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The Particle as a Statistical Ensemble of Events in Stueckelberg–Horwitz–Piron Electrodynamics

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Abstract: Stueckelberg–Horwitz–Piron (SHP) electrodynamics formalizes the distinction between coordinate time (measured by laboratory clocks) and chronology (temporal ordering) by defining 4D spacetime events x^μ as functions of an external evolution parameter τ. Classical spacetime events $x^\mu(\tau)$, evolving as τ grows monotonically, trace out particle worldlines dynamically and induce the *five* U(1) gauge potentials through which events interact. Since Lorentz invariance imposes time reversal symmetry on x^0 but not τ, the formalism resolves grandfather paradoxes and related problems of irreversibility. Nevertheless, the causal structure of the 5D Green's function introduces singularities associated with the τ-dependence of the induced fields, which are regularized by generalizing the action to include a higher order kinetic term for the fields. The resulting theory remains gauge and Lorentz invariant, and the related QED is super-renormalizable. The field equations are Maxwell-like but τ-dependent and sourced by a current that represents a statistical ensemble of events distributed along the worldline. The width of the distribution defines a mass spectrum for the photons that carry the interaction. As the width becomes very large, the photon mass goes to zero and the field equations become τ-independent Maxwell's equations. Maxwell theory thus emerges as an equilibrium limit of SHP. Particles and fields can exchange mass in the SHP theory, however on-shell particle mass is restored through self-interaction.

Keywords: classical gauge theory; pair creation/annihilation; temporal paradoxes

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1. Introduction

In developing his interpretation of antiparticles as particles traveling backward in time, Stueckelberg [1,2] sought to demonstrate that pair creation/annihilation processes appear naturally in a thoroughly deterministic and classical relativistic Hamiltonian mechanics. He described a particle as the worldline traced out by a spacetime event $x^\mu(\tau)$ evolving dynamically as the Poincaré invariant parameter proceeds monotonically from $\tau = -\infty$ to $\tau = \infty$. Stueckelberg argued that pair annihilation is observed when the coordinate time $x^0(\tau)$ reverses direction, because for some values $x^0_2 > x^0_1$ of the laboratory clock there will be two solutions to $x^0(\tau) = x^0_1$ but no solution to $x^0(\tau) = x^0_2$. Such trajectories must overcome the mass-shell constraint which by keeping $\dot{x}^2$ from changing sign, prevents the event from crossing the spacelike region that separates future-oriented trajectories from past-oriented trajectories. Horwitz and Piron [3] extended these ideas in constructing a canonical relativistic mechanics for the two-body problem with a scalar interaction, and Horwitz et. al. [4–8] found relativistic generalizations of the standard central force problems. Study of radiative transitions [9–11] indicates that both the scalar potential and 4-vector potential are required to account for known phenomenology. Sa'ad, Horwitz, and Arshansky [12] proposed a unification of the five potentials by extending gauge invariance to include τ-dependent transformations. Adopting the conventions

$$\mu,\nu = 0,1,2,3 \qquad \alpha,\beta,\gamma = 0,1,2,3,5 \qquad g_{\alpha\beta} = \text{diag}(-1,1,1,1,\pm 1) \tag{1}$$

and in analogy with the notation $x^0 = ct$, making the formal designations $x^5 = c_5\tau$ and $\partial_5 = \dfrac{1}{c_5}\partial_\tau$, the event dynamics of Stueckelberg-Horwitz-Piron (SHP) are defined by the Lagrangian

$$L = \frac{1}{2}M\dot{x}^\mu \dot{x}_\mu + \frac{e}{c}\dot{x}^\alpha a_\alpha(x,\tau) = \frac{1}{2}M\dot{x}^\mu \dot{x}_\mu + \frac{e}{c}\dot{x}^\mu a_\mu(x,\tau) + e\frac{c_5}{c}a_5(x,\tau) \tag{2}$$

