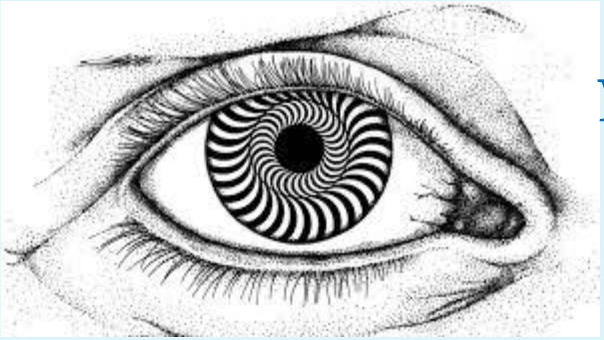


Your bait of falsehood takes this carp of truth

W. Shakespeare, "Hamlet"

"*Novel Approach: Information Quantity for Calculating Uncertainty of Mathematical Model* "

Dr. Boris Menin

Over the past two decades, the considerable efforts are made to develop methods allowing the design of the mathematical models with the lowest discrepancy from the observed material object.

Numerous methods and criteria have been proposed to achieve this goal. However, all of them are focused on identifying *a posterior* uncertainty caused by the ineradicable gap between model and a physical system.

The present approach is focused to formulate the *a priori* interaction between the level of detailed descriptions of the material object (the number of recorded variables) and the lowest achievable total experimental uncertainty of the main researched parameter.

The very act of measurement process already presupposes the existence of the physical-mathematical model describing the phenomenon under investigation.

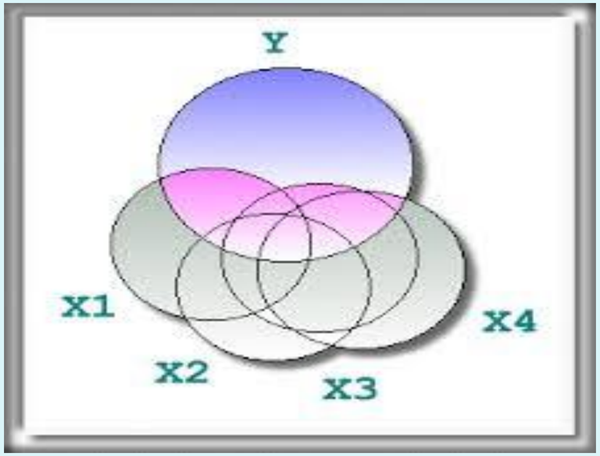
Measurement theory focuses on the process of measuring the experimental determination of the values by using special hardware called measuring instruments. This theory covers only the aspects of data analysis and measurement procedures of the variable observed or after formulating a mathematical model.

Thus the problem that there is uncertainty *before* experimental or computer simulation and caused by limited number of variables recorded in the mathematical model is generally ignored in the measurement theory.

Human intuition and experience tell a simple, at first glance, the truth. For a small number of variables, the researcher gets a rough picture of the process under study.

In turn, the huge number of variables recorded can allow a deep and complete understanding of the structure of the phenomenon. However, this apparent attractiveness of each variable brings its own uncertainty in the integrated (theoretical or experimental) uncertainty of the model or experiment. Moreover, the complexity and cost of computer modeling and field tests increase tremendously.

Thus, a rational or optimal number of variables that is specific to each of the studied processes need to be considered in order to evaluate the physical-mathematical model.



Let's start with a simple example. We see the position of the point x on the segment of length S (range of observation) with uncertainty Δx. We introduce the definition:

- absolute uncertainty is Δx, (1)
- relative uncertainty is $r = \Delta x/x$, (2)
- comparative uncertainty is $\varepsilon = \Delta x/S$. (3)

The accuracy of the experiment ω can be defined as the value inverse to:

$$\omega = 1/\varepsilon = S/\Delta x \quad (4)$$

This definition satisfies the condition that greater accuracy corresponds to the lower comparative uncertainty. Absolute and relative uncertainties are familiar to physicists. Regarding the comparative uncertainty, it is rarely mentioned. Nevertheless, this value is of great importance in the application of information theory to physics and engineering sciences.

Addressing Comparative Uncertainty in Multi-Variable Systems

Level

- System of Primary variables (SI)
- Class of Phenomena
- Structure of Model
- Computerization of Model or Test Realization

Types

- Absolute Uncertainty
- Relative uncertainty
- Comparative uncertainty

Limitations

- Knowledge of the observation interval is required
- An equiprobable choice of variables is affirmed

Applications

- Technological
- Industrial
- Physical

Analytical result of the comparative uncertainty is a surprisingly simple relation

If all the events are equiprobable, the quantity of information obtained by observing the object ΔZ is equal to

$$\Delta Z = \boldsymbol{k_b} \cdot \ln(\mathbf{S}/\Delta x) = -\boldsymbol{k_b} \cdot \ln\varepsilon = \boldsymbol{k_b} \ln\omega, \qquad (5)$$

where $\mathbf{S}/\Delta x$ is the number of events, $\boldsymbol{k_b}$ is the Boltzmann constant, 1.38 m² kg/(s² K).

If the range of observation **S** is not defined, the information obtained during the observation/measurement cannot be determined, and the entropic price becomes infinitely large.

In turn, the efficiency Q of experimental observation, on the assumption that the some perturbation is added in the system under study, may be defined as the ratio of the obtained information ΔZ to a value equal to the increase in entropy ΔH accompanying observation:

$$Q = \Delta Z / \Delta H \qquad (6)$$

It follows from all the foregoing that the modeling is an information process in which information about the state and behavior of the observed object is obtained by the developed model. This information is the main subject of interest of modeling theory.

De facto, the physical-mathematical model formulation is based on two guidelines:

Observation is framed by a System of Primary Variables (SPV)

The harmonic construction of modern science is based on a simple consensus that any physical laws of micro- and macro-physics are described by quite certain dimensional variables: primary and derived (secondary) quantities. Taking variable as a fundamental generally means that it can be assigned as a standard of measurement, which is independent from the standard that chosen for the other fundamental variable. The primary variables are selected arbitrarily, while the

secondary variables are chosen to satisfy discovered physical laws or relevant definitions. The variables are selected within a pre-agreed system of primary variables (SPV) such as SI (International system of units) or CGS (centimeter–gram–second system of units). The SPV is a set of dimensional variables, which are primary and can generate secondary variables. They are necessary and sufficient to describe the known laws of nature, as in the quantitative physical content.

Number of variables taken into account in the physical-mathematical model is limited

SPV includes the primary and secondary variables used for descriptions of different classes of phenomena (*CoP*).

In other words, the additional limits of the description of the studied material object are caused due to the choice of *CoP* and the number of secondary parameters taken into account in the mathematical model. For example, in mechanics SI (International system of units) uses the basis {***L***– length, ***M***– mass, ***T***– time}, i.e. $CoP_{SI} \equiv \boldsymbol{LMT}$. Basic accounts of electromagnetism here add the magnitude of electric current ***I***. Thermodynamics requires the inclusion of thermodynamic temperature $\boldsymbol{\Theta}$. For photometry it needs to add ***J***– force of light. The final primary variable of SI is a quantity of substance ***F***.

