Optimal sensor placement through Bayesian experimental design: effect of measurement noise and number of sensors

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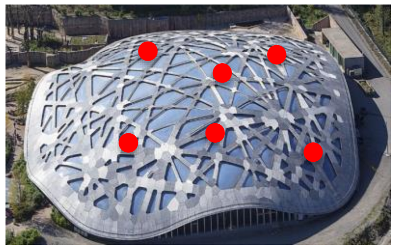
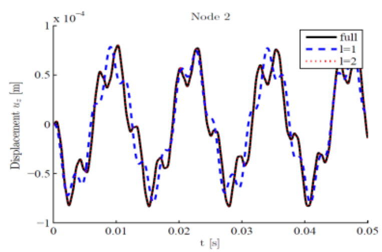
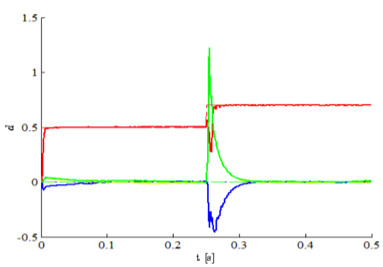
Motivation

Structural Health Monitoring can be conceptually divided in three stages: in our work, we will focus on the design of the sensor network







SHM system design
$\boldsymbol{d}$

⇩

Data collection
$\boldsymbol{y}$

⇩

Parameters estimation
$\boldsymbol{\theta}$

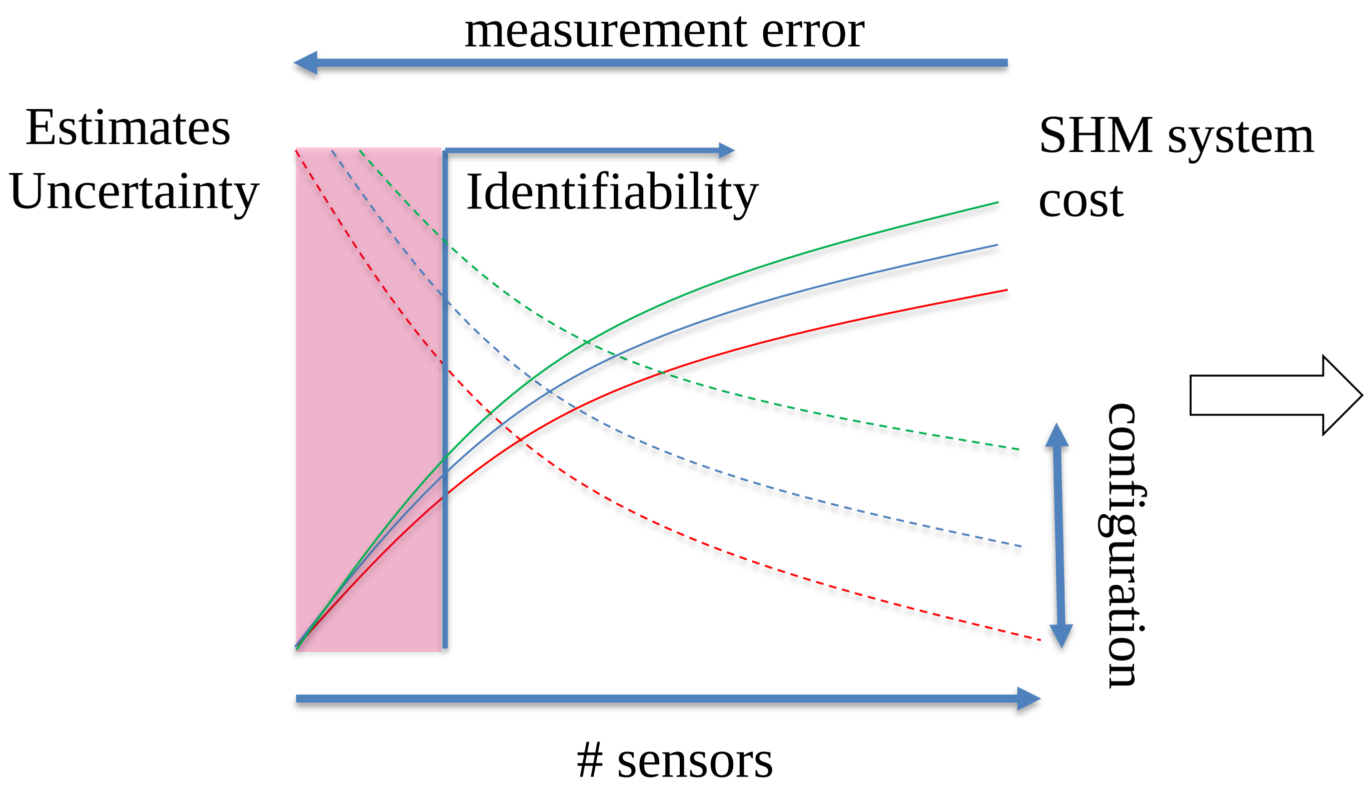