which leads to the Lorentz force

$$M\ddot{x}^\mu = \frac{e}{c}f^\mu{}_\alpha(x,\tau)\dot{x}^\alpha \qquad\qquad \frac{d}{d\tau}(-\tfrac{1}{2}M\dot{x}^2) = g_{55}\, e\frac{c_5}{c}f^{5\mu}(x,\tau)\dot{x}_\mu \tag{3}$$

where the 10 field strengths are $f^\alpha{}_\beta = \partial^\alpha a_\beta - \partial_\beta a^\alpha$. The second of (3) permits mass exchange between particles and fields, thus permitting pair processes. The Lagrangian (2) is unique up to the τ-dependent gauge transformations

$$a_\alpha(x,\tau) \to a_\alpha(x,\tau) + \partial_\alpha \Lambda(x,\tau) \tag{4}$$

and in the associated quantum mechanics the wavefunction is invariant under U(1) phase transformations generated by $\Lambda(x,\tau)$. It has been shown [13] that this system describes the most general classical interaction in flat spacetime consistent with the quantum commutation relations

$$[x^\mu, x^\nu] = 0 \qquad\qquad m\,[x^\mu, \dot{x}^\nu] = -i\hbar g^{\mu\nu}(x)\,. \tag{5}$$

To complete the dynamical picture we define the current

$$\begin{gathered}\dot{X}^\alpha a_\alpha \to \int d^4x\; \dot{X}^\alpha(\tau)\delta^4\left(x - X(\tau)\right) a_\alpha(x,\tau) = \frac{1}{c}\int d^4x\; j^\alpha(x,\tau) a_\alpha(x,\tau) \\ j^\alpha(x,\tau) = c\dot{X}^\alpha(\tau)\delta^4\left(x - X(\tau)\right)\end{gathered} \tag{6}$$

and make some choice of kinetic action term for the fields, the most obvious candidate being

$$S_{\text{field}} = \frac{1}{4c}\int d\tau d^4x\; f^{\alpha\beta}(x,\tau) f_{\alpha\beta}(x,\tau)\,. \tag{7}$$

The resulting action leads to Maxwell-like field equations that admit a wave equation and a Green's function, which can be used to derive the fields for specific event trajectories. But one is immediately confronted by conceptual difficulties in attempting to describe even the simple case of low energy Coulomb scattering. The potential induced by a 'static' particle, an event evolving uniformly along the x^0 axis, contains singularities of the form $\delta\left((\tau - t)^2 - |\mathbf{x}|^2/c^2\right)$, rendering comparison with known phenomenology nearly impossible. In the following section, we introduce a higher order term for S_{field} that removes singularities of this type.

2. Non-local field kinetics $\Leftrightarrow$ ensemble of events

The singularities can be repaired by writing the action with a slightly less obvious candidate for the field kinetic term, the non-local form

$$S_{\text{em}} = \int d^4x d\tau \left\{ \frac{e}{c^2}\, j^\alpha(x,\tau) a_\alpha(x,\tau) - \int \frac{ds}{\lambda}\, \frac{1}{4c}\left[f^{\alpha\beta}(x,\tau)\Phi(\tau - s) f_{\alpha\beta}(x,s)\right]\right\} \tag{8}$$

where λ is a parameter with dimensions of time and the field interaction kernel is

$$\Phi(\tau) = \delta(\tau) - \left(\frac{\lambda}{2}\right)^2 \delta''(\tau) = \int \frac{d\kappa}{2\pi}\left[1 + \left(\frac{\lambda\kappa}{2}\right)^2\right] e^{-i\kappa\tau}. \tag{9}$$

We write the inverse function of the interaction kernel as

$$\varphi(\tau) = \lambda \Phi^{-1}(\tau) = \lambda \int \frac{d\kappa}{2\pi} \frac{e^{-i\kappa\tau}}{1 + (\lambda\kappa/2)^2} = e^{-2|\tau|/\lambda} \tag{10}$$

which satisfies

$$\int \frac{ds}{\lambda}\, \varphi(\tau - s)\, \Phi(s) = \delta(\tau) \qquad\qquad \int \frac{d\tau}{\lambda}\, \varphi(\tau) = 1. \tag{11}$$

