Conference Proceedings Paper

Integrated Regional Enstropy as a Measure of Kolmogorov Entropy

Anthony R. Lupo [1,*] and Andrew D. Jensen [2]

Published: 19 July 2016

[1] Department of Soil, Environmental, and Atmospheric Sciences, University of Missouri, Columbia, MO, USA
[2] Department of Mathematics and Meteorology, Northland College, Ashland, WI, USA
* Correspondence: lupoa@misosuri.edu

Abstract: Enstrophy in a fluid relates to the dissipation tendency in a fluid that has use in studying turbulent flows. It also corresponds to vorticity as kinetic energy does to velocity. Earlier work showed that the Integrated Regional Enstrophy (IRE) was related to the sum of the positive Lypunov Exponents. Lyapunov Exponents are the characteristic exponent(s) of a dynamic system or a measure of the divergence or convergence of system trajectories that are initially close. Relatively high values of IRE derived from an atmospheric flow field in the study of atmospheric blocking was identified with the onset or demise of blocking events, but also transitions of the large-scale flow in general. Kolmogorv Entropy (KolE) also known as metric entropy is related to the sum of the positive Lyapunov Exponents as well. This quantity can be thought of as a measure of predictability (higher values less predictability) and will be non-zero for a chaotic system. Thus, the measure of IRE is related to KolE as well. This study will show that relatively low (high) values of IRE derived from atmospheric flows correspond to more stable (transitioning) large-scale flow a greater (lesser) degree of predictability and KolE. The transition is least predictable and should be associated with higher IRE and KolE.

Keywords: enstrophy; entropy; Lyapunov Exponents; blocking

---

1. Introduction

Blocking was described by [1] as a meridional perturbation that destabilizes the zonal flow. The study of [1] derived a quantity called Integrated Regional Enstrophy (IRE) which can be used as a measure for change in the zonal flow that may indicate blocking. IRE conjectures that the blocking domain sum of the flow enstrophy is related to the sum of the positive eigenvalues of the linearization operator of the barotropic voricity equation [2,3]. As such, IRE can also be regarded as a flow stability indicator [2,3]. The work of [2] demonstrated for three case studies of Southern Hemisphere blocking, that IRE increased locally with the decay of each blocking event, and was relatively low during the maintenance period. This study and those of [4] and [5] suggest blocking does not survive the transition of large-scale flow regimes. The work of [2] examined more than 100 cases of northern hemisphere blocking over a three year period and found that local increases in IRE can be associated with the onset of termination of blocking.

Later this quantity was used to examine the dynamics of the blocking episode associated with the Russian Heat Wave and drought of summer 2010 [6]. The blocking episode involved four individual case studies of strong and persistent blocking from 1 May to 16 August, 2010 over Russia and Eastern Europe which contributed to the drought conditions [2,7]. The onset and decay of each blocking case with local maxima in the IRE as was shown by [2] and [3], and then later by [8], seemed to indicate this quantity was a reliable indicator of blocking. Then [9] examined a persistent and strong Pacific Region case during January and February 2014, and demonstrated this event survived the transition between on phase and another in the Pacific North American (PNA) pattern.

Enstrophy in a geophysical fluid relates to the dissipation tendency, and this quantity has use in studying turbulent flows as show by the studies cited above. Entsrophy is a quantity also corresponds to vorticity as kinetic energy does to velocity. Many studies have shown IRE is related to the sum of the positive Lypunov Exponents, or flow stability. Lyapunov Exponents are the characteristic exponent(s) of a dynamic system or a measure of the divergence or convergence of system trajectories that are initially close. Although relatively high values of IRE derived from an atmospheric flow field in the study of atmospheric blocking was identified with the onset or demise of blocking events, IRE can also be associated with transitions of the large-scale flow in general. Kolmogorov (Kolmogorov-Sinai) Entropy (abbreviated here as KolE) also known as metric entropy is related to the sum of the positive Lyapunov Exponents as well. This quantity can be thought of as a measure of predictability (higher values less predictability) and will be non-zero for a chaotic system, and we'll demonstrate that KolE and IRE are related. Other investigators (e.g. [10]) have used KolE to study the stability and predictability found in atmospheric flows.

