i(y_i) = C_t(U_{1,t}, U_{2,t}, U_{3,t} \dots U_{N,t})\left(\prod_{i=1}^{N}\frac{1}{\beta_{i,t}}\left(1+\frac{\xi_{i,t}(y-u_{i,t})}{\beta_{i,t}}\right)^{\left(1-\frac{1}{\xi_{i,t}}\right)}\right)$$

The endogenous linkage mechanism between the extreme losses and dependence structures is

$$L = \sum_{t=1}^{T}\left\{\log C_t\left(U_{i,t}\colon \omega + \alpha\theta_{t-1} + \gamma\sum_{i=1}^{N}\omega_i I_{i,t}(Y_{i,t}-U_{i,t})\right) + \sum_{i=1}^{N}\log\left(\frac{1}{\beta_{i,t}}\left(1+\frac{\xi_{i,t}(y-u_{i,t})}{\beta_{i,t}}\right)^{\left(1-\frac{1}{\xi_{i,t}}\right)}\right)\right\}$$

Based on this we derived some important key components of copula evolution, dependence copula, marginal value at risk (MVaR), expected shortfall (ES), system level loss, conditional value at risk (CVar), marginal expected shortfall (MES), Tail dependencies and extreme risk index (ERI). After this we used two techniques for the estimation purpose. Those are Inference function for margins (IFM) and Canonical maximum likelihood (CMLE). The derived equations are:

$$-\frac{n_t}{\beta_t} + \frac{1+\xi_{i,t}x}{\beta_{i,t}^2}\sum_{t=1}^{T}\left(\frac{\beta_{i,t}x_{i,t}}{\beta_{i,t}+\xi_{i,t}x_{i,t}}\right) = 0 \text{ and } (\hat{\omega}, \hat{\alpha}, \hat{\beta}, \hat{\gamma}) = argmax\sum_{t=1}^{T}\log\left(C_t(U_{1,t}, U_{2,t}, U_{3,t} \dots U_{N,t}\colon \theta_t)\right)$$

Here we use the recent five years dataset for the practical implementation purpose from SENSEX, NIFTY 50, BANK NIFTY, S&P 500 and NASDAQ. First, we estimate the GPD marginal parameters. The results are given in Table.1:

Table.1: Estimated GPD Marginal Parameters

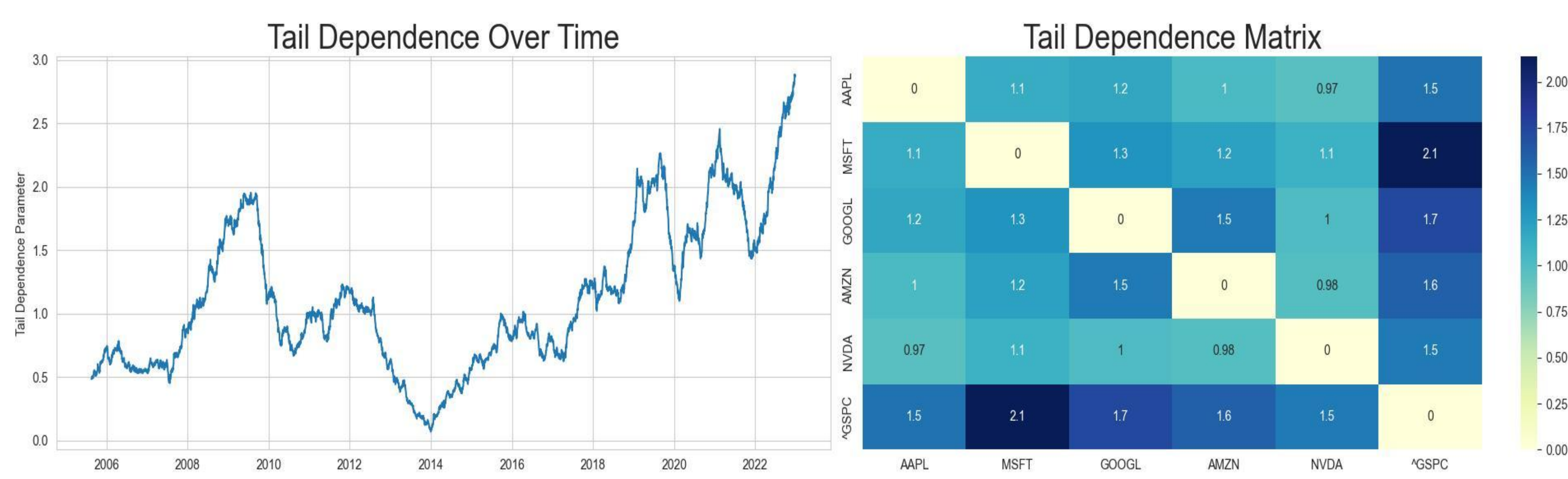
Index	Threshold u	Shape ξ	Scale β	Tail dependencies
Nifty 50	2.35%	0.182	1.247	0.82
Bank Nifty	2.95%	0.241	1.486	0.88
Sensex	2.82%	0.175	1.196	0.54
S&P 500	3.50%	0.213	1.352	0.61
NASDAQ	2.99%	0.267	1.621	0.91