⇩

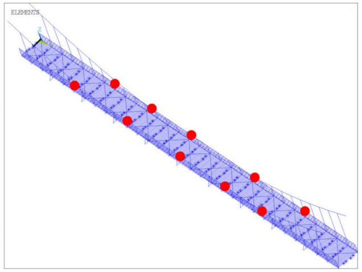
Decision making

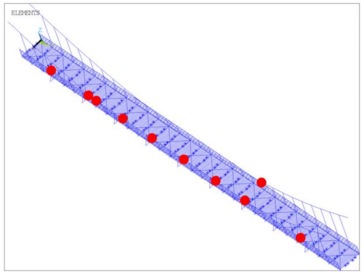
Motivation

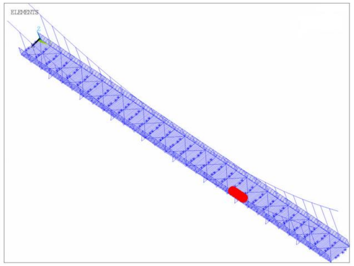
The usefulness of the sensor network depends on the number, type and location of the sensors. Therefore, we need a method to quantify the information obtained by the acquisition system.

Optimal SHM system design





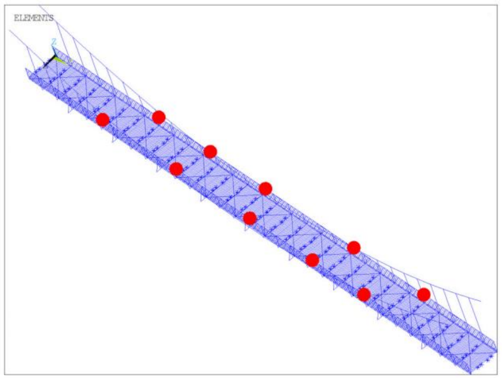




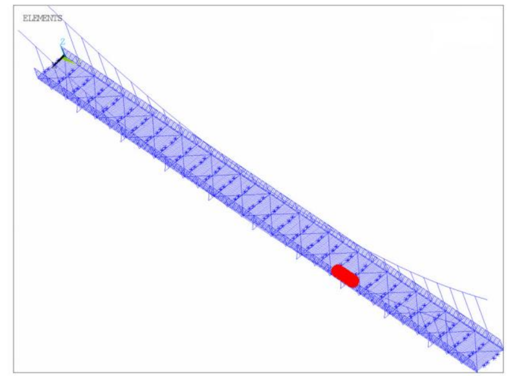
Optimal sensor placement: deterministic methods

The existing approaches does not take into account the measurement noise, i.e. the sensors accuracy.