If SPV and *CoP* are not given, then the definition of "information about researched object" loses its force. Without SPV, the modeling of phenomenon is impossible.

In SI there are $\xi = 7$ primary variables: ***L*** is the length, ***M*** is the mass, ***T*** is time, ***I*** is the electric current, ***Θ*** is the thermodynamic temperature, ***J*** is the force of light, ***F*** is the number of substances.

The dimension of any secondary variable ***q*** can only be expressed as a unique combination of dimensions of the main primary variables to different powers (1):

$$\boldsymbol{q} \supset \boldsymbol{L}^{l}\, \boldsymbol{M}^{m}\, \boldsymbol{T}^{t}\, \boldsymbol{I}^{i}\, \boldsymbol{\Theta}^{\Theta}\, \boldsymbol{J}^{j}\, \boldsymbol{F}^{f}. \tag{7}$$

l, *m*... *f* are exponents of the variables, the range of each has a maximum

and minimum value. The exponents of variables can take only integer values and they change in the following ranges:

$$-3 \le l \le +3,\ \ -1 \le m \le +1,\ \ -4 \le t \le +4,\ \ -2 \le i \le +2,$$

$$-4 \le \Theta \le +4, \quad -1 \le j \le +1, \quad -1 \le f \le +1. \tag{8}$$

So, the number of choices of dimensions for each variable, according to (8), is the following:

$$\boldsymbol{e}_l = 7;\ \boldsymbol{e}_m = 3;\ \boldsymbol{e}_t = 9;\ \boldsymbol{e}_i = 5;\ \boldsymbol{e}_\theta = 9;\ \boldsymbol{e}_j = 3;\ \boldsymbol{e}_f = 3. \tag{9}$$

- The total number of dimension options of physical variables equals

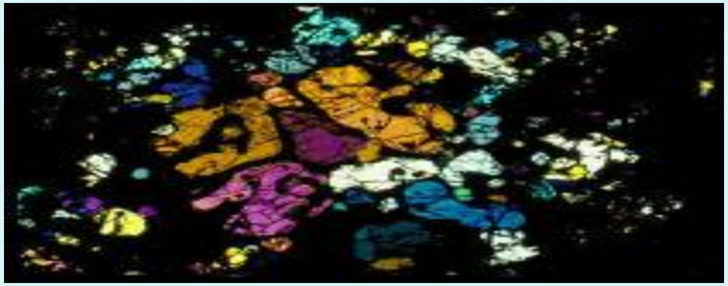
$\boldsymbol{\Psi}° = \boldsymbol{e}_l \cdot \boldsymbol{e}_m \cdot \boldsymbol{e}_t \cdot \boldsymbol{e}_i \cdot \boldsymbol{e}_\theta \cdot \boldsymbol{e}_j \cdot \boldsymbol{e}_f - 1 = 7 \cdot 3 \cdot 9 \cdot 5 \cdot 9 \cdot 3 \cdot 3 - 1 = 76\,544,$ (10)

where "-1" corresponds to the case where all exponents of the primary variables in the formula (7) are treated to zero dimension.

Ψ° includes both required, and inverse variables (for example, L^1 is the length, L^{-1} is the running length). The object can be judged knowing only one of its symmetrical parts, while others structurally duplicating this part may be regarded as information empty. Therefore, the number of options of dimensions may be reduced by **ω** = 2 times. This means that the total number of dimension options of physical variables without inverse variables equals **Ψ** = **Ψ°**/2 = 38,272.

According to **π**-theorem, the number $\aleph_{SI}$ of possible dimensionless complexes (criteria) with ξ = 7 main dimensional variables for SI will be

$\aleph_{SI} = \boldsymbol{\Psi} - \xi = 38{,}272 - 7 = 38{,}265.$ (11)

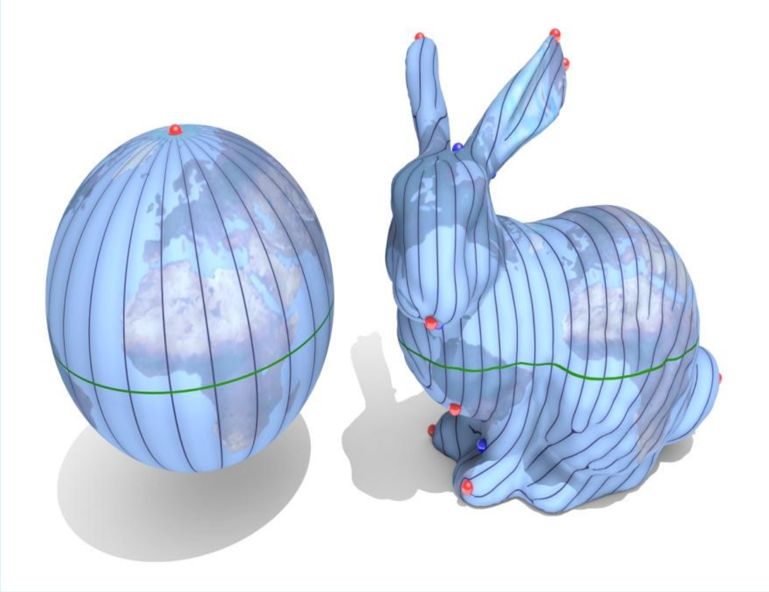


We denote Δ_{pmm} as the uncertainty in determining the dimensionless theoretical field ***u***, "embedded" in a physical-mathematical model and caused only by its dimension that is the property of the model to reflect a certain number of characteristics of researched phenomena, its external and internal connections:

$$\Delta_{pmm} = \Delta_{pmm}' + \Delta_{pmm}'', \qquad (12)$$

where Δ_{pmm}' is the uncertainty due to *CoP*, which is associated with the reduction in the number of recorded primary variables compared with SPV; and Δ_{pmm}'' is the uncertainty due to the choice of the number of recorded influencing variables within the framework of the set of *CoP*.

The equation (12) is an expression of the fact that during modeling of any phenomenon or technological process and equipment there is a gap between the researched object and its theoretical representation in physical-mathematical form due to choosing only *CoP* and a number of variables recorded by the conscious observer due to his knowledge, experience and intuition.



An overall uncertainty of the model including inaccurate input data, physical assumptions, the approximate solution of the integral-differential equations, etc., will be larger than Δ_{pmm}. Thus, Δ_{pmm} is only one component of a possible mismatch of real object and its modeling results. In turn, Δ_{pmm}'' cannot be defined without declaration of the chosen *CoP* (Δ_{pmm}'). So, according to its nature, Δ_{pmm} will be equal to the sum of two terms.

When comparing different models (according to a value of $\mathbf{\Delta}_{pmm}$) describing the same object, preference should be given to the model for which $\mathbf{\Delta}_{pmm}/\mathbf{\Delta}_{exp}$ is closer to 1. The uncertainty Δ_{exp} is the estimated uncertainty in the determination of the generalized objective function (similarity criterion) during an experiment or computer simulation. It will be always larger than Δ_{pmm}.