The block intensity (BI) is a quantity first proposed by [11] which is calculated as the mean lifetime value of the daily normalized central height values of the event. The normalization is proportional to the upstream and downstream height gradients, and [12] standardized the estimate of the gradients. They [12] also demonstrated that BI increased nearly linearly with increases in the upstream and downstream height gradient, and thus had use as a flow regime blocking diagnostic. The goal of this work is to demonstrate that IRE, KolE, and BI all show similar behavior over the lifetime of a blocking event and can be through of as measures of predictability. Section 2 will describe the data and methods used. Section three will show the analysis, and section four will highlight the main conclusions of this work.

2. Data and Methods

2.1. Data

In order to meet our objective, the National Centers for Environmental Prediction / National Center for Atmospheric Research (NCEP/NCAR) re-analyses [13], archived at the NCAR research facilities in Boulder, CO (http://www.esrl.noaa.gov/psd/data/reanalysis/reanalysis.shtml) can be used which provide for large-scale meteorological data at various resolutions from 1° x 1° to 2.5° x 2.5° latitude-longitude grids. We will use the 500 hPa height data (m) at 1200 UTC each day, since these contain the most observational data.

2.2. Methods

In this paper, we use the work of [1], who postulated that if the atmosphere is barotropic, the positive Lyapunov Exponent(s) in the atmosphere can be expressed as the area integrated regional enstrophy (IRE):

$$\sum_{i>0} \lambda_i \approx \int_A \varsigma^2 dA \tag{1}$$

where λ_i is the ith lyapunov greater than zero in a dynamic system, ζ is the vorticity, or the curl of the wind vector and the quantity squared is called enstrophy, which is the dissipation tendency of a fluid. Many studies ([2,3,6–9]), demonstrated the utility of this quantity in identifying the onset and termination of atmospheric blocking and flow regime transformation.

As a positive Lyapunov Exponent, this quantity relates to predictability and can also be related to the production of system information, e.g. KolE (Kolmogorov–Sinai Entropy / or metric entropy) (e.g. [14,15]). The larger the IRE, the less predictable the atmosphere, as trajectories of two initial conditions would diverge rapidly. Also, relatively low (high) values of IRE derived from atmospheric flows correspond to more stable (transitioning) large-scale flow a greater (lesser) degree of predictability and KolE. The transition is least predictable and should be associated with higher IRE and KolE. The expression for KolE is:

$$KolE = \sum_{\sigma_i > 0} \sigma_i \tag{2}$$

where σ_i are the positive Lyapunov Exponents. This value was calculated using a R-program calculate "correlation entropy" which is a lower bound of the KolE [14]. Thus, it is trivial to demonstrate that KolE can be represented in terms of Equation (1) to get:

$$KolE \approx IRE \approx \int_A \varsigma^2 dA \tag{3}$$

The blocking event studied here onset at 0000 UTC 23 January, 2014, and terminated at 0000 16 February, 2014, and was studied in [9]. This event and all events since July, 1968 can be found at the University of Missouri Blocking Archive (http://weather.missouri.edu/gcc). During the block lifetime, the block intensity was calculated daily for this event at 1200 UTC from onset to termination following [12], and the formula for intensity is given by Equation (4):

$$BI = 100\left(\frac{z_m}{RC} - 1.0\right) \tag{4}$$

where Zm is the central height value (m) and RC the mean contour representing the upstream and downstream gradients;

$$RC = \frac{(z_u + 2z_m + z_d)}{4} \tag{5}$$

where Zu and Zd are the lowest (trough axis) heights upstream and downstream of the block. Thus, the normalization value represents a one-pass Shapiro-type filter [16] on the wave length or large scale component of the flow. Thus, BI can be thought of as the relative strength of the synoptic-scale component of the flow, which will be demonstrated below. The mean BI for the Northern (Southern) Hemisphere since 1970 is 3.04 (2.80), and the distribution of BI is Gaussian [11],[12]. These works define a strong (weak) blocking event as an event that is one standard deviation greater (less) than these values.