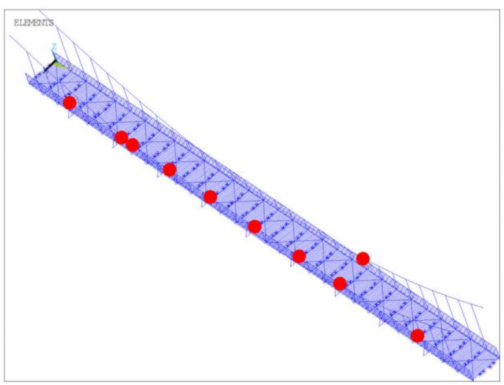
EFI



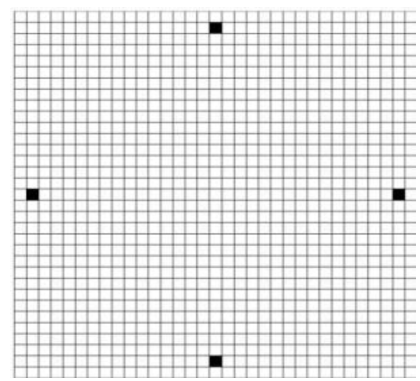
EVP



KE



Sensitivity to damage



M. **Meo**, G. **Zumpano**, (2005), M. **Bruggi**, S. **Mariani**, (2013), **Leyder**, C., **Ntertimanis**, V., **Chatzi**, E., **Frangi**, A. (2015).

Optimal sensor placement: Bayesian framework

In a Bayesian sense, the optimal spatial configuration $\boldsymbol{d}^*$ of the sensor network can be found by maximizing the Shannon information gain. In order to compute it, we use a Monte Carlo approximation.

$$\boldsymbol{d}^* = \arg\max_{\boldsymbol{d}\in\boldsymbol{D}} U(\boldsymbol{d})$$

Expected gain in Shannon information

$$U(\boldsymbol{d}) = \int_{\boldsymbol{Y}} \int_{\Theta} u(\boldsymbol{d}, \boldsymbol{y}, \boldsymbol{\theta}) p(\boldsymbol{\theta}, \boldsymbol{y} | \boldsymbol{d}) d\boldsymbol{\theta} d\boldsymbol{y}$$

⇓

Monte Carlo sampling

Prior: $\boldsymbol{\theta} \sim p(\boldsymbol{\theta})$

Likelihood: $\boldsymbol{y} \sim p(\boldsymbol{y} | \boldsymbol{\theta}, \boldsymbol{d})$

$$U(\boldsymbol{d}) \approx \frac{1}{n_{out}} \sum_{i=1}^{n_{out}} \left\{ \ln\left[p\left(\boldsymbol{y}^i | \boldsymbol{\theta}^i, \boldsymbol{d}\right)\right] - \ln\left[\frac{1}{n_{in}} \sum_{j=1}^{n_{in}} p\left(\boldsymbol{y}^i | \boldsymbol{\theta}^j, \boldsymbol{d}\right)\right]\right\}$$

X. **Huan**, Y. M. **Marzouk**, (2013).

Model evaluation

The measurements are related to the mechanical parameters to be estimated through a FEM-based forward model. The sensor accuracy is taken into account through a fictitious measurement noise.

- **Evaluation of the likelihood**

$$p\left(\boldsymbol{y}^i \middle| \boldsymbol{\theta}^j, \boldsymbol{d}\right) = p_{\boldsymbol{\epsilon}}\left(\boldsymbol{y}^i - \boldsymbol{G}\left(\boldsymbol{\theta}^j, \boldsymbol{d}\right)\right)$$

- **Forward model**

$$\boldsymbol{y} = \boldsymbol{G}(\boldsymbol{\theta}, \boldsymbol{d}) + \boldsymbol{\epsilon}$$

Measurement noise

$$\boldsymbol{G}(\boldsymbol{\theta}, \boldsymbol{d}) = \boldsymbol{L}(\boldsymbol{d})\boldsymbol{K}(\boldsymbol{\theta})^{-1}\boldsymbol{F}$$

Observation matrix

$$\boldsymbol{K}(\boldsymbol{\theta}) = \sum_{i=1}^{n_{\boldsymbol{\theta}}} \frac{E_i}{E-1}\left(\boldsymbol{K}_{und} - \boldsymbol{K}_i\right)$$

Optimization

In order to reduce the computational cost of the forward model, a cheaper surrogate model is built.