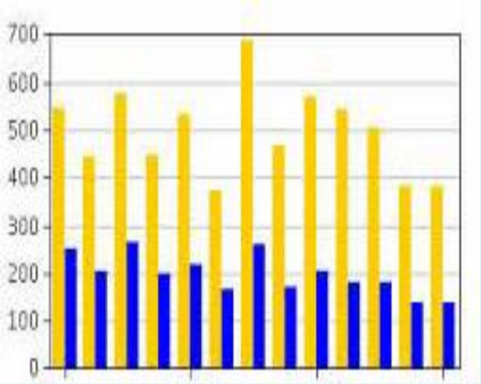
Many different models may describe essentially the same object, where two models are considered to be essentially the same if they are indistinguishable from a value of Δ_{pmm}.

For the purposes of our research, with some physical intuition thrown in, assume that the recognized material object has a huge number of properties (variables, complexes) that characterize its content and interaction with the environment. Then we assume that each dimensionless complex represents the original readout through which some information on the dimensionless researched field ***u*** (recognized object) can be obtained by the observer.

In addition, the modeler takes into account the relatively small number of variables than the current reality due to constraints of time, technical and financial resources. Therefore, the "image" of the object being studied is shown in the model with a certain uncertainty, which depends primarily on the number of variables taken into account. In addition, the object can be addressed by different groups of researchers, who use different approaches for solving specific problems and, accordingly, different groups of variables, which differ from each other in quality and quantity of the contents of variables. Such, for example, happened when studying the motion of an electron, like particle or wave. Thus, for any physical or technical problem, the occurrence of a particular variable in the model can be considered as a random process.

It is supposed that the accounting of readouts (complexes/variables) is equiprobable. We want to emphasize that use of the concept "readout" in examining some object at the stage of the model development is due to the expediency of the vector (positional) ways of representing information of the observed phenomena.

When there are a large numbers of components (a large-dimensional vector space) it is possible to distinguish only two states of the vector component: for example, presence or absence of a signal, in our case the appearance or lack of a readout-variable.



It should be noted that the approval of the equiprobable occurrence of readout is justified by the purpose of the research – finding the absolute uncertainty Δ_{pmm} stipulated by the level of the detail of the researched object. Indeed, any other distribution of readouts yields less information, which leads to a larger uncertainty of the model in comparison with an uncertainty calculated at the uniform distribution of readouts.

This approach completely ignores the human evaluation of information. In other words, a set of 100 notes played by chimpanzees, and a melody of Mozart's 100 notes in his Piano Concerto No.21-Andante movement, have exactly the same amount of information. Let there be $\aleph_{SI}$ readouts, such that there is an uncertainty directly related to $\aleph_{SI}$. That is, the larger the $\aleph_{SI,}$ the greater the uncertainty. Its measured numerical value is called entropy, and may be calculated by the formula:

$$\mathbf{H} = \boldsymbol{k_b} \cdot \ln \aleph_{SI}, \qquad (13)$$

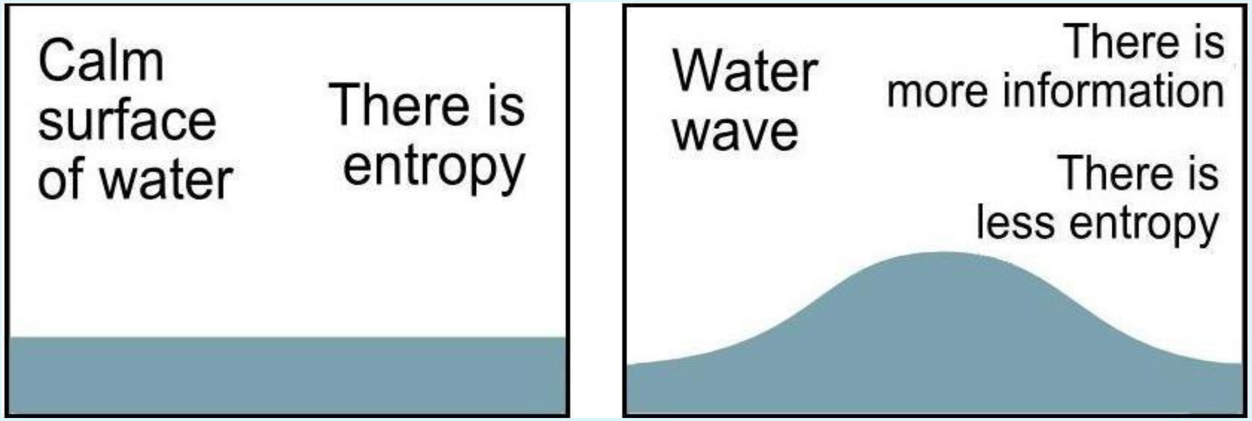
where $\boldsymbol{k_b}$ is the Boltzmann's constant.

When a researcher chooses the influencing factors (the conscious limitation of the number of variables describing an object), the mathematical model entropy is decreased *a priori*. It is natural to measure the entropy change by a parameter:

$$\Delta\mathbf{H} = \mathbf{H}_{pr} - \mathbf{H}_{ps}, \qquad (14)$$

where $\Delta\mathbf{H}$ is entropy difference between the two cases, pr – "*a priori*",

ps - "*a posterior*".



If one considers that the efficiency Q of the passive mental method equals to one because just a thought experiment is conducted and no distortion is brought into the real system (modeler is thinking only), then one can write according to (14):

$$\boldsymbol{\Delta A} = \mathbf{Q} \cdot \mathbf{\Delta H} = \mathbf{H}_{pr} - \mathbf{H}_{ps}, \qquad (15)$$

where **ΔA** is the *a priori* information quantity about the material object.

Using Equations (13)-(15) and imposing symbols: **z**' being the number of physical dimensional values in the selected *CoP*, **β**' is the number of primary physical dimensional values in the selected *CoP*, we obtain:

$$\boldsymbol{\Delta A'} = \mathbf{Q} \cdot (\mathbf{H'}_{pr} - \mathbf{H'}_{ps}) = 1 \cdot [\boldsymbol{k_b} \cdot \ln \aleph_{SI} - \boldsymbol{k_b} \cdot \ln(\mathbf{z'} - \boldsymbol{\beta'})] = \boldsymbol{k_b} \cdot \ln[\aleph_{SI} / (\mathbf{z'} - \boldsymbol{\beta'})], \qquad (16)$$

where $\boldsymbol{\Delta A'}$ is the *a priori* amount of information quantity about the observed object due to the choice of *CoP*.

The value $\boldsymbol{\Delta A'}$ is linked to $\boldsymbol{\Delta}_{pmm}'$ and **S** (the dimensionless interval of supervision of a field ***u***) by the dependence (Brillouin):

$$\boldsymbol{\Delta}_{pmm}' = \mathbf{S} \exp(-\boldsymbol{\Delta A'} / \boldsymbol{k_b}). \qquad (17)$$

where $\boldsymbol{\Delta}_{pmm}'$ is the *a priori* uncertainty of the observed object model cased only due to the choice of *CoP*.

