EUROfusion

Pattern recognition in nuclear fusion data by means of geometric methods in probabilistic spaces

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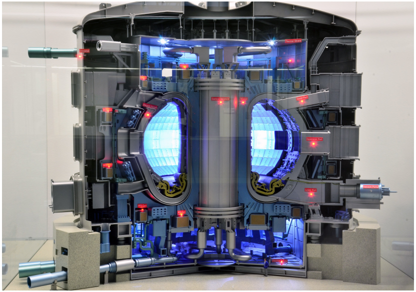
Overview

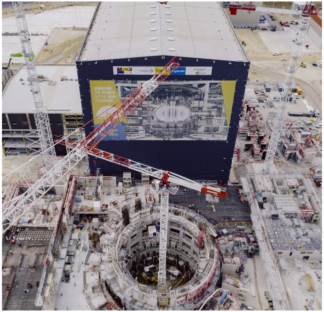
1. Stochastic uncertainty in fusion plasmas
2. Pattern recognition in probabilistic spaces
3. Geodesic least squares regression
4. Application in fusion science: edge-localized plasma instabilities
5. Application in astronomy: Tully-Fisher scaling
6. Conclusion

Overview

Fusion energy

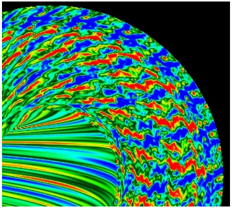
- ‘Star on earth’
- Clean, safe, inexhaustible energy source
- Magnetic confinement fusion: tokamak, stellarator, . . .
- Confine hot hydrogen isotope plasma with magnetic fields
- *ITER*: next-generation international tokamak
- Complex physical system, turbulent transport
- Difficult to probe $\rightarrow$ uncertainty in measurements and models





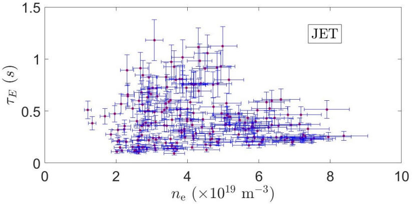
Uncertainty in fusion plasmas

- Sources of statistical uncertainty:
    - Fluctuation of system properties
    - Measurement noise



Plasma turbulence (PPPL)

Edge-localized modes (MAST)

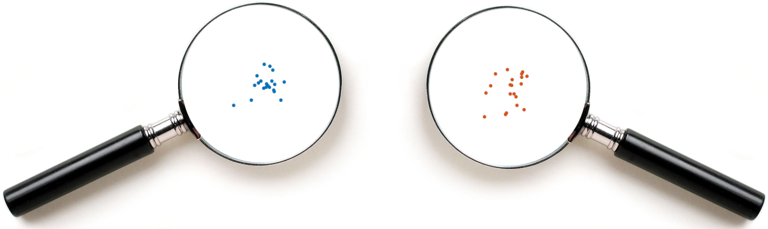


Overview

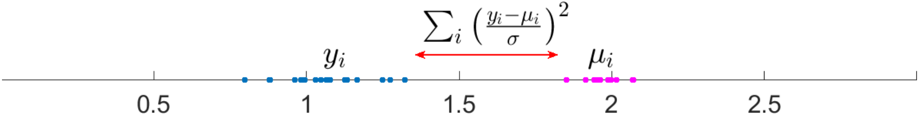
Difference/distance between points

Patterns $\leftrightarrow$ distances

Zooming in...



Mahalanobis distance



Information geometry

- Family of probability distributions $\rightarrow$ differentiable manifold
- Parameters $=$ coordinates
- Metric tensor: ***Fisher information*** matrix

Parametric probability model: $p\left(\boldsymbol{x}|\boldsymbol{\theta}\right) \Longrightarrow$

$$g_{\mu\nu}\left(\boldsymbol{\theta}\right) = -\mathbb{E}\left[\frac{\partial^2}{\partial\theta^\mu \partial\theta^\nu} \ln p\left(\boldsymbol{x}|\boldsymbol{\theta}\right)\right], \quad \mu,\nu = 1,\ldots,m$$