Varying the action (8) with respect to the potentials, leads to field equations

$$\partial_\beta f_\Phi^{\alpha\beta}(x,\tau) = \partial_\beta \int \frac{ds}{\lambda}\, \Phi(\tau - s) f^{\alpha\beta}(x,s) = \frac{e}{c}\, j^\alpha(x,\tau) \tag{12}$$

describing the non-local superposition of fields $f_\Phi^{\alpha\beta}$ sourced by the instantaneous event current $j^\alpha(x,\tau)$. Using (11) to remove $\Phi(\tau)$ from the LHS we obtain equations for the local field sourced by a non-local superposition of event currents,

$$\partial_\beta f^{\alpha\beta}(x,\tau) = \frac{e}{c} \int ds\, \varphi(\tau - s)\, j^\alpha(x,s) = \frac{e}{c}\, j_\varphi^\alpha(x,\tau) \tag{13}$$

which are formally similar to 5D Maxwell's equations. Expanding to 4D tensor, vector and scalar components and including the Bianci identity, we obtain the pre-Maxwell equations

$$\begin{aligned} &\partial_\nu f^{\mu\nu} - \frac{1}{c_5}\partial_\tau f^{5\mu} = \frac{e}{c}\, j_\varphi^\mu && \partial_\mu f^{5\mu} = \frac{e}{c}\, j_\varphi^5 = \frac{c_5}{c}\, e\rho_\varphi \\ &\partial_\mu f_{\nu\rho} + \partial_\nu f_{\rho\mu} + \partial_\rho f_{\mu\nu} = 0 && \partial_\nu f_{5\mu} - \partial_\mu f_{5\nu} + \frac{1}{c_5}\partial_\tau f_{\mu\nu} = 0 \end{aligned} \tag{14}$$

which may be compared with the 3-vector form of Maxwell's equations. Shifting the integral in the source of the inhomogeneous equation (13) as

$$j_\varphi^\alpha(x,\tau) = \int ds\, \varphi(\tau - s)\, j^\alpha(x,s) = \int ds\, e^{-2|s|/\lambda}\, j^\alpha(x,\tau - s) \tag{15}$$

we recognize $j_\varphi^\alpha(x,\tau)$ as a weighted superposition of currents originating at events $X^\mu(\tau - s)$ displaced from $X^\mu(\tau)$ by an amount s along the worldline. We regard this superposition as the current produced by an ensemble of events in the neighborhood of $X^\mu(\tau)$, a view encouraged by the particular weight function $\varphi(s)$. Given a Poisson distribution describing the occurrence of independent random events with a constant average rate of $1/\lambda$ events per second, the average time between events is λ and the probability at τ that the next event will occur following a time interval $s > 0$ is just $e^{-s/\lambda}/\lambda$. Extending the displacement to positive and negative values, the ensemble is constructed by assembling a set of event currents $j^\alpha(x,\tau - s)$ along the worldline, each weighted by $\varphi(s)$, the probability that the occurrence of this event is delayed from τ by an interval of at least $|s|$. Causality relations embedded in the Green's function will select the one event from this ensemble at lightlike separation from the interacting event.