Substituting (16) into (17):

$$\mathbf{\Delta_{pmm}' = S\,(z' - \beta')/\aleph_{SI}}. \qquad (18)$$

Following the same reasoning, it can be shown that **Δ**'' is the following:

$$\mathbf{\Delta_{pmm}'' = S\,(z''- \beta'')/(z'- \beta')}, \qquad (19)$$

where Δ_{pmm}'' is the *a priori* uncertainty of the observed object model cased only due to the choice of the number of recorded dimensionless variables in a model; **z**'' is the number of physical dimensional variables recorded in a mathematical model; **β**'' is the number of primary physical dimensional variables recorded in a model. Then, summarizing $\mathbf{\Delta_{pmm}}'$ and $\mathbf{\Delta_{pmm}}''$, one can estimate the value $\mathbf{\Delta_{pmm}}$.

All of the above could be summarized as follows in the form of **ℵ** –hypothesis: *Let during a model formulation the chosen system of primary variables with the total number of the dimensional physical variables be denoted by **Ψ**, ξ of which are of independent dimension. In the framework of the class of phenomena (the total number of the dimensional variables **z**', the number of the primary dimensional variables **β**') there is a dimensionless field **u** raised in a given range of values **S**. Then the absolute uncertainty of **u** calculation Δ_{pmm} (for a given number of the recorded physical dimensional variables **z**'', of which **β**'' is the number of the recorded primary physical dimensional variables) can be determined from the relationship*:

$$\mathbf{\Delta_{pmm} = S \cdot [(z' - \beta')/(\Psi - \xi) - (z'' - \beta'')/(z' - \beta')]}, \qquad (20)$$

where $\varepsilon = \mathbf{\Delta_{pmm}/S}$ is the comparative uncertainty.

Using formula (20), one can find the recommended uncertainty value with the theoretical analysis of the physical phenomena. Moreover, equation (20) also can inform a limit on the advisability of obtaining an increase of the measurement accuracy in conducting pilot studies or computer simulation. It is not a purely mathematical abstraction and it represents an intrinsic property of the model caused only by the number of selected variables and the chosen *CoP*. Equation (20) has physical meaning. This relationship testifies that in nature there is a fundamental limit to the accuracy of measuring any observed material object, which cannot be surpassed by any improvement of instruments, methods of measurement and the model's computerization. The value of this limit is much more than the Heisenberg uncertainty relation provides and places severe restrictions on the micro-physics.

The overall uncertainty model including additional uncertainties associated with inaccurate input data, physical assumptions, the approximate solution of the integral-differential equations, etc., will be larger than $\mathbf{\Delta}_{\mathrm{pmm}}$.

Factually, equation (20) can be regarded as the uncertainty principle for the model development process. Namely, any change in the level of the detailed description of the observed object (z''-β''; z'-β') causes a change in the uncertainty model Δ_{pmm} and the accuracy of each main variable characterizing the properties of the object internal structure.

Equating the derivative of $\boldsymbol{\Delta u}/\boldsymbol{S}$ (20) with respect to z'-β' to zero, we obtain the condition for achieving the minimum comparative uncertainty for a particular *CoP*:

$$(z'-\beta')^2/(\boldsymbol{\Psi}-\underline{\boldsymbol{\xi}})\ ) = (z''-\beta'') \qquad (21)$$

By usage of (21) we can find values of the lowest achievable comparative uncertainty of different CoP_{SI} :

1. For mechanics processes ($CoP_{SI}\equiv \boldsymbol{LMT}$), taking into account the aforementioned explanations and (21), the lowest comparative uncertainty $\varepsilon_{\boldsymbol{LMT}}$ can be reached at the following conditions:

$$(z'-\beta') = e_l \cdot e_m \cdot e_t -1)/2-3=(7\cdot 3\cdot 9-1)/2-3=91, \qquad (22)$$

$$(z''-\beta'')= (z'-\beta')^2/ (\boldsymbol{\Psi}-\underline{\boldsymbol{\xi}}) = 91^2 /38{,}265=0.2164<1, \qquad (23)$$

where "-1" corresponds to the case when all the primary variable exponents are zero in formula (7); dividing by 2 indicates that there are direct and inverse variables, e.g., L^1 is the length, L^{-1} is the run length, and 3 corresponds to the three primary variables ***L***, ***M***, ***T***.

According to (22) ε_{LMT} equals

$$\varepsilon_{LMT} = (\boldsymbol{\Delta u}/\boldsymbol{S})_{LMT} = 91/38{,}265+0.2164/91=0.0048. \qquad (24)$$

In other words, according to (23), even one dimensionless main variable does not allow one to reach the lowest comparative uncertainty. Therefore, in the frame of the suggested approach, nobody can realize the original first-born comparative uncertainty using any mechanistic model ($CoP_{\mathrm{SI}} \equiv \boldsymbol{LMT}$). Moreover, the greater the number of mechanical parameters, the greater the first-born embedded uncertainty. In other words, for example, the Cavendish method, in the frame of the suggested approach, is not recommended for measurements of the Newtonian gravitational constant.

Such statements appear to be highly controversial, and one might even say, very unprofessional, not credible and far from current reality. However, as we shall see below, the proposed approach allows the obvious conclusions to be made consistent with practice.

2. For electromagnetic processes ($COP_{SI} \equiv$ ***LMTI***), taking into account (9), the lowest comparative uncertainty can be reached at the following conditions:

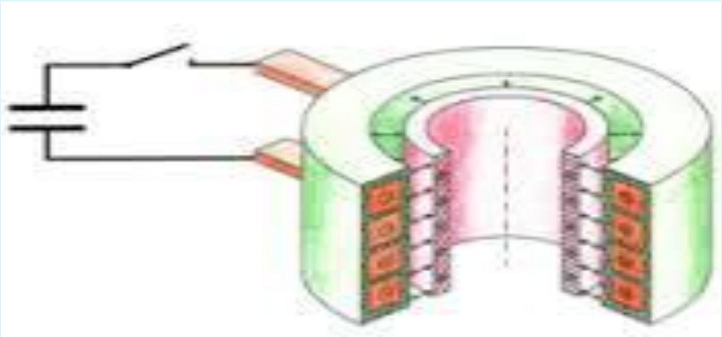
$$(z'-\beta') = e_l \cdot e_m \cdot e_t \cdot e_i -1)/2-4=(7\cdot3\cdot9\cdot5-1)/2-3=468, \qquad (25)$$

$$(z''-\beta'')= (z'-\beta')^2/ (\boldsymbol{\Psi}- \underline{\xi}) = 468^2 /38{,}265=5.723873\approx6, \qquad (26)$$

where "-1" corresponds to the case when all the primary variable exponents are zero in formula (7); dividing by 2 indicates that there are direct and inverse variables, e.g., L^1 is the length, L^{-1} is the run length, and 4 corresponds to the four primary variables ***L***, ***M***, ***T***, ***I***.