- **Surrogate model: polynomial chaos expansion**

$$\boldsymbol{\theta}_i \sim p(\boldsymbol{\theta}), \boldsymbol{d}_i \sim \mathcal{U}(\boldsymbol{\mathcal{D}})$$
$$\boldsymbol{X}_i = \left[\boldsymbol{\theta}_i^T \;\; \boldsymbol{d}_i^T\right]$$

$$\Downarrow$$

$$\boldsymbol{y}_i{}^{FE} = \boldsymbol{G}(\boldsymbol{\theta}_i, \boldsymbol{d}_i)$$

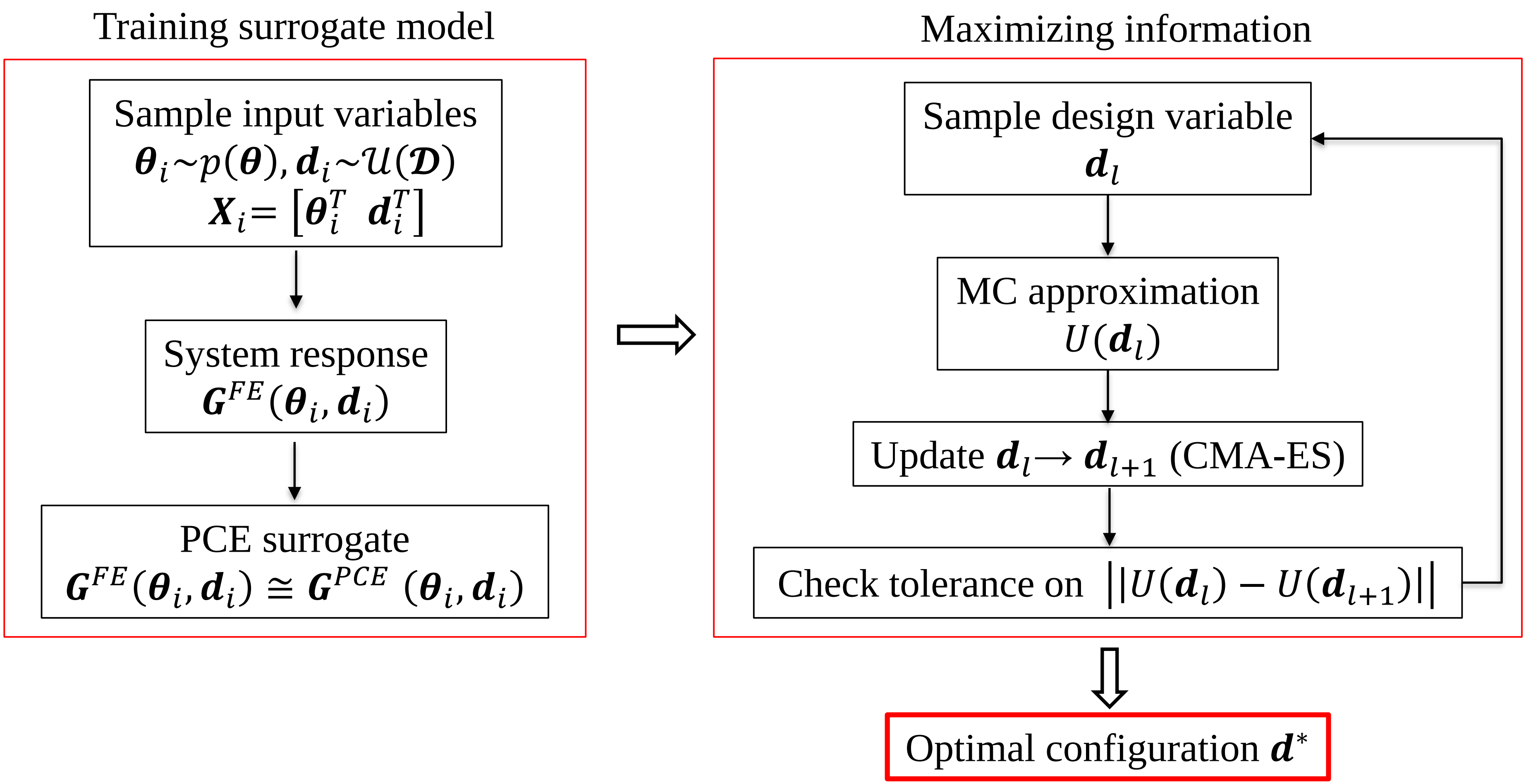
$$\Rightarrow \boldsymbol{y}^{PCE} = M(\boldsymbol{X}) = \sum_{\boldsymbol{\alpha} \in \mathbb{N}^M} y_\alpha \psi_\alpha (\boldsymbol{X})$$