The pre-Maxwell equations in Lorenz gauge lead to the wave equation and Green's function [14]

$$\partial_\beta \partial^\beta a^\alpha = \left(\partial_\mu \partial^\mu + \left(g_{55}/c_5^2\right)\, \partial_\tau^2\right) a^\alpha = -\frac{e}{c}\, j_\varphi^\alpha(x,\tau) \tag{16}$$

$$G_P(x,\tau) = -\frac{1}{2\pi}\delta(x^2)\delta(\tau) - \frac{c_5}{2\pi^2}\frac{\partial}{\partial x^2}\frac{\theta(-g_{55}g_{\alpha\beta}x^\alpha x^\beta)}{\sqrt{-g_{55}g_{\alpha\beta}x^\alpha x^\beta}} = G_{Maxwell} + G_{Correlation}. \tag{17}$$

The contribution from $G_{Correlation}$ is smaller than that of $G_{Maxwell}$ by c_5/c and drops off as $1/|\mathbf{x}|^2$, so it may be neglected at low energy [15]. The δ-functions in $G_{Maxwell}$ have support only at the retarded time τ_R that solves $(x - X(\tau_R))^2 = 0$ and $x^0 > X^0(\tau_R)$. Thus, while the current that sources the pre-Maxwell field represents an ensemble of events along the worldline, the retarded causality of the Green's function selects the one member of the ensemble that intersects the lightcone of the observation point. We find the standard Liénard-Wiechert potential multiplied by $\varphi(\tau - \tau_R)$

$$a^\alpha(x,\tau) = \frac{e}{2\pi}\int ds\ \varphi(\tau - s)\,\dot{X}^\alpha(s)\,\delta\left[(x - X^\alpha(s))^2\right] = \frac{e}{4\pi}\frac{\varphi(\tau - \tau_R)\,\dot{X}^\alpha(\tau_R)}{\left|(x^\mu - X^\mu(\tau_R))\,\dot{X}_\mu(\tau_R)\right|}. \tag{18}$$

The remaining τ-dependence of the fields resides in the finite function φ and expresses the relative time synchronization between the source and a test event experiencing the potential at the spacetime point x at the chronological time τ. To find the Coulomb potential we specify the event trajectory $X(\tau) = \big(c(\tau + \tau_0), \mathbf{0}\big)$ and (18) becomes

$$a^0(x,\tau) = \frac{e}{4\pi|\mathbf{x}|}\varphi\left(\tau + \tau_0 - \left(t - \frac{|\mathbf{x}|}{c}\right)\right) \qquad a^5(x,\tau) = \frac{c_5}{c}a^0(x,\tau). \tag{19}$$

For a test event evolving along a parallel trajectory at $x(\tau) = (c(\tau + \tau_0), \mathbf{x})$

$$a^0(x,\tau) = \frac{e}{4\pi|\mathbf{x}|}e^{-|\mathbf{x}|/\lambda c} \tag{20}$$

which has the form of a Yukawa-type potential with photon mass $m_\gamma \sim \hbar/\lambda c^2$. For large λ, the current ensemble spreads along the worldline, the potential becomes Coulomb-like, the information entropy decreases and the photon mass is small[1]. The factor λ plays a similar role in QED. Although dimensional considerations suggest that photon loops in 5D would render SHP non-renormalizable, quantization of the higher order field kinetic term inserts a mass cut-off into the photon propagator factor

$$\left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right]\frac{-i}{k^2 + g_{55}\kappa^2 - i\epsilon}\ \frac{1}{1 + \lambda^2\kappa^2} \tag{21}$$

making the theory super-renormalizable at second order.

Following an argument by Stueckelberg, Saad et. al. noticed [12] that by requiring $j^5_\varphi(x,\tau)$ and $f^{5\mu}(x,\tau)$ to vanish pointwise in x as $\tau \to \pm\infty$, integration of the pre-Maxwell equations over τ recovers Maxwell's equations, where Maxwell fields and currents are identified as the τ integrals of corresponding τ-dependent SHP quantities. In particular, integration of (15) with (6) using (11)

$$J^\mu(x) = c\int d\tau \int ds\ \varphi(\tau - s)\,\dot{X}^\mu(s)\delta^4\left(x - X(s)\right) = c\int ds\ \dot{X}^\mu(s)\delta^4\left(x - X(s)\right) \tag{22}$$