$\boldsymbol{\theta}$ = m-dimensional parameter vector

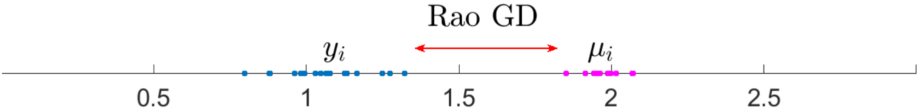
- Line element:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}\theta^\mu \mathrm{d}\theta^\nu$$

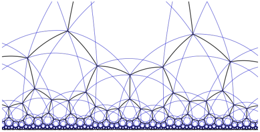
- Minimum-length curve: ***geodesic***
- ***Rao geodesic distance*** (GD)

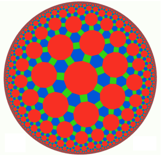
Pattern recognition in probabilistic spaces

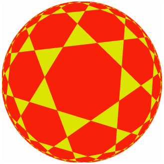
- Pattern recognition:
  - Classification, clustering
  - Regression analysis
  - Dimensionality reduction, visualization
- Observation/prediction (structureless number)
  $\rightarrow$ distribution (structured object)
- More information, more flexibility



The univariate Gaussian manifold







- PDF:

$$p(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

- Line element:

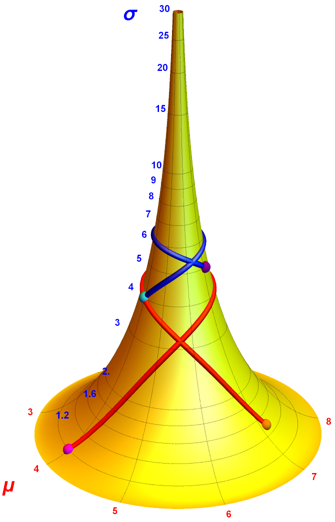
$$ds^2 = \frac{d\mu^2}{\sigma^2} + 2\frac{d\sigma^2}{\sigma^2}$$

- Hyperbolic geometry: Poincaré half-plane, Poincaré disk, Klein disk, ...
- Analytic geodesic distance

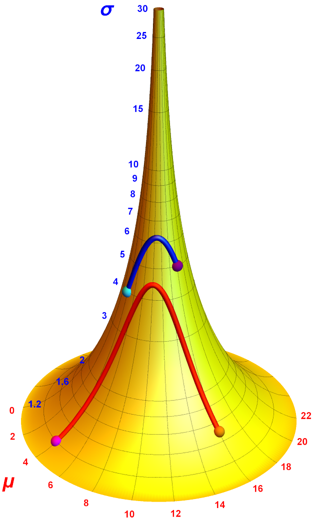
https://www.youtube.com/watch?v=i9IUzNxeH4o

The pseudosphere (tractroid)