- **Optimization: Covariance Matrix Adaptation Evolution Strategy (CMA-ES)**

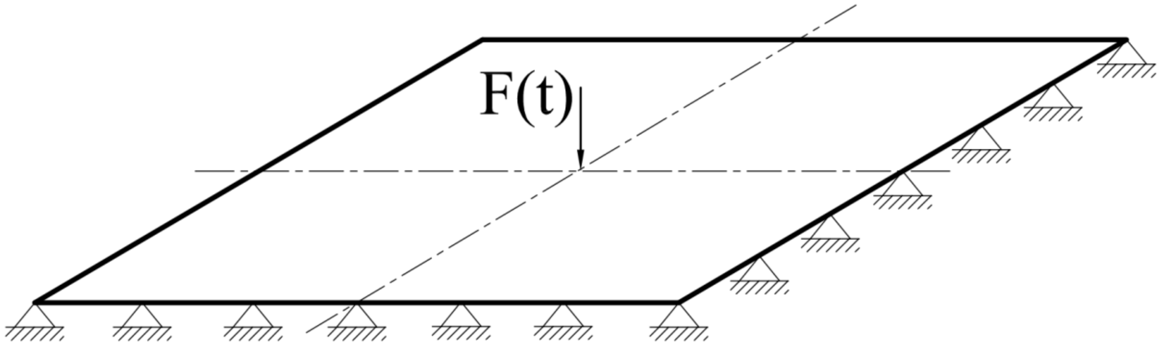
1. $\boldsymbol{d}_i \sim \boldsymbol{m} + \sigma \mathcal{N}_i(\mathbf{0}, \boldsymbol{C})$ $\quad\quad$ $\boldsymbol{m} \in \mathbb{R}^{n_d}, \boldsymbol{C} \in \mathbb{R}^{n_d \times n_d}$
2. $\boldsymbol{m}$ and $\boldsymbol{C}$ are updated through cumulation
3. Check the tolerance on $U(\boldsymbol{d})$

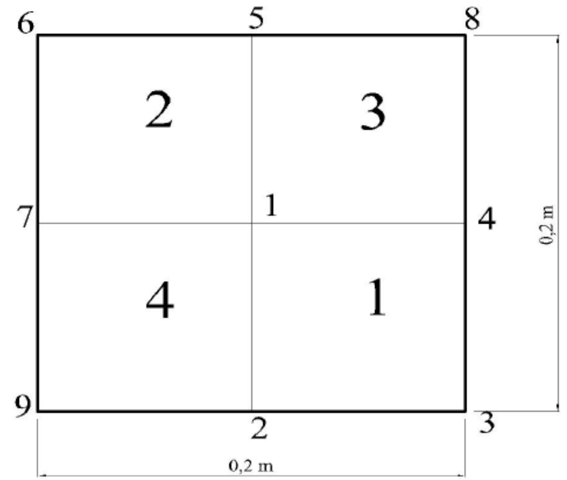
N. **Hansen**, S.D. **Müller**, P. **Koumoutsakos**, (2003).

Bayesian OSP framework



Application: simply supported plate





10x10 mesh: 726 d.o.f.

Displacement measurements

4 zones: $\boldsymbol{\theta} = [E_1, E_2, E_3, E_4]$

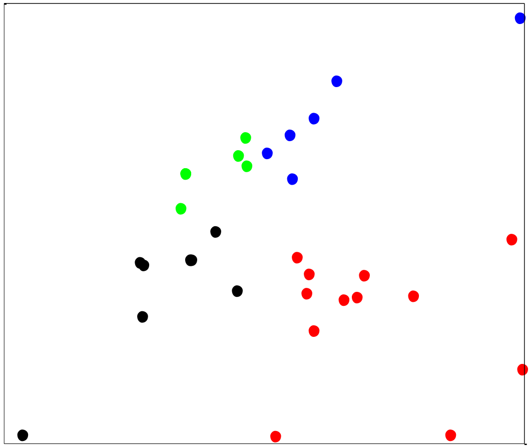
Application: simply supported plate
Choice of prior distribution $p(\boldsymbol{\theta})$