recovers the Maxwell current in standard form. This integration has been called concatenation and is understood as aggregation over chronological time τ of all events that occur at some spacetime point x^μ. Another approach [15] to retrieving Maxwell theory from SHP is to slow the τ-evolution to zero by taking $c_5/c \to 0$, thus freezing the microscopic system into a static equilibrium[2]. Under this condition the homogeneous pre-Maxwell equation (14) imposes

$$c_5\left(\partial_\nu f_{5\mu} - \partial_\mu f_{5\nu}\right) + \partial_\tau f_{\mu\nu} = 0 \xrightarrow[c_5 \to 0]{} \partial_\tau f_{\mu\nu} = 0 \tag{23}$$

requiring that the field strength $f^{\mu\nu}$ be τ-independent in this limit. As seen in the Liénard-Wiechert potential, the τ-dependence of the fields resides in $\varphi(\tau - \tau_R)$ and can only be suppressed by taking

1 Taking m_γ as the experimental error on the mass of the photon ($10^{-18} eV/c^2$) we may estimate $\lambda > 10^{-2}$ seconds.
2 Asymmetry between the cross-sections for elastic particle-particle and particle-antiparticle scattering depends on a factor $(1 \pm c_5/c)$ and so a dynamical SHP theory requires $0 < c_5/c \ll 1$.

$\lambda \to \infty$. In this limit $\varphi(\tau) \to 1$, all field components become τ-independent and the photon mass $m_\gamma \sim \hbar/\lambda c^2$ vanishes. Assigning equal weight to all event currents $j^\alpha(x,\tau)$ in the ensemble $j^\alpha_\varphi(x,\tau)$ effectively collapses the entire worldline into a single source event. The standard Maxwell particle current is recovered[3] as

$$j^\alpha_\varphi(x,\tau) = \int ds\, 1 \cdot j^\alpha(x,s) = J^\alpha(x) \quad \Rightarrow \quad \partial_\mu j^\mu_\varphi(x,\tau) + \frac{1}{c_5}\partial_\tau j^5_\varphi(x,\tau) = \partial_\mu J^\mu(x) = 0. \qquad (24)$$

One sees in (14) that $f^{\mu\nu}$ decouples from $f^{5\mu}$ and satisfies Maxwell's equations.

While mass exchange must be present in any classical theory of pair processes and must also be small to account for standard electromagnetic phenomenology, such a compromise cannot explain the fixed masses of elementary particles. Nevertheless, there are indications [16] that under certain circumstances a self-interaction induced by $G_{Correlation}$ has the effect of restoring on-shell evolution in event trajectories and thus returning the particle worldline to the observed fixed mass. A more general approach is found in the statistical mechanics of the many-event system. While the model presented here describes a particle as a weighted ensemble of events $\varphi(s)X^\mu(\tau - s)$ along a single worldline, Horwitz has modeled [17] a particle as an ensemble of n independent spacetime events $X^\mu_i(\tau)$, $i = 1, 2, \ldots, n$ defined at a given τ. He has shown that the total particle mass is determined by a chemical potential. Following collisions governed by a general class of interactions that includes pair processes, particles return to their equilibrium mass values. These developments indicate that the statistical mechanics of event ensembles will be a fruitful way to understand mass.

3. Discussion

While defining the system in an unconstrained 8D phase space relaxes the *a priori* mass shell relation $\dot{x}^2 = c^2$ and thus permits classical trajectories that reverse the direction of their time evolution, it also eliminates reparametrization invariance[4]. In SHP electrodynamics the evolution parameter τ cannot be identified as the proper time of the motion, but is a dynamical quantity proportional to it through

$$c^2\, ds^2(\tau) = -g_{\mu\nu} dx^\mu dx^\nu = -\dot{x}^2(\tau)\, d\tau^2. \qquad (25)$$