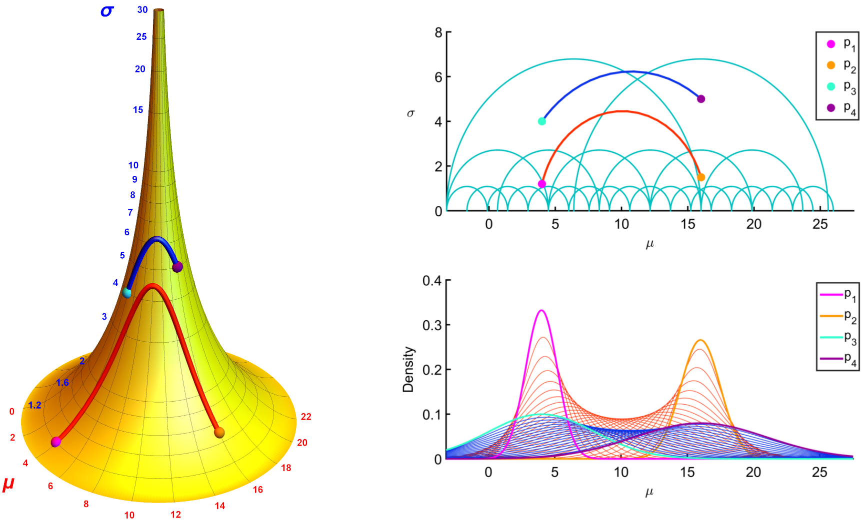
Original



Compressed

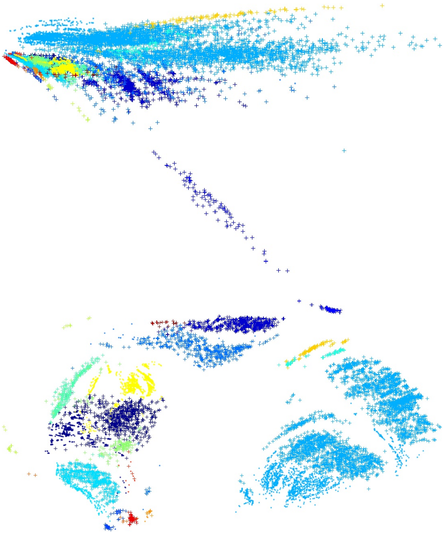


Geodesics on the Gaussian manifold



Data visualization with uncertainty

Plasma energy confinement time w.r.t. global plasma parameters

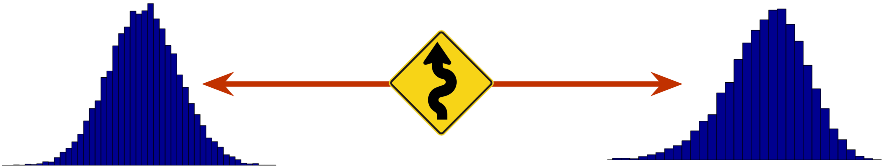


Overview

Challenges in regression analysis

- Data uncertainty: measurement error, fluctuations, ...
- Model uncertainty: missing variables, linear vs. nonlinear, Gaussian vs. non-Gaussian, ...
- Heterogeneous data and error bars
- Uncertainty on response (y) *and* predictor (x_j) variables
- Atypical observations (outliers)
- Near-collinearity of predictor variables
- Data transformations, e.g.

$$\ln(y) = \ln(\beta_0) + \beta_1 \ln(x_1) + \beta_2 \ln(x_2) + \ldots + \beta_p \ln(x_p)$$



Least squares and maximum a posteriori

- Workhorse: ordinary least squares (OLS)
- Maximum likelihood (ML) / maximum *a posteriori* (MAP):

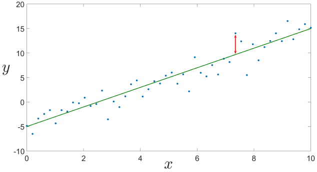
$$p(y_i|x_i,\boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{y_i \overset{\leftrightarrow}{-} \mu_i}{\sigma}\right)^2\right]$$

$$\mu_i = f_i(x_i,\boldsymbol{\theta}) \overset{\text{e.g.}}{=} \beta_0 + \beta_1 x_i$$

Michigan, circa 1890s.

- Need ***flexible*** and ***robust*** regression
- Parameter estimation $\rightarrow$ distance minimization:

Expected $\leftrightarrow$ Measured



The minimum distance approach

- ***Minimum distance estimation*** (Wolfowitz, 1952):

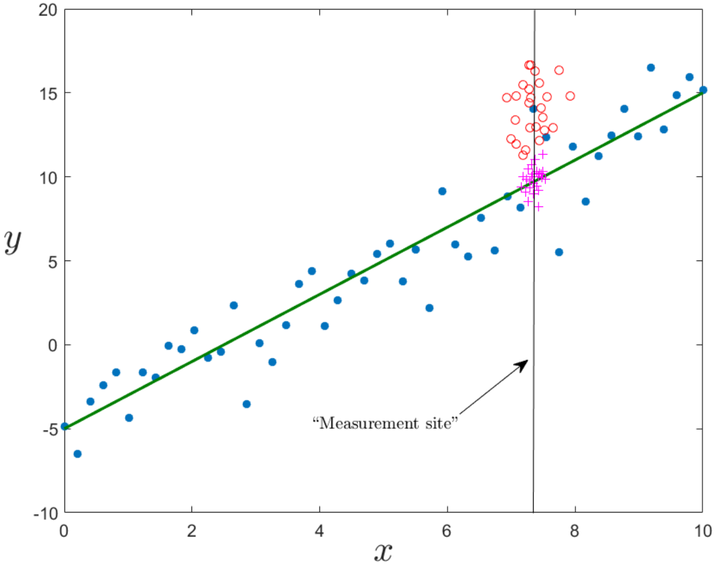
  Which distribution does the model predict?

  vs.