Optimal position of $n_s = 4$ sensors, results of 10 algorithm runs

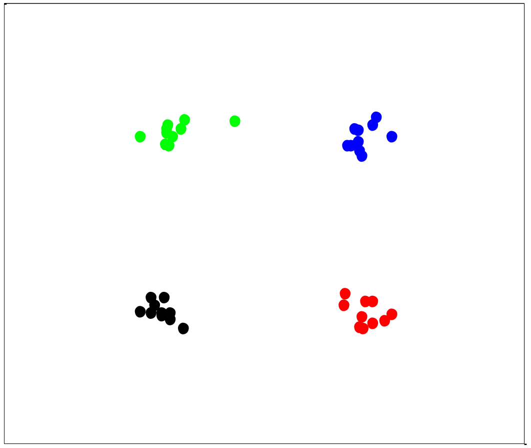
$\boldsymbol{\theta} = [E_1\ E_2\ E_3\ E_4]$

$N_s = 4, N^{PCE} = 10^4,$
$p = 10, N^{MC} = 5 \cdot 10^3$

N_s: # sensors
N^{PCE}: # PCE samples
p: PCE polynomial degree
N^{MC}: # MC samples



$p(\boldsymbol{\theta}) \sim \mathcal{U}[0, E]$



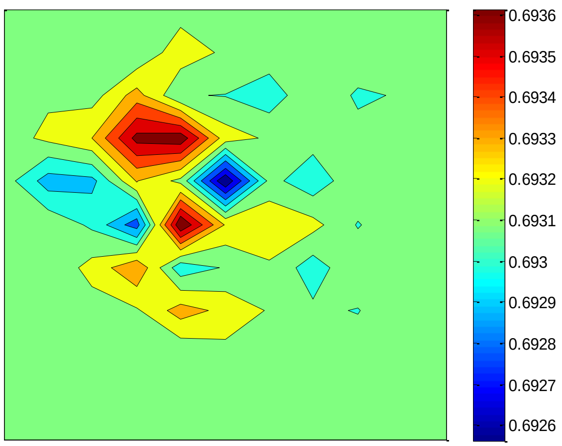
$p(\boldsymbol{\theta}) \sim \mathcal{U}\left[\frac{2\,E}{3}, E\right]$

Application: simply supported plate
Effect of σ_ϵ

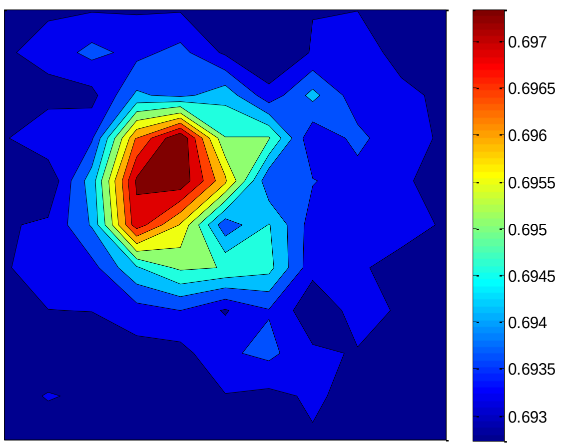
Contour of the objective function with one sensor for each possible location on the plate with different standard deviations of the measurement noise.

$\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$
$\boldsymbol{\theta} = [E_2]$
$N_s = 1, N^{PCE} = 10^4,$
$p = 10, N^{MC} = 5 \cdot 10^3$

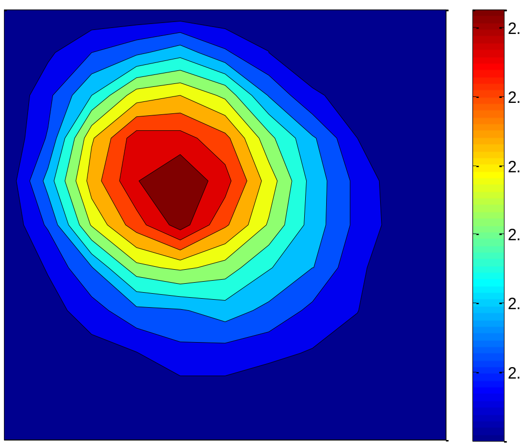
N_s: # sensors
N^{PCE}: # PCE samples
p: PCE polynomial degree
N^{MC}: # MC samples