Then, one can calculate the minimum achievable comparative uncertainty ε_{LMTI}

$$\varepsilon_{LMTI} = (\Delta u/S)_{LMTI}= 468/38{,}265+5.723873/468=0.0244. \qquad (27)$$



3. For combined heat and electromagnetic processes ($COP_{SI} \equiv \boldsymbol{LMT\Theta I}$), taking into account (9), the lowest comparative uncertainty can be reached at the following conditions:

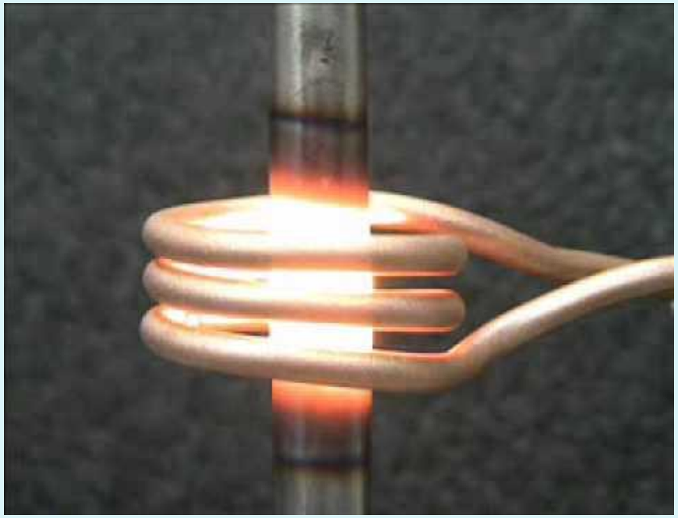
$$(z'-\beta') = e_l \cdot e_m \cdot e_t \cdot e_\Theta \cdot e_i -1)/2-4=(7\cdot 3\cdot 9\cdot 9\cdot 5-1)/2-5=4{,}247, \qquad (28)$$

$$(z''-\beta'')= (z'-\beta')^2/ (\boldsymbol{\Psi}- \boldsymbol{\xi}) = 4{,}247^2 /38{,}265\approx 471, \qquad (29)$$

where "-1" corresponds to the case when all the primary variable exponents are zero in formula (7); dividing by 2 indicates that there are direct and inverse variables, e.g., L^1 is the length, L^{-1} is the run length, and 4 corresponds to the four primary variables ***L***, ***M***, ***T***, *Θ*, ***I***.

Then, one can calculate the minimum achievable comparative uncertainty $\varepsilon_{LMT\Theta I}$

$$\varepsilon_{LMT\Theta I} = (\Delta u/S)_{LMT\Theta I} = 4{,}247/38{,}265+471/4{,}247=0.2219. \qquad (30)$$



4. For heat processes (COP_{SI} = ***LMTΘ***), taking into account (9), the lowest comparative uncertainty can be reached at the following conditions:

$$(z'-\beta') = e_l \cdot e_m \cdot e_t \cdot e_{\Theta} -1)/2-4=(7\cdot 3\cdot 9\cdot 9-1)/2-5=846, \quad (31)$$

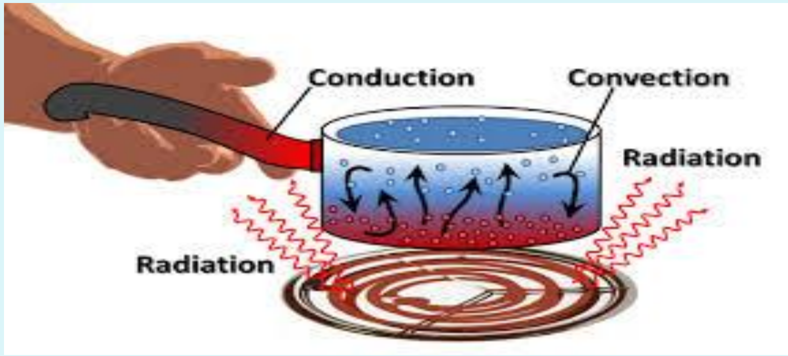
$$(z''-\beta'')= (z'-\beta')^2/ (\boldsymbol{\Psi}- \underline{\xi}) = 846^2 /38{,}265\approx 19, \quad (32)$$

where "-1" corresponds to the case when all the primary variable exponents are zero in formula (7); dividing by 2 indicates that there are direct and inverse variables, e.g., L^1 is the length, L^{-1} is the run length, and 4 corresponds to the four primary variables ***L***, ***M***, ***T***, *Θ*.

Then, one can calculate the minimum achievable comparative uncertainty $\varepsilon_{LMT\Theta I}$

$$\varepsilon_{LMT\Theta} = (\Delta u/S)_{LMT\Theta} = 846/38{,}265+19/846=0.0442. \quad (33)$$

Let us now try to apply the aforementioned method for the analysis of the accuracy of the mathematical models of heat transfer processes and measurement accuracy of fundamental physical constants.



APPLICATIONS OF THE COMPARATIVE UNCERTAINTY METRIC

1.Freezing

The process of freezing and sub-cooling of the paste material layer posted onto a moving cooled cylinder wall has been investigated [1] in two-dimensional space on a closed rectangular region is described. According to analysis of the recorded variables dimensions, the model is classified by CoP_{SI}≡$\boldsymbol{LMT\Theta}$, (z'-β')=846 (28), (z"-β")=19 (29), $\varepsilon_{\boldsymbol{LMT\Theta}}$ = $(\boldsymbol{\Delta u/S})_{\boldsymbol{LMT\Theta}}$ =0.0442 (30).

There were recorded 18 (z*) input dimensional variables and 5 (β*) primary physical variables, such that we obtain z*-β*=18-5=13 for the dimensionless criteria.

A study of the developed model by computer simulation using the random balance method has been conducted. As the objective function, the final dimensionless temperature of the outer surface of the material $\boldsymbol{\Theta}$ was selected.

The declared achieved discrepancy between the experimental and computational data in the range of admissible values of the similarity criteria and dimensionless conversion factors did not exceed 8%.

It can be shown that an absolute total dimensionless uncertainty of the indirect measurement $(\Delta\Theta_s)_{exp}$, reached in the experiment equals

$$(\Delta\Theta_s)_{exp} = 0.066. \qquad (34)$$

From equation (20), using calculated values $\aleph_{SI}$ (11), $z'-\beta'$ (28), and $(z''-\beta'')$ (29), one obtains a dimensionless uncertainty value $(\Delta_s)_{pmm}$ of the chosen model:

$$(\Delta\Theta_s)_{pmm} \leq \Theta_{smax} \cdot ((z'-\beta')/\aleph_{SI} + (z^*-\beta^*)/(z'-\beta')) =$$

$$= 0.93 \cdot [846/38{,}265+13/846] \approx 0.038, \qquad (35)$$

where $\Theta_{smax}= 0.93$ is the dimensionless given range of changes of the dimensionless final temperature allowed by the chosen model [1].

From (34) and (35) we get $(\Delta\Theta_s)_{exp} > (\Delta\Theta_s)_{pmm}$, i.e., an actual uncertainty in the experiment is 1.7 times (0.066/0.038) larger than the possible minimum. It means, at the recorded number of dimensionless criteria the existing accuracy of the dimensional variable's measurement is insufficient. In addition, the number of the chosen dimensionless variables $z^*-\beta^*=13$ is less than the recommended ≈19 (39) that corresponds to the lowest comparative uncertainty at $CoP_{SI} \equiv \boldsymbol{LMT\Theta}$. That is why, for further experimental work it is required to use devices of a higher class of accuracy sufficient to confirm/clarify a new model designed with many dimensionless variables.