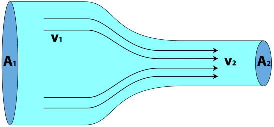
  Which distribution do you observe?

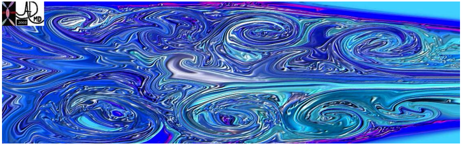
- Gaussian case: different means *and* standard deviations
- Hellinger divergence (Beran, 1977)
- Empirical distribution: kernel density estimate

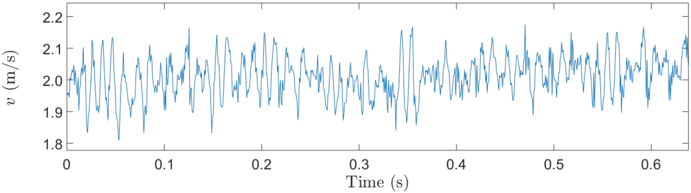
Modeled and *observed* distribution



Example: fluid turbulence







Geodesic least squares

$$\frac{1}{\sqrt{2\pi\left(\sigma_y^2+\sum_{j=1}^m \beta_j^2\sigma_{x,j}^2\right)}}\exp\left\{-\frac{1}{2}\frac{\left[y-\left(\beta_0+\sum_{j=1}^m \beta_j x_{ij}\right)\right]^2}{\sigma_y^2+\sum_{j=1}^m \beta_j^2\sigma_{x,j}^2}\right\}$$

Modeled distribution

σ_{mod}^2

Rao GD

Observed distribution

$$\frac{1}{\sqrt{2\pi}\,\sigma_{\text{obs}}}\exp\left[-\frac{1}{2}\frac{(y-y_i)^2}{\sigma_{\text{obs}}^2}\right]$$

- Model-based approach: regression on probabilistic manifold
- To be estimated: $\sigma_{\text{obs}}, \beta_0, \beta_1, \ldots, \beta_m$
- iid data: minimize sum of squared GDs

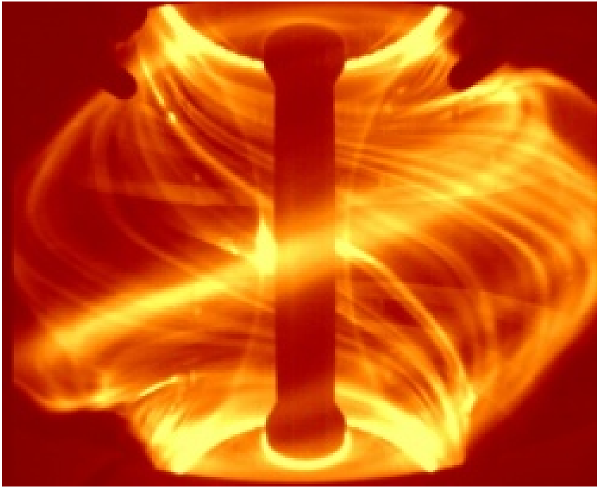
  $\Longrightarrow$ ***geodesic least squares (GLS)*** regression
- If $\sigma_{\text{mod}} = \sigma_{\text{obs}} \to$ Mahalanobis distance