3. Results—Blocking Case Study

The blocking event examined here occurred during 23 January to 16 February 2014 and was located over the Eastern Pacific near the Gulf of Alaska and near the West Coast of North America (130^o W). This event formed out of a very long-lived ridge over the same area. The event dominated a significant period of the winter season, and was likely responsible for the cold winter over North America that year (e.g. [9,17]). The work of [9] describes this event in more detail and the 500 hPa heights during the intensification stage of the blocking event which is shown in Fig. 1. This event was noteworthy as it survived a large-scale flow regime change during early February 2014 (4th-7th), as the Pacific North American (PNA) teleconnection pattern change from positive to negative during early February 2014. Earlier work (e.g. [2,18]) suggested that blocking events would not survive a transition in the large-scale flow regime, and [9] showed that, under certain conditions, these events could survive. Additionally, this event is noteworthy for the longevity and persistence.

Examining the block intensity (BI) [12] for this case (Figure 2a) demonstrated that the block was more intense just following the onset of the event, and then intensified in early February (3rd-6th) near the same time that the phase of the PNA flipped from positive to negative indicating flow regime change. The work of [11] demonstrated the connection between block onset and cyclogenesis, and then [19] demonstrated that rapid height rises, and thus BI, follows upstream cyclogenesis during the mid-life block intensification. Then, BI was markedly less indicating a weaker block until the decay period. The IRE diagnostic also followed a similar evolution during the block lifecycle (Fig. 2b), with the IRE maximizing during onset, intensification, and termination. Note that during the onset and mid-life intensification periods (Table 1), both the BI and IRE were higher than greater than that during the block lifecycle. Table 1 also shows that the KolE was also relatively high during these two intensification periods.

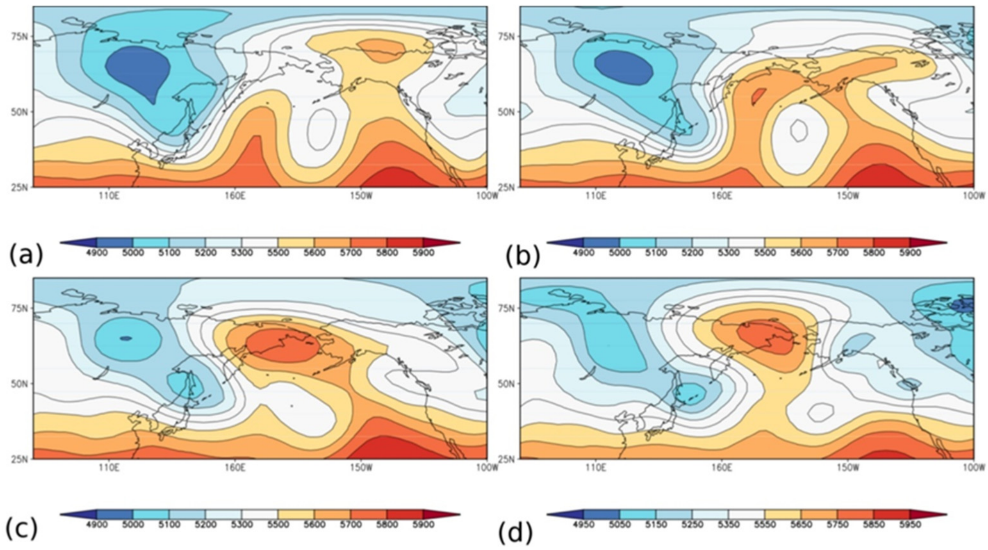

Figure 1. Adapted from [16], the 500 hPa heights derived from the NCEP/NAR reanalyses over the Pacific Ocean basin at 1200 UTC for (**a**) 4 February, (**b**) 5 February, (**c**) 6 February, and (**d**) 7 February in 2014.