$\sigma_\epsilon = 10^{-3}$ m



$\sigma_\epsilon = 10^{-4}$ m



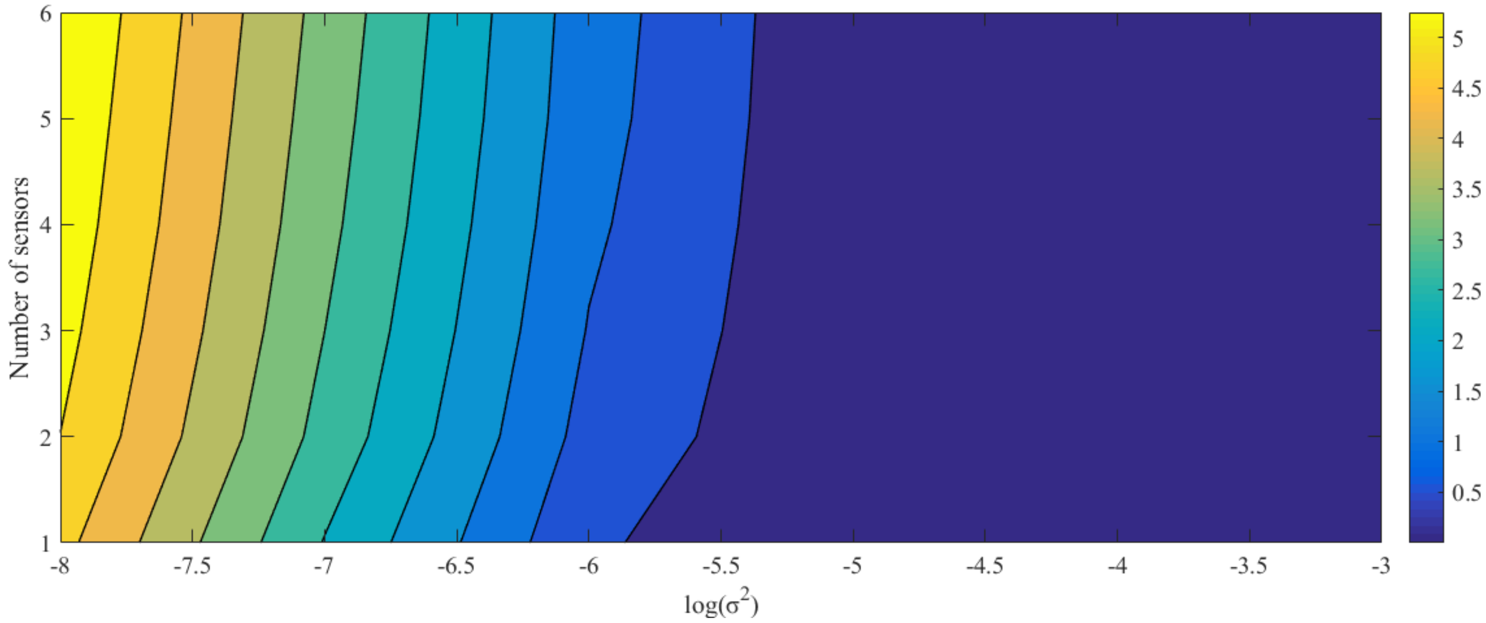
$\sigma_\epsilon = 10^{-5}$ m

Application: simply supported plate
Effect of σ_ϵ and number of sensors

Contour of the objective function with one sensor for different standard deviations and number of sensors.

$\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$
$\boldsymbol{\theta} = [E_2]$
$N^{PCE} = 10^4$,
$p = 10$, $N^{MC} = 5 \cdot 10^3$

N_s: # sensors
N^{PCE}: # PCE samples
p: PCE polynomial degree
N^{MC}: # MC samples



Conclusions

- Optimal sensor placement and SHM system design
- Take into account: ⟸ **Bayesian optimal experimental design**
  - Measurements uncertainties
  - Number of sensors
- **Maximization of expected information gain** between prior and posterior
- Use of **surrogate model (PCE) for MC** approximation and **stochastic optimization (CMA-ES)** methods for computational speed-up
- Future developments: larger number of sensors, larger number of parameters, application to complex cases

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