G. Verdoolaege *et al.*, Entropy **17**, 4602, 2015

Overview

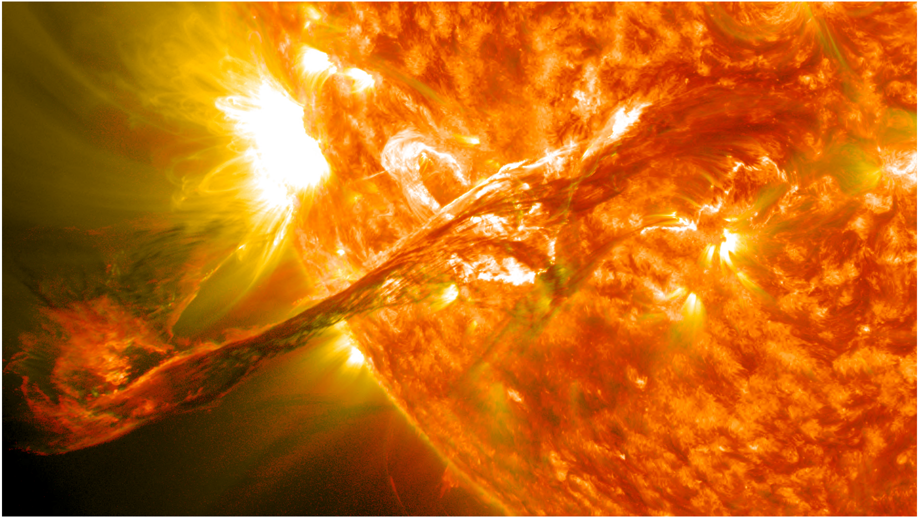
Edge-localized modes (ELMs)

- Repetitive instabilities in plasma edge
- Magnetohydrodynamic origin



MAST, Culham Centre for Fusion Energy, UK

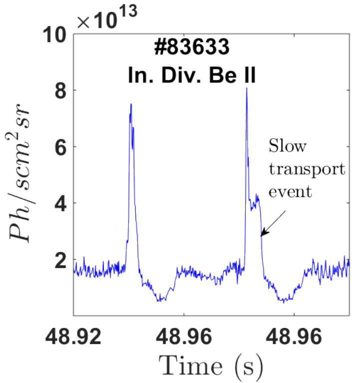
Analogy 1: Solar flares



Analogy 2: Cooking pot

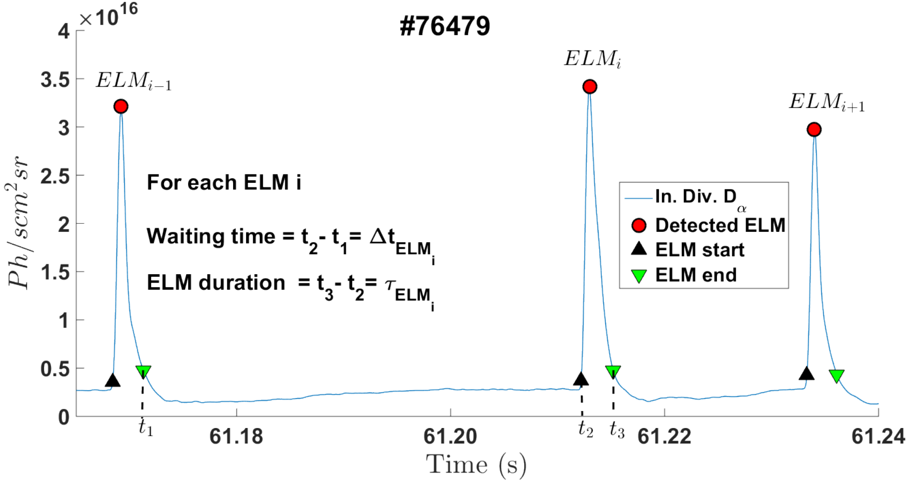
Importance of ELMs

- Confinement loss
- Potential damaging effects
- Impurity outflux
- $\rightarrow$ ELM control/mitigation
- Energy $\propto$ (frequency)$^{-1}$



