Breakage and entropy

Fractal distribution in nature

Questions arisen from literature

- *Why fractal distribution is so frequent?*
- *It is true that fractal dimension is always less than 3?*
- *Is there an ultimate distribution?*
- *Why fractal distribution with a fractal dimension of around 2.5–2.6 seems to be steady-state or stable?*

Entropy notions

- classical entropy of thermodynamics (S_c) (Clausius (1865))
- statistical formulation of the classical entropy of thermodynamics (S_f),
- statistical entropy (S_s)
- entropy of information theory (S_I)
- grading entropy

Entropy principle

The directional properties of natural or spontaneous processes ('entropy principle'):

- In the classical sense, the second law of thermodynamics states: "In any closed system the entropy remains constant or increases during the ongoing processes " (Breuer, 1993).

- *It is true that grading entropy can be related to the entropy principle?*

- **Basic assumption: Largest fraction does not disappear**

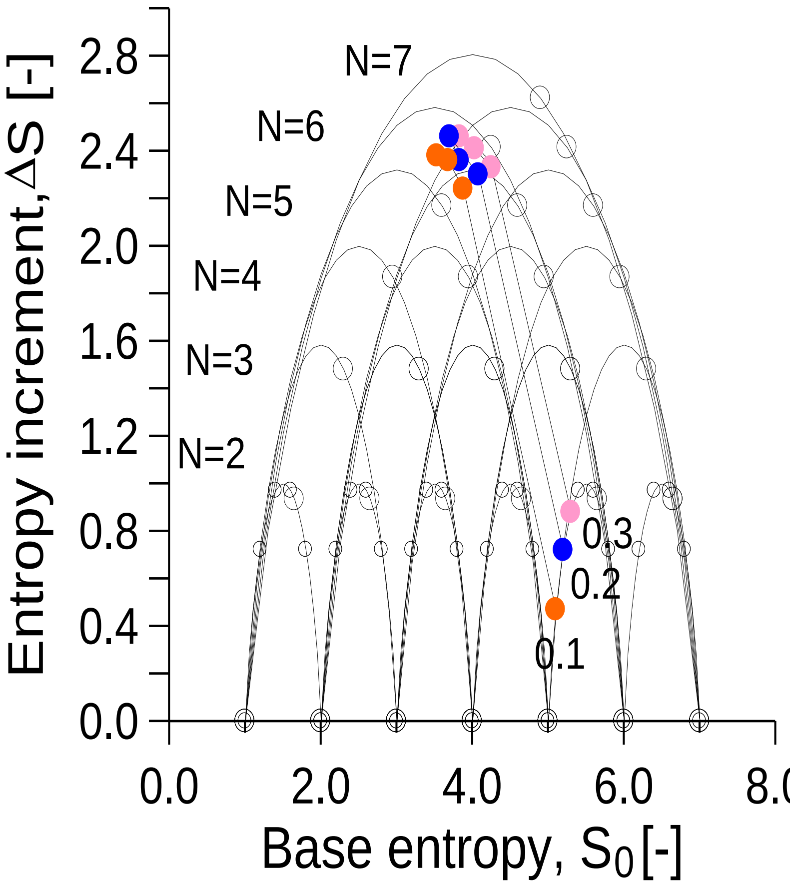
Hard grains

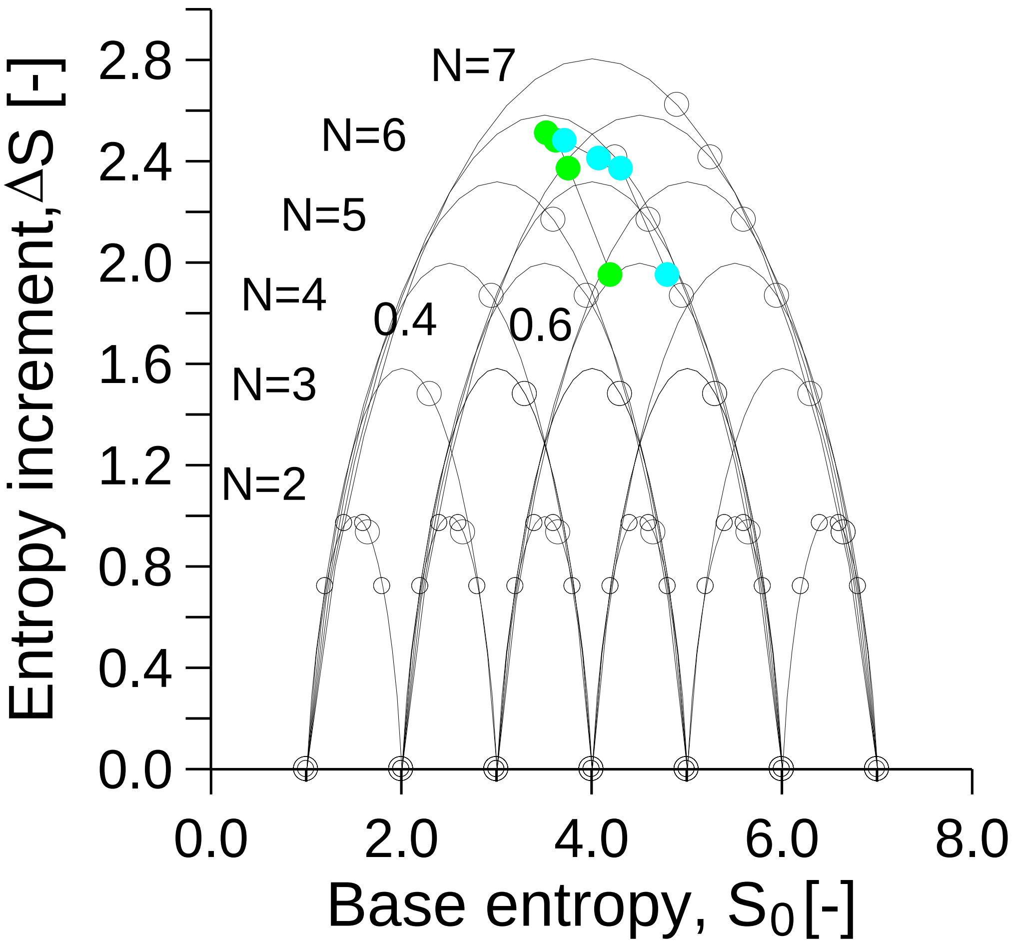
Sand samples:

- One fraction sand
- Concave fractal
- Convex fractal
- Silica, carbonate
- Each sample was subjected to a series of crushing treatments using a special reinforced crushing pot, made at the Geotechnical Department with the dimensions: diameter: 50 mm, height: 70 mm, wall thickness: 3mm.

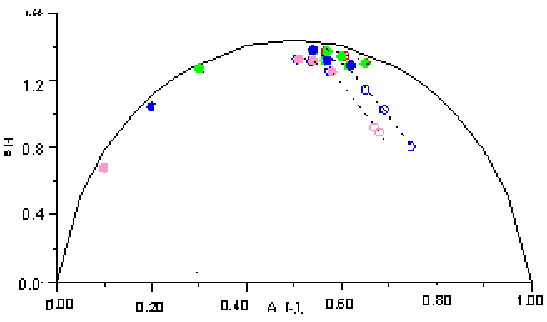
Oedometer pot

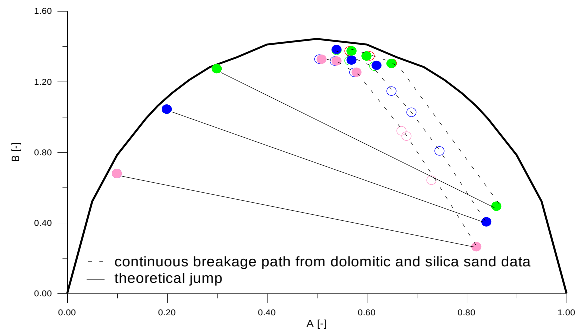
The continuous grading curve path in the non-normalised diagram



