THE HOLOGRAPHIC BOUND IN NEWTONIAN COSMOLOGY

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Summary

Introduction

The continuum description of spacetime breaks down at short length scales and/or high curvatures.

A continuum description emerges after coarse graining some unknown, underlying degrees of freedom.

Thermodynamical approach: ignore large amounts of detailed knowledge, concentrate on a few coarse–grained averages.

Emergent approach to spacetime: gravity is an entropic force.

We do not know the fundamental degrees of freedom of gravity, but their coarse–grained effect is to drive the system in the direction of increasing entropy.

Gravitational equipotential surfaces can be identified with isoentropic surfaces.

The (baryonic and dark) matter content of a hypothetical Newtonian Universe is regarded as a density of particles $|\psi|^2$, where ψ satisfies the Schroedinger equation

$$H\psi = E\psi$$

Given the gravitational potential U, the expectation value $\langle\psi|U|\psi\rangle$ measures the gravitational entropy of the Universe when the matter is in the state ψ.

Methods

Newtonian cosmology:

Gravity described by the Poisson Eq.,

$$\nabla^2 U = 4\pi G \rho,$$

matter described by continuity and Euler Eqs. (ideal fluid):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \qquad \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} + \frac{1}{\rho}\nabla p - \mathbf{F} = 0.$$

Hubble's law:

$$\mathbf{v} = H_0 \mathbf{r}, \quad H_0 = \mathrm{Hubble's\ constant}$$

This implies a repulsive harmonic potential

$$U_{\mathrm{Hubble}}(\mathbf{r}) = -\frac{H_0^2}{2}\mathbf{r}^2$$

Madelung: Factorising ψ into amplitude and phase,

$$\psi = \exp\left(\frac{\mathcal{S}}{2k_B} + \mathrm{i}\frac{\mathcal{I}}{\hbar}\right),$$

Schroedinger quantum mechanics becomes a fluid mechanics:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\,\mathbf{v} + \frac{1}{m}\nabla\mathcal{Q} + \frac{1}{m}\nabla V = 0,$$

$$\mathcal{Q} := -\frac{\hbar^2}{2m}\left[(\nabla S)^2 + \nabla^2 S\right], \quad S := \frac{\mathcal{S}}{2k_B}, \quad \mathbf{v} = \frac{1}{m}\nabla\mathcal{I}$$

$\mathcal{Q}$ is the quantum potential, V the external potential in $H\psi = E\psi$.

Both Newtonian cosmology and Schroedinger quantum mechanics are fluid mechanics:

	Euler	Madelung
volume density	ρ	$\exp(2S)$
velocity	$\mathbf{v}$	$\nabla\mathcal{I}/m$
pressure term	$\nabla p/\rho$	$\nabla\mathcal{Q}/m$
external forces	$\mathbf{F}$	$-\nabla V/m$

Thus Newtonian cosmology can be regarded as a nonrelativistic quantum mechanics. Mass m_V contained within a volume V:

$$m_V = m\int_V \mathrm{d}^3x|\psi|^2$$

The observable Universe has a (baryonic and dark) mass m within a sphere of radius R_0.

What is the Hamiltonian of the (matter content of a Newtonian) Universe?

First approximation: the free Hamiltonian

$$H_{\rm free} = -\frac{\hbar^2}{2m}\nabla^2$$

Eigenfunctions: free spherical waves with $l = 0, m_l = 0$

$$\psi_{\kappa 00}(r, \theta, \varphi) = \frac{1}{\sqrt{4\pi R_0}} \frac{1}{r} \exp\left(\mathrm{i}\kappa r\right), \qquad \kappa \in \mathbb{R},$$

Second approximation: the Hubble Hamiltonian

$$H_{\rm Hubble} = -\frac{\hbar^2}{2m}\nabla^2 - \frac{k_{\rm eff}}{2}\mathbf{r}^2, \qquad k_{\rm eff} = mH_0^2$$

governs the Hubble expansion of the Universe.

Hubble eigenfunctions: $H_{\mathrm{Hubble}}\psi = E\psi$ with $l = 0, m_l = 0$:

$$\psi_\alpha^{(1)}(r,\theta,\varphi) = \frac{N_\alpha^{(1)}}{\sqrt{4\pi}} \exp\left(\frac{\mathrm{i}\beta^2 r^2}{2}\right) {}_1F_1\left(\frac{3}{4} - \frac{\mathrm{i}\alpha}{4}, \frac{3}{2}; -\mathrm{i}\beta^2 r^2\right)$$

and

$$\psi_\alpha^{(2)}(r,\theta,\varphi) = \frac{N_\alpha^{(2)}}{\sqrt{4\pi}} \frac{1}{r} \exp\left(\frac{\mathrm{i}\beta^2 r^2}{2}\right) {}_1F_1\left(\frac{1}{4} - \frac{\mathrm{i}\alpha}{4}, \frac{1}{2}; -\mathrm{i}\beta^2 r^2\right).$$

$$\alpha := \frac{2E}{\hbar H_0}, \qquad \beta^4 := \frac{m^2 H_0^2}{\hbar^2},$$

$N_\alpha^{(1)}$ $N_\alpha^{(2)}$ radial normalisations, ${}_1F_1$ confluent hypergeometric function.

Results and discussion

The gravitational entropy operator

$$\mathcal{S}_g := \mathcal{N} \frac{k_B m H_0}{\hbar} \mathbf{R}^2$$

is suggested by Verlinde's entropic gravity and by Hubble's law.
$\mathcal{N}$: undetermined dimensionless factor.
For the free eigenfunctions:

$$\langle \psi_{\kappa 00} | \mathcal{S}_g | \psi_{\kappa 00} \rangle = 10^{123} k_B, \quad \mathcal{N} = 3/2.6$$

This saturates the holographic bound.
For the Hubble eigenfunctions:

$$\langle \psi_\alpha^{(1)} | \mathcal{S}_g | \psi_\alpha^{(1)} \rangle = 10^{120} k_B = \langle \psi_\alpha^{(2)} | \mathcal{S}_g | \psi_\alpha^{(2)} \rangle, \quad \mathcal{N} = 1/6$$

Three orders of magnitude below the holographic upper bound.

Conclusions

The holographic principle: $\mathcal{S}_{\max} \simeq 10^{123} k_B$ for the whole Universe.

Phenomenological estimates: $\mathcal{S}_{\text{measured}} \simeq 10^{104} k_B$.

Gravitational entropy (black holes) are the largest single contributors to the entropy budget.

Even without black holes, our toy model captures some key elements: the holographic principle is respected by free waves, Hubble waves do not even saturate it.

A fully relativistic description will improve these theoretical estimates.

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