2.Gravitational constant G

In none of the current experiments of the calculation of NGC value has the prospective interval been *declared*, in which its true value can be placed. In other words, the exact trace of the placement of G is lost somewhere. Therefore, in order to apply our stated approach, as a possible measurement interval of G, we choose the difference of its value reached by the experimental results of two projects: G_{min} =6.6719199·m^3 $kg^{-1}s^{-2}$ [2] and G_{max} = 6.6755927·$m^3kg^{-1}s^{-2}$ [3].

Then, the possible observed range S* of G variations equals

$$S_G^* = G_{max} - G_{min} = 6.6755927 \cdot 10^{-11} - 6.6719199 \cdot 10^{-11} = $$
$$= 3.6728 \cdot m^3\, kg^{-1}\, s^{-2}. \qquad (36)$$

Taking into account (36), we analyzed several publications and CODATA (*Committee on Data for Science and Technology*) recommendations over the past 15 years (2000–2016) from the position of the reached relative and comparative uncertainty values. These data are summarized in Figures 1, 2.

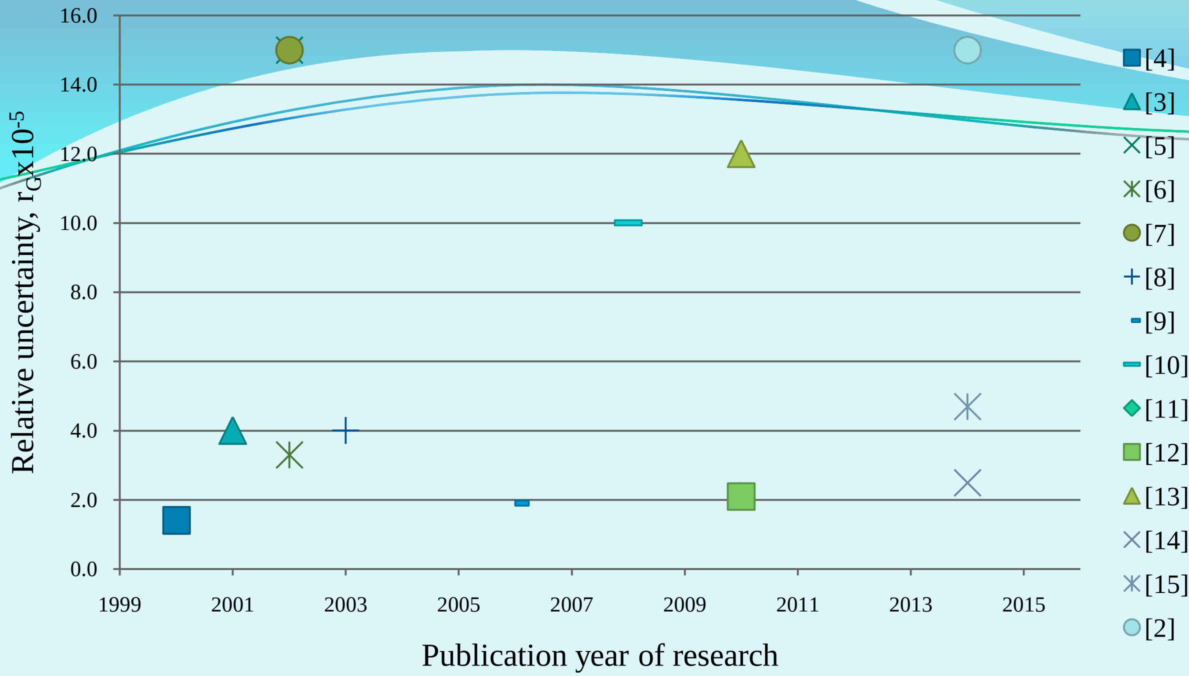


Figure 1. A graph summarizing the partial history of measurement of Newtonian gravitational constant measurements by view of the reached relative uncertainty r_G

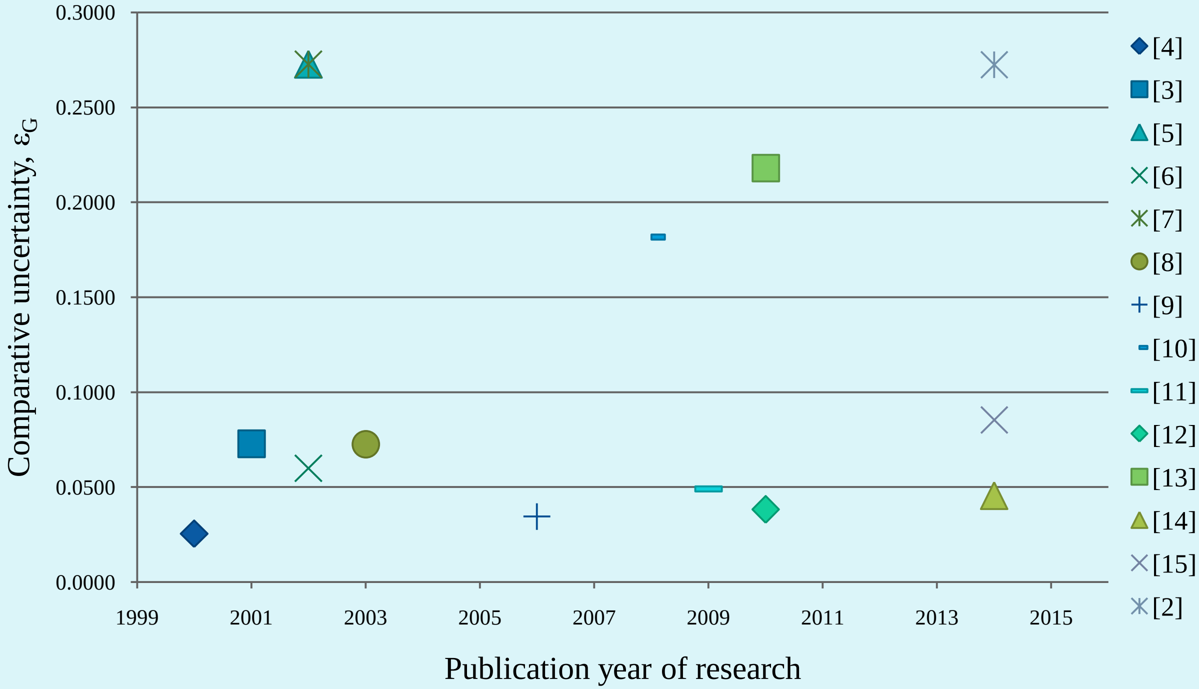


Figure 2. A graph summarizing the partial history of measurement of Newtonian gravitational constant measurements by view of the reached comparative uncertainty ε_{LMTI}

It is seen from the data given in Figures 1 and 2 that there was not a dramatic improvement of the accuracy of the measurement of G during the last 15 years. This is true when based on the calculation of the relative uncertainty, the possible achievable lowest value of which was not mentioned.