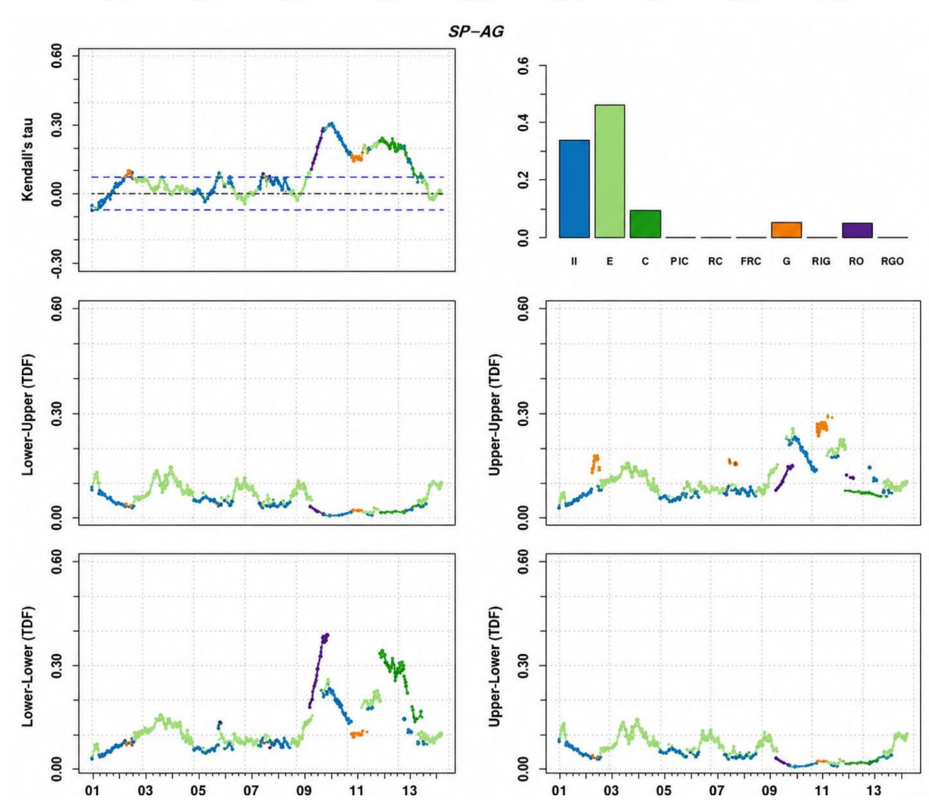
Table.1 shows that all five stock indices exhibit heavy-tailed loss behavior, as indicated by the positive GPD shape parameters ($\xi > 0$), confirming the presence of extreme market losses. Among them, NASDAQ has the highest shape ($\xi = 0.267$) and scale ($\beta = 1.621$) parameters, suggesting the greatest severity of extreme losses, while Nifty 50 and Sensex exhibit comparatively lower tail risk. Furthermore, NASDAQ (0.91) and Bank Nifty (0.88) display the strongest tail dependencies, indicating a higher likelihood of joint extreme market movements during periods of financial stress.

Table 2. Comparative systemic risk measures

Index	VaR (95%)	ES (95%)	MES	CVaR	ERI
Nifty 50	3.87	5.44	5.76	6.32	0.64
Bank Nifty	5.12	7.38	8.11	8.96	0.82
Sensex	3.74	5.12	5.38	6.01	0.61
S&P 500	4.41	6.31	6.98	7.55	0.73
NASDAQ	5.63	8.27	9.15	10.42	0.91

Fig.1: Dynamic Tail Dependence and Cross-Market Dependence Matrix.





The results indicate that tail dependence increases significantly during periods of market stress, confirming stronger co-movements and contagion effects among financial assets under extreme conditions. Furthermore, the tail dependence matrix and TDF analysis reveal substantial interconnectedness and asymmetric dependence, particularly among major technology stocks and the S&P 500, highlighting the importance of dynamic EVT–copula models for systemic risk assessment and extreme-loss transmission. The Kendall's tau and Tail Dependence Function (TDF) plots demonstrate time-varying and asymmetric dependence, with upper-tail dependence generally exceeding lower-tail dependence, implying that markets tend to experience extreme gains simultaneously.

CONCLUSIONS

The proposed Dynamic EVT–Copula framework integrates extreme value modelling, time-varying dependence structures, and endogenous feedback effects into a unified systemic risk modelling approach. By allowing extreme losses to influence the evolution of dependence among financial institutions, the framework effectively captures financial contagion, crisis amplification, and systemic interconnectedness. The model provides a more realistic representation of market dynamics during periods of stress, where tail events strengthen dependence and increase systemic vulnerability.

Its ability to quantify extreme risk transmission and evolving interdependencies makes it a valuable tool for systemic risk assessment, stress testing, and macroprudential regulation. Overall, the framework enhances the understanding of financial crises and offers improved capabilities for monitoring and managing systemic risk in complex financial systems.

FUTURE WORK/ REFERENCES/ACKNOWLEDGMENT

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