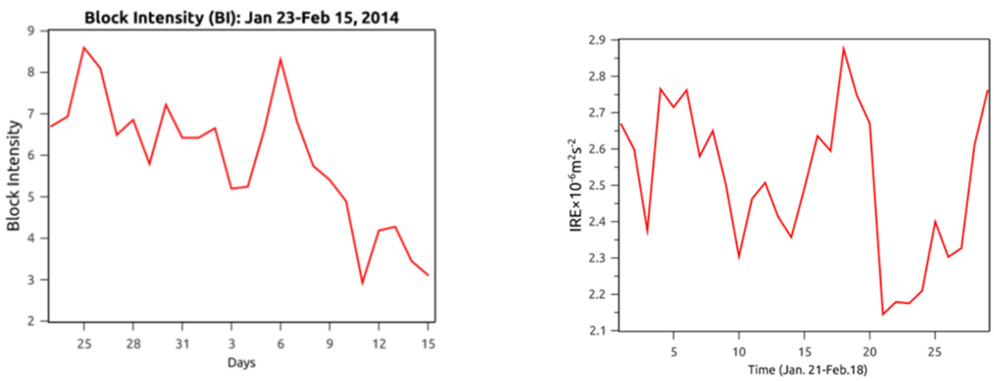

Figure 2. Adapted from [9], (**a**) the Block Intensity (BI) from [12] (ordinate) for the event studied in winter 2014 versus date in January and February 2014 (abscissa) (left), and (**b**) the IRE(s^{-2}) (ordinate) and days following the onset period in January and February 2014 (abscissa) (right).

Table 1. Values for BI, IRE, and KolE during the lifecycle of the blocking event studied here, and the onset period as well as the mid-life intensification.

Block Period	BI	IRE ($\times10^{-6}$ s^{-2})	KolE
23 January–16 February 2014 (lifetime) 24 Days	5.93	2.49	0.75
14–24 January 2014 (onset)	6.90	2.56	0.86
30 January–9 February (intensification)	6.46	2.57	1.39

4. Discussion and Conclusions

The objectives of this study were to determine whether IRE, KolE, and BI provide the same information about the predictability of blocking. A very strong blocking event that occurred in the Pacific Ocean Basin during the winter of 2013–2014 was selected for study to demonstrate the utility of the methods used here. Blocking can develop very rapidly and models often fail to anticipate their onset and/or decay, especially after about ten days [20]. These results corroborate such studies, but also show that mid-life intensification would also be associated with lower predictability. The work here demonstrated that for this blocking event, IRE, KolE, and BI are all relatively high during block onset and intensification. This may be an inherent characteristic of blocking, which indicates that we can expect limited progress in blocking predictability based on today's understanding of these events as well as the modeling currently available to the community. The next step would be to study several blocking events and examine the behavior of BI, IRE, and KolE, and examine the ability of models to capture the evolution of BI during a forecast period.

Acknowledgments: The authors would like to acknowledge the anonymous reviewers of this work.

Conflicts of Interest: The authors declare no conflict of interest. The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, and in the decision to publish the results.

