Braess Paradox in Electrical Networks – When more might mean less

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Abstract: Electrical grids are part of the network of connections maintaining a city alive nowadays. Most of these connections are wired in parallel in order to guarantee a sustainable flow of electricity plus being robust enough against failures. The Braess Paradox states that in a congested network, it may happen that adding a new path between destinations can increase the level of congestion. In this communication the Braess Paradox is investigated for several network configurations. Special interest is dedicated to the Wheatstone bridge and to those networks containing such configurations as part of their structural elements. The flow across the network as well as the overall resistance are computed and expressed in terms of network characteristics.

Keywords: *Braess Paradox, Wheatstone bridge, electrical networks, Smart Grid, adjacency matrix, geodesic path, clustering coefficient.*

Introduction

Electrical grids are part of the network of connections maintaining a city alive nowadays. Many times we see a limited amount of lines and poles, as well as, supportive local electrical transformer points (LETP). Most of these connections are wired in parallel in order to guarantee a sustainable flow of electricity plus being robust enough against failures. Why are we not making the system redundant and increasing the number of grid points and cabling? How the performance of a power grid network can be assessed from its connectivity pattern? In answering the first question, we hypothesized that despite of the economic cost of such approach there is a counter intuitive fact known as the Braess Paradox, which states that in a congested network, it may happen that adding a new path between destinations can increase the level of congestion [1 – 4]. This fact is extremely important when you are designing Smart Grids and Cities. The electrical grid can be mapped into a graph or network, where the hubs are vertices and power lines are edges. The idea is to optimize the topology of the network and make it a smart grid. In transportation networks, the phenomenon results from the decisions of network participants who selfishly seek to optimize their own performance metrics. In an electric power distribution network, an analogous increase in congestion can arise as a consequence of Kirchhoff's laws. To address the second question, we also hypothesized that power grid performance might be assessed through a combination of indices characterizing the networks and enabling quantify the easiness of connecting two distant points by walking the shortest path.

In this communication, the Braess Paradox is investigated for several network configurations. Special interest is dedicated to the Wheatstone bridge and to those networks containing such configurations as part of their structural elements. The flow across the network as well as the overall resistance are computed and expressed in terms of network characteristics.

Model and Results

Braess Paradox might be understood by appealing to the concept of signed graphs, in which positive edges (+) represent arcs along which the current flow encounters the least resistance, while negative edges (-) represent arcs with high resistance load. The collection of positive edges forms the shortest path in the model graph. Even though the results shown below are done for 10 different vertices with a varying amount of edges, it might be extended to the analysis of realistic electrical grids. The above algorithm was implemented in Wolfram Language and Mathematica and results are presented below.

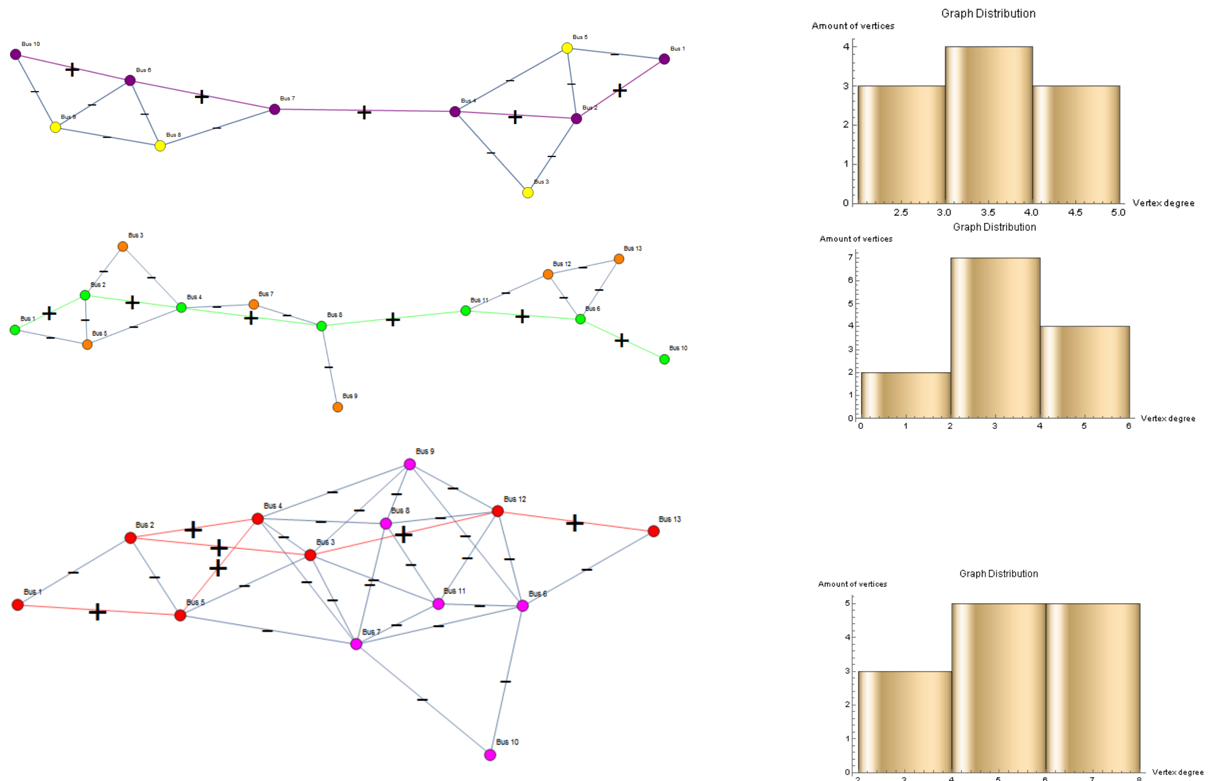

Fig 1: (upper panel) The reduced network graph (10 vertices) where the highlighted lines represent the shortest and convenient path from one node to any other that traverses the least amount of resistances present. These are labeled with "+" while all others are indicated with a "-". (middle panel) Upon creating the 13 - bus system with the added edges to the reduced network, we can see that based on the distribution of "+" & "-" lines in both network, the reduced network is balanced, but the 13 - bus network is unbalanced. Now, by comparing the values of the total resistance traversed between both networks, we see that the reduced network' s path was more efficient in avoiding the larger resistances. (lower panel) After creating a 13-bus network with 31 connections. Actually, the more edges you add to the network, the more unbalanced the system becomes.

Conclusions

Braess Paradox, a concept originally involved with traffic networks and its counterintuitive approach of adding an additional road to alleviate congestion, can be extended to its implications with electrical grids. Specifically, the consequences of erroneously increasing the number of grid points and cablings for better transmission of electrical power in a network may actually decrease the networks level of performance and lead to detrimental loses in electrical power flow and ultimately cause power outages across the grid.

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Conflicts of Interest

The authors declare no conflict of interest.

References and Notes

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