In addition, judging the data by the comparative uncertainty according to the proposed approach, one can see that the measurement accuracy had not significantly changed either. Perhaps this situation has arisen as a result of unaccounted systematic errors in these experiments.

At the same time, it must be mentioned that, most likely, the exactness of G as other fundamental physical constants, cannot be infinite, and, in principle, must be calculable. Therefore, the development of a larger number of designs and an improvement of the various experimental facilities for the measurement of G by using schemes combining a torsion balance and electromagnetic equipment (electrostatic servo control) is absolutely necessary in order to obtain closer results to the minimum comparative uncertainty $(\boldsymbol{\varepsilon}_{\min})_{LMTI}$.=0.0244 (27) .

Applying the present approach, we can argue about the order of the desired value of the relative uncertainty $(r_{min})_{LMTI}$. For this purpose, we take into account the following variables: $(\boldsymbol{\varepsilon}_{min})_{LMTI} = 0.0244$ (27), $S_G{}^* = 3.6728 \cdot 10^{-14}$ (36). Then, the lowest possible absolute uncertainty for $CoP_{SI} \equiv \boldsymbol{LMTI}$ equals

$$(\Delta_{min})_{LMTI} = (\boldsymbol{\varepsilon}_{min})_{LMTI} \cdot S^* = 0.0244 \cdot 3.6728 \cdot 10^{-14} = \\ = 8.961632 \cdot m^3\, kg^{-1}\, s^{-2}. \quad (37)$$

In this case, the lowest possible relative uncertainty $(r_{min})_{LMTI}$ for $CoP_{SI} \equiv \boldsymbol{LMTI}$ is as follows:

$$(r_{min})_{LMTI} = (\Delta_{min})_{LMTI} /((G_{max} + G_{min}) /2) = \\ = 8.961632 \cdot 10^{-16} / 6.673756 \cdot 10^{-11} = 1.353823 \cdot 10^{-5} \approx 1.35 \cdot 10^{-5}. \quad (38)$$

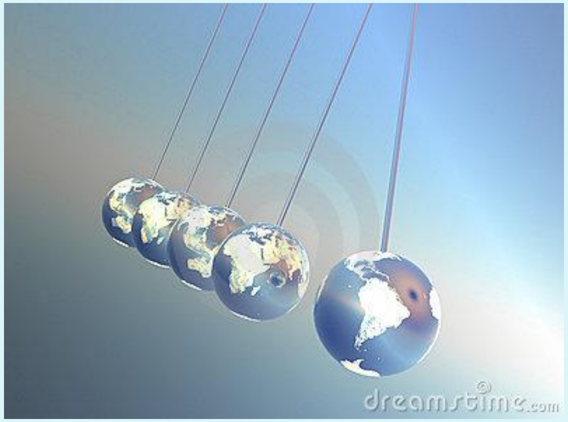
This value is in excellent agreement with the recommendations mentioned in [10] of $1.4 \cdot 10^{-5}$ and could be particularly relevant in the run-up to the adoption of new definitions of SI units.

3.Boltzmann constant

Analysis of the Boltzmann constant measurements made during 2007–2015 shows that none of the current experimental measurements that calculate k_b have declared an uncertainty interval in which the true value can be placed. Therefore, in order to apply the stated approach, as the estimated interval of k_b changes, we choose the difference of its value reached by the experimental results of two projects: $k_{\boldsymbol{b}\max} = 1.38065511 \cdot 10^{-23}$ m² kg/(s² K) [16] and $k_{\boldsymbol{b}\min} = 1.380640 \cdot 10^{-23}$ m² kg/(s² K) [17]. In this case, the possible observed range S_k of k_b variation is equal

$$S_k = k_{\boldsymbol{b}\max} - k_{\boldsymbol{b}\min} = 1.501 \cdot 10^{-28} \text{ m}^2 \text{ kg/(s}^2 \text{ K)}. \quad (39)$$

We studied several scientific publications and CODATA recommendations over eight years from the perspective of the achieved relative and comparative uncertainties values. The data are summarized in Figures 3-5. By analyzing



theoretical methods and experimental schemes, one can declare that results were obtained using $CoP_{SI} \equiv \boldsymbol{LMT\Theta}$ or $CoP_{SI} \equiv \boldsymbol{LMT\Theta I}$.

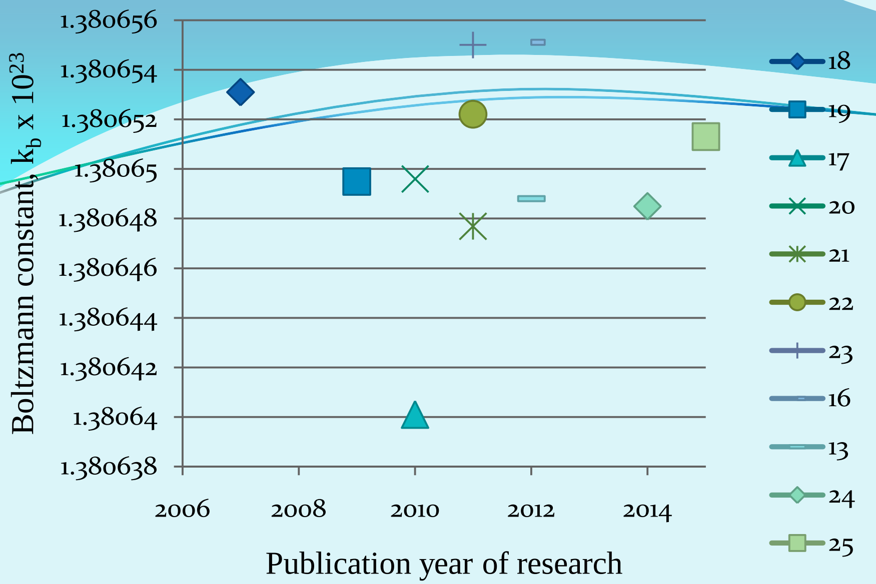


Figure 3. Graph summarizing the partial history of Boltzmann constant measurements by view of the reached its value.

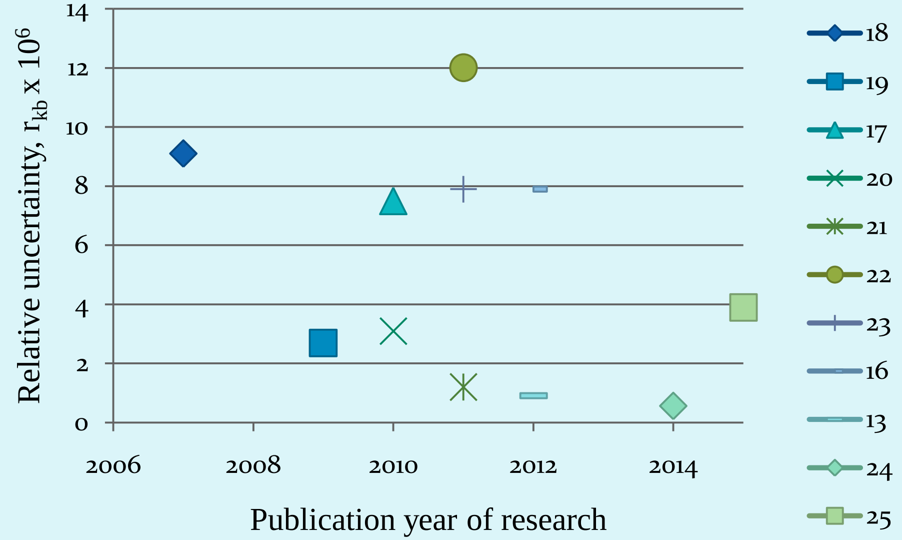


Figure 4. Graph summarizing the partial history of Boltzmann constant measurement, displaying changes in the relative uncertainty.