References

1. Dymnikov VP, Kazantsev YV, Kharin VV., "Information entropy and local Lyapunov exponents of barotropic atmospheric circulation." *Izv. Atmos. Oceanic Phys.*, **28**, 425–432, **1992**.
2. Lupo, A.R., Mokhov, I.I., Dostoglou, S., Kunz, A.R., Burkhardt, J.P., "Assessment of the impact of the planetary scale on the decay of blocking and the use of phase diagrams and enstrophy as a diagnostic," *Izv., Atmos. Oc. Phys.*, **43:1**, 45–51, **2007**
3. Athar, H., Lupo, A.R., "Scale Analysis of Blocking Evens from 2002 to 2004: A case study of an unusually persistent blocking events leading to a heat wave in the Gulf of Alaska during August 2004. *Adv. Meteor.*, **2010**, article ID 610263, 15 pp.
4. Haines, K., Holland, A.J., "Vacillation cycles and blocking in a channel," *Quart. J. Roy. Meteor. Soc.*, **124,** 873–897, **1998**.
5. Colucci, S.J., Baumhefner, D.P., "Numerical prediction of the onset of blocking: A case study with forecast ensembles," *Mon. Wea. Rev.*, **126**, 773–784, **1998.**
6. Lupo, A.R., Mokhov, I.I., Akperov, M.G., Chernokulsky, A.V., and Athar, H., "A dynamic analysis of the role of the planetary and synoptic-scale in the summer of 2010 blocking episodes over the European Part of Russia. *Adv. Meteor.*, **2012**, Article ID 584257, 11 pp.
7. Lupo, A.R., Mokhov, I.I., Chendev, Y.G., Lebedeva, M.G., Akperov, M.G., and Hubbart, J.A., "Studying summer season drought in western Russia". *Adv. Meteor., Special Issue: Large scale dynamics, anomalous flows, and teleconnections Atmospheric Science*, **2014**, 9pp, Article ID 942027.
8. Jensen, A.D., Lupo, A.R., "Using enstrophy advection as a diagnostic to identify blocking-regime transition. *Quart. J. Roy. Meteor. Soc.*, **2013,** DOI:10.1002/qj.2248, 7 pp.
9. Jensen, A.D., "A dynamic analysis of a record breaking winter season blocking event." *Adv. Meteor., Special Issue: Large scale dynamics, anomalous flows, and teleconnections Atmospheric Science, 2015*, **2015**, 9pp, Article ID 634896.
10. Zeng, X., Pielke, R.A., Eykholt, R., "Estimating the fractal dimension and the predictability of the atmosphere." *J. Atmos. Sci*, **49,** 649 – 659, **1992.**
11. Lupo, A.R, Smith, P.J., "Climatological features of blocking anticyclones in the Northern Hemisphere," *Tellus, Series A*, **47:4,** pp. 439–456, **1995**
12. Wiedenmann, J.M., Lupo, A.R., Mokhov, I.I., Tikhonova, E.A., "The climatology of blocking anticyclones for the Northern and Southern Hemispheres: block intensity as a diagnostic," *J. Clim*, **15,** 3459–3473, **2002.**
13. Kalnay, E., Coauthors, "The NCEP/NCAR 40-year reanalysis project." *Bull. Amer. Meteor. Soc.*, **77,** 437–471, 1996.
14. Ekmann, J.P., Ruelle, D., "Ergodic theory of chaos", *Rev. Mod. Phys.*, **57:3**, 617-656, **1985**.

15. Ott, E., "*Chaos in Dynamical Systems*." New York: Cambridge University Press. 138 pp, **1993.**
16. Shapiro, R., "Smoothing, filtering, and boundary effects." *Rev. Geophys.*, **8,** 359-387, **1970.**
17. Lupo, A.R., Colucci, S.J., Mokhov, I.I>, Wang, Y., "Large-scale dynamics, anomalous flows, and teleconnections 2015". *Adv. Meteor., Special Issue: Large scale dynamics, anomalous flows, and teleconnections Atmospheric Science, 2015*, **2015**, 3pp, Article ID 1893468.
18. Haines, K., Holland, A.J., "Vacillation cycles and blocking in a channel," *Quart. J. Roy. Meteor. Soc.*, **124,** 873–897, **1998**.
19. Lupo, A.R., "A diagnosis of two blocking events that occurred simultaneously in the mid-latitude Northern Hemisphere. *Mon. Wea. Rev.*, **125,** 1801 – 1823, **1997**.
20. Matsueda, M., "Predictability of Euro-Russian blocking in summer of 2010," Geophys. Res. Lett., **38:6,** article ID L06801, 6 pages, **2011**.

© 2016 by the authors; licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons by Attribution (CC-BY) license (http://creativecommons.org/licenses/by/4.0/).