Therefore, the parameter τ plays the role of an irreducible chronological time, independent of the spacetime coordinates and similar to the external time t in nonrelativistic Newtonian mechanics. It determines the temporal ordering of events — the order of their physical occurrence — which may differ from the order of observed coordinate times x^0 registered by laboratory clocks as the events appear in measuring apparatus. As Horwitz has observed, grandfather paradoxes may be resolved by noticing that the return trip to a past coordinate time x^0 must take place while the chronological time τ continues to increase. The *occurrence* of event $x^\mu(\tau_1)$ at τ_1 is understood to be an irreversible process that cannot be changed by a subsequent event occurring at the same spacetime location, $x^\mu(\tau_2) = x^\mu(\tau_1)$ with $\tau_2 > \tau_1$. This absence of closed timelike curves similarly applies in SHP quantum electrodynamics [18] where the particle propagator $G(x_2 - x_1, \tau_2 - \tau_1)$ vanishes[5] unless $\tau_2 > \tau_1$, thus preventing divergent matter loops, when $x_2 = x_1$. This explicit distinction between chronological and coordinate time [21] describes a microscopic event dynamics in which a covariant Hamiltonian generates evolution of a 4D block universe defined at τ to an infinitesimally close 4D block universe defined at $\tau + d\tau$. Standard Maxwell electrodynamics emerges as an equilibrium limit in which the system becomes τ-independent, and the 4D block universe remains static.

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3 The current $(1/c_5)\, j^5_\varphi(x,\tau)$ remains finite because $j^5(x,\tau)$ includes the factor $\dot{X}^5 = c_5$.

4 The mass shell constraint and reparametrization invariance are related features of a Lagrangian that is homogeneous of first degree in the velocities, which is not the case for (2).

5 The equivalence of τ-retarded causality to the Feynman contour in evaluating propagators was first observed by Feynman in connection with the path integral for the Klein-Gordon equation [19,20]. In SHP QED it also emerges from the vacuum expectation value of τ-ordered operator products.

For Stueckelberg, pair processes provide empirical evidence that time must be understood as two distinct physical phenomena, chronology and coordinate, and so must be formalized through independent quantities τ and $(x^0, \mathbf{x})$ in a physically reasonable theory. As a result, there is no static configuration in SHP — a particle may only remain at the origin in its rest frame for all coordinate time x^0 if its underlying microscopic event continually and uniformly evolves along its time axis, as $x = \left(c(\tau + \tau_0), \mathbf{0}\right)$. The coordinate $x^0 = c\tau_0$ at $\tau = 0$ is not simply an artifact of initializing a system clock, because the field (20) induced by this event trajectory depends explicitly on the constant τ_0. The irreversible concatenation performed on (20) by a measuring apparatus recovers the familiar Coulomb potential with no dependence on τ_0 or even on the details of the weight function $\varphi(\tau)$. Similarly, τ_0 plays no role in the quantized theory where sharply defined mass-momentum states retain no information about the initial conditions of coordinates. Nevertheless, in classical SHP the microscopic event dynamics are determined by the Lorentz force (3) and so a test event at $x(\tau) = (c(\tau + \tau_0'), \mathbf{x})$ will experience a Coulomb force depending on the synchronization $\tau_0 - \tau_0'$. The higher order kinetic term $\left(\partial_\tau f^{\alpha\beta}\right)\left(\partial_\tau f_{\alpha\beta}\right)$ smooths the current and potential by associating with an event $X^\mu(\tau)$ an ensemble whose members are of the form $\varphi(s)X^\mu(\tau - s)$, where the weight $\varphi(s)$ is the probability that a process generating independent random events at a constant average rate will produce an event occurring at displacement s from time τ. A single member of the ensemble is selected by the causal properties of the Green's function when determining the potential induced by the event trajectory. Exclusion of the higher order term from the electromagnetic action is equivalent to taking $\varphi(\tau) = \lambda\delta(\tau)$, which does recover the concatenated Coulomb force but renders the Lorentz force difficult, if not impossible, to reconcile with known phenomenology. A term of this type has also been considered by Pavsic for brane interactions [22].

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