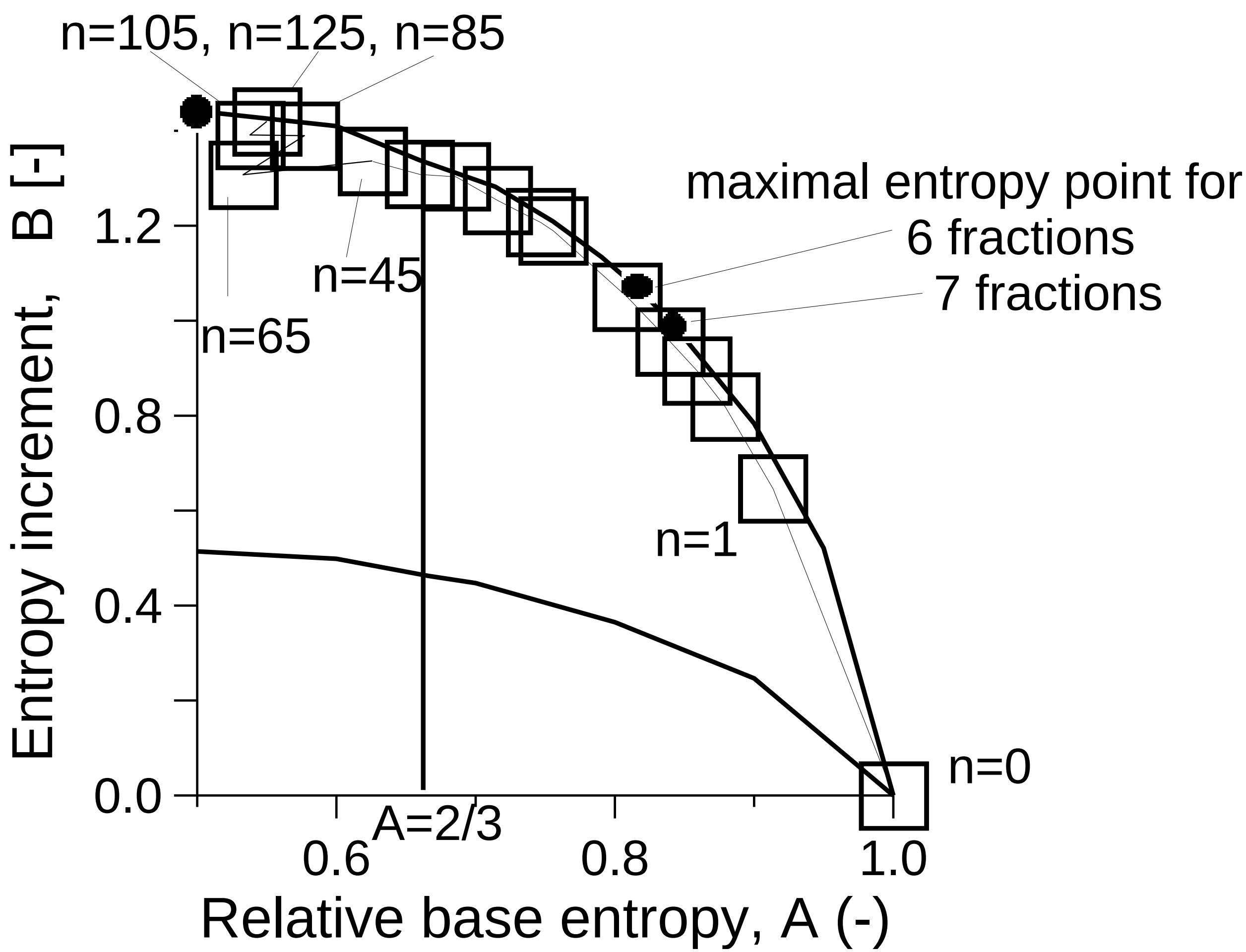


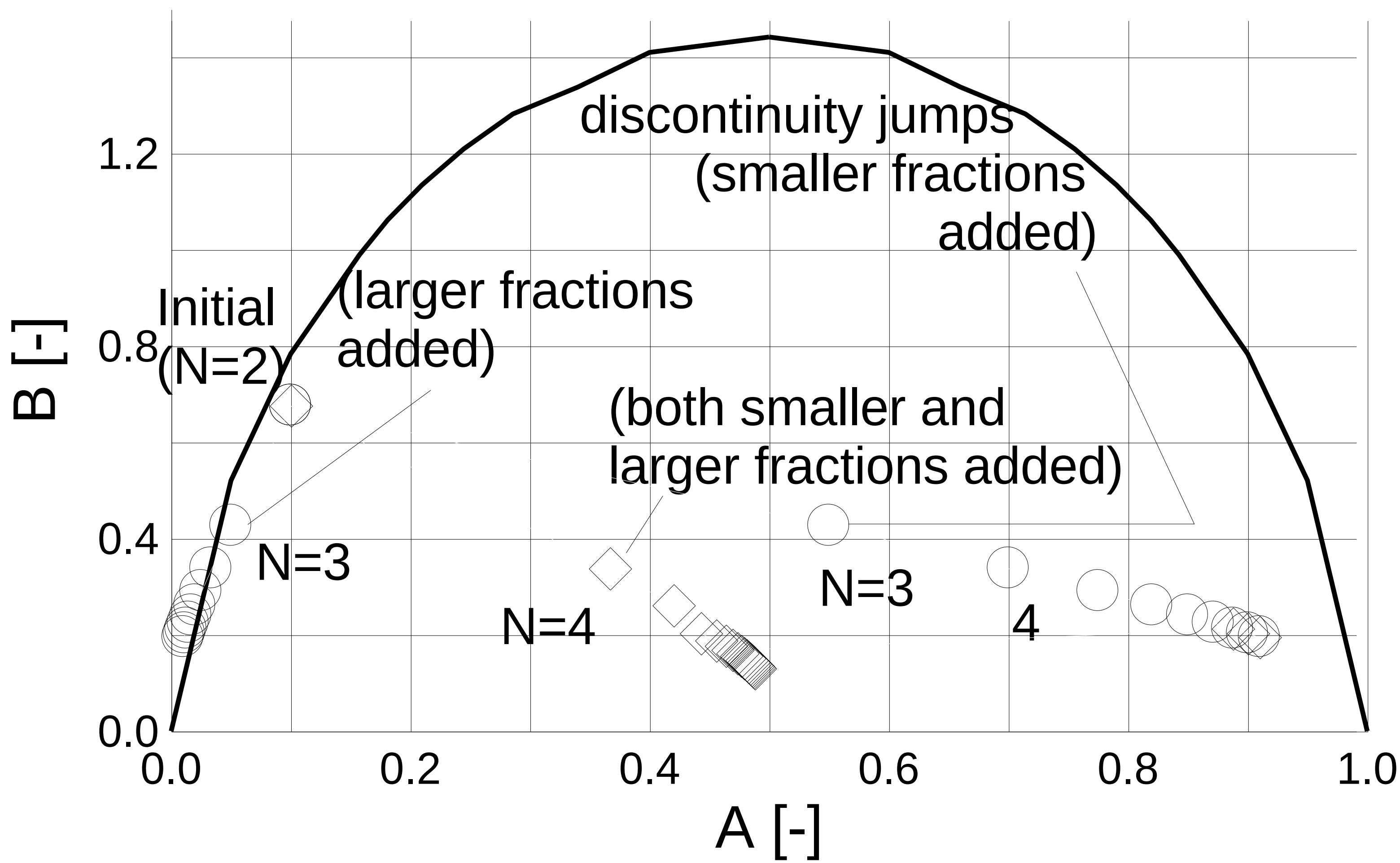
The discontinuous grading curve path in the normalised diagram