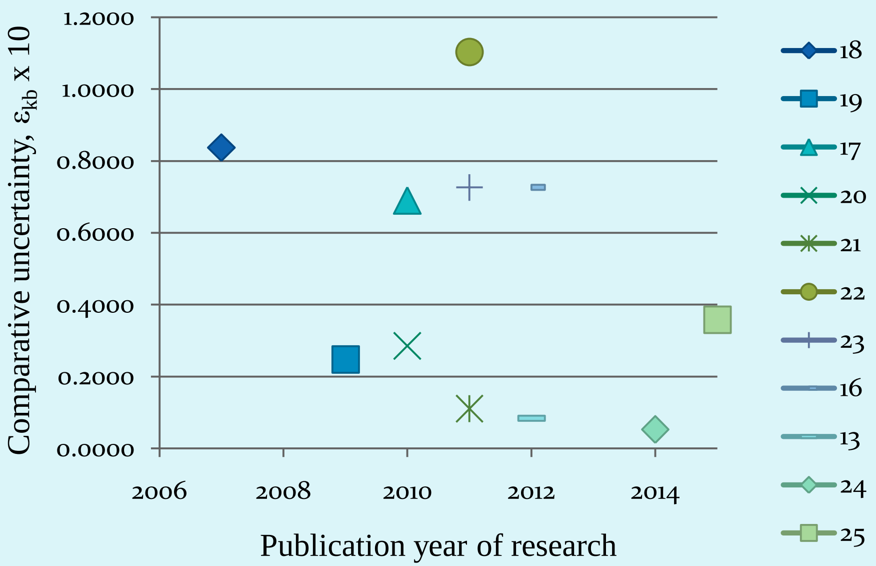


Figure 5. Graph summarizing the partial history of Boltzmann constant measurement displaying changes in the comparative uncertainty.

It can be seen from the data given in Figures 3–5 that a dramatic improvement in the accuracy of measurement of the Boltzmann constant has not been achieved during the last decade, judging the data by both relative and comparative uncertainties and two different CoP_{SI} : ***LMTΘ***, ***LMTΘI***.

Despite the fact that the authors of the mentioned studies stated on account of all possible sources of uncertainty, the value of the absolute and relative uncertainties can differ by more than twenty times. A similar situation exists for the spread of the value of the comparative uncertainty.

Without going into analysis of the uncertainties nature, a part of which the researchers have already identified, we can say with great confidence that, under the proposed approach, one of the reasons for the created unsatisfactory situation is a number of variables taken into account in the measurement or the chosen model for calculation of the Boltzmann constant.

So, for $CoP_{SI} \equiv \boldsymbol{LMT\Theta}$, in order to achieve the minimum comparative uncertainty there must be taken into account 19 variables (33), and for $CoP_{SI} \equiv \boldsymbol{LMT\Theta I}$ already 471 (30) variables. In all these works the number of variables taken into account is much smaller. Thus, to improve the accuracy of measurement of the Boltzmann constant there need to complicate experimental stands. To realize this goal, scientists must be prepared to spend sufficient resources.

However, the key data that provide the 2010 recommended value of k_b would appear to be close to meeting CODATA requirements [13]. At the same time, the development of a larger number of designs and improvements of various experimental facilities for the measurement of the Boltzmann constant is required in order to bring the results closer to the minimum comparative errors $(\boldsymbol{\varepsilon}_{\min})_{LMT\theta}$ or $(\boldsymbol{\varepsilon}_{\min})_{LMT\theta I}$.

We can argue about the order of the desired value of the relative uncertainty of $CoP_{SI} \equiv \boldsymbol{LMT\Theta}$ that is usually used for measurements of the Boltzmann constant. For this purpose, we take into account the following data: $(\boldsymbol{\varepsilon}_{min})_{LMT\theta}$ =0.0442 (33), S_k =1.501·10^{-28} m² kg/(s² K) (39). Then, the lowest possible absolute uncertainty for $CoP_{SI} \equiv \boldsymbol{LMT\Theta}$ equals

$$(\Delta_{min})_{LMT\theta} = (\boldsymbol{\varepsilon}_{min})_{LMT\theta} \cdot S_k = 0.0442 \cdot 1.501 \cdot 10^{-28} =$$
$$= 0.066344 \cdot 10^{-28} \text{ m}^2 \text{ kg/(s}^2 \text{ K)}. \quad (40)$$

In this case, the lowest possible relative uncertainty $(r_{min})_{LMT\theta}$ for $CoP_{SI} \equiv \boldsymbol{LMT\Theta}$ is as follows:

$$(r_{min})_{LMT\theta} = (\Delta_{min})_{LMT\theta} / ((k_{bmax} + k_{bmin})/2) =$$
$$= 0.066344 \cdot 10^{-28} / 1.380648 \cdot 10^{-28} = 4.8 \cdot 10^{-7}. \quad (41)$$

This value is in excellent agreement with the recommendations mentioned in [25] (5.7·10^{-7}), and can be used for the new definition of the Kelvin and a significant revision of the International System of Units (SI).

The concept of relative uncertainty was used when considering the accuracy of the achieved results (absolute value and absolute uncertainty of the separate variables and criteria) during the measurement process in different applications. However, this method for identifying the measurement accuracy does not indicate the direction of deviation from the true value of the main variable. In addition, it involves an element of subjective judgment. That is why, for the purposes of this approach, along with a relative uncertainty, this study recommends a comparative uncertainty for analyzing published results.

The introduced novel analysis is intended to help physicists and designers to determine the most simple and reliable way to select a model with the optimal number of recorded variables calculated according to the minimum achievable value of the model uncertainty.

The information approach and its presented results can be used for the prediction of the model's discrepancy of physical phenomenon and technological process for the practical problems of macro- and microphysics.

One important remark about the physical meaning of the proposed information approach. Any physical process, from quantum mechanics to palpitation, can be viewed by the observer only through the idiosyncratic "lens". Its material is alloy of not only mathematical equations, but also, without fail, regardless of the researcher's desire, his intuition, experience and knowledge. They, in turn, are framed by a system of primary variables, which is also chosen by the universal agreement of human individuals. Thus, the aberration in modeling (distortion of reality) is inherent, before the formulation of any physical, and even more so, mathematical statement. The degree of depravity of the image of a true real object depends precisely on the chosen class of phenomena and the number of variables considered.

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Thank you