Initially one fraction soil



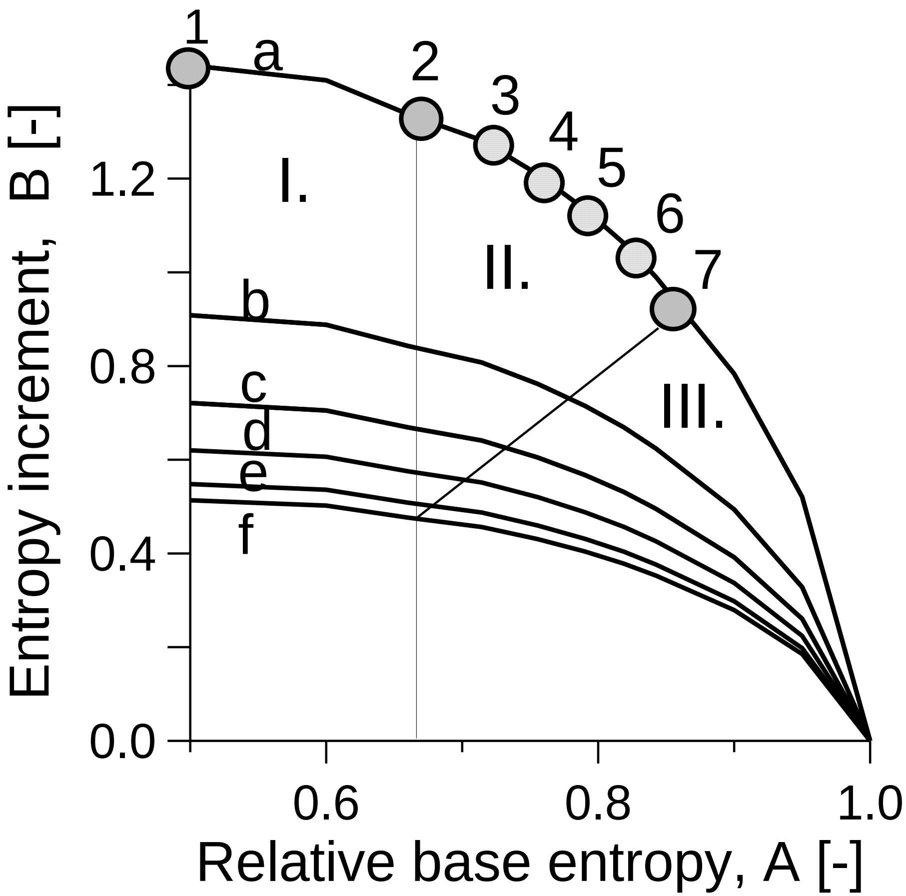


discontinuity jumps
(smaller fractions
added)
Initial
(N=2)
(larger fractions
added)
(both smaller and
larger fractions added)
N=3
N=4
N=3
4
B [-]
A [-]
0.0
0.4
0.8
1.2
0.0
0.2
0.4
0.6
0.8
1.0

Comments on the normalized entropy path

- *Why fractal distribution is so frequent?*
- **If *N* is constant, the path goes towards the maximum *B* line, where all grading curves have finite fractal distribution.**
- On the *A*>0.5 side of the maximum *B* line, the fractal dimension *n* varies between 3 and -infinity.
- On the *A*<0.5 side of the maximum normalized entropy increment line, the fractal dimension *n* varies between 3 and plus infinity.

Normalized
grading entropy diagram
stability rule
and fractal dimension



1: maximum entropy increment point where the fractal dimension is n=3 independently of N.

2, 3, 4…7 : maximum entropy points for N = 2, 3, 4…7, the fractal dimension is n=2 since dependent on N except in point 1.

I, II, III: Grain structure stability domains for N=7
I: piping, II: stable, III: stable with suffosion.

Some conclusions – hard grains

- The base entropy S_o decreases, the entropy increment ΔS strictly monotonically increases. The breakage speed is different for silica and carbonate sand, the path is similar.

It is true that grading entropy can be related to the entropy principle?

- **It follows from the measured data that the entropy principle is valid for the ΔS entropy coordinate.**
- (The base entropy S_o decrease can be explained by the decrease of the mean grain diameter during breakage.)
-

- *Why fractal distribution with n<3 is so frequent?* **If a smaller fraction appear, the normalized entropy path has a discontinuity, increasing the *A* value and decreasing *B* value.**
- As a result, the normalised entropy path is drifted into the right side of the diagram n <3, A>0.5. That is why the structurally stable grading curves (A>2/3) occur more frequently than the structurally unstable ones.
- This can be used in rock testing.

- *Is there an ultimate state?*
- Applying the entropy principle to the non-normalized grading entropy *B*, a theoretical ultimate state is resulted, the symmetry point of the maximum line, with fractal dimension $n = 3$.
- This can be reached by tests with topology change only (applied here) on condition that ***N*** **stops to increase and large grains remain.**
- Further research is suggested on this and on soft rocks where the largest fraction does not remain.