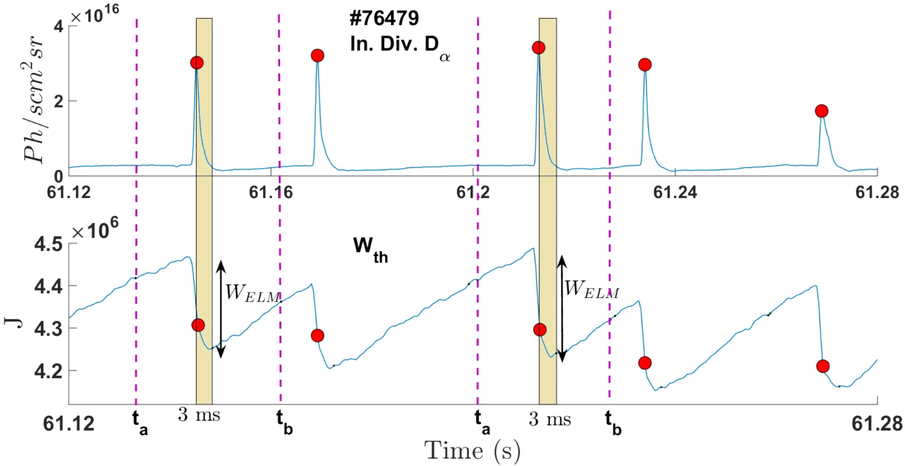
Data extraction: waiting times

- 32 recent JET discharges
- Waiting time: time before ELM burst

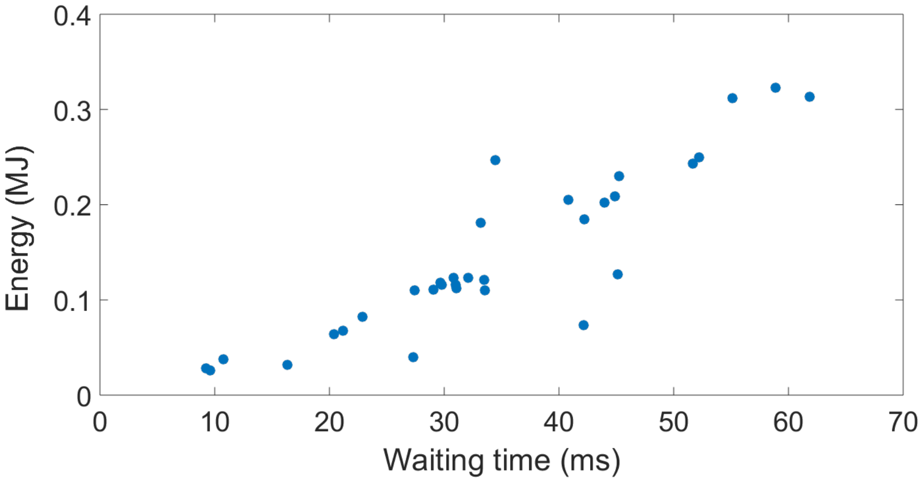


Data extraction: energies

- Energy carried from the plasma by an ELM

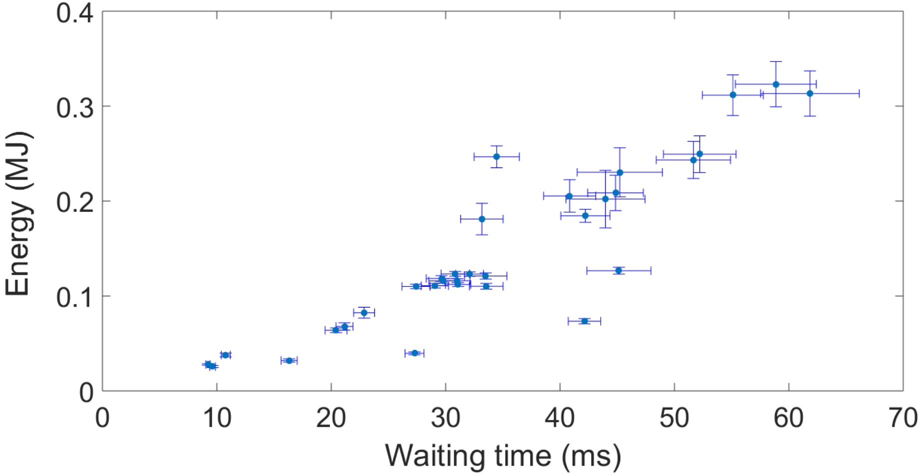


Average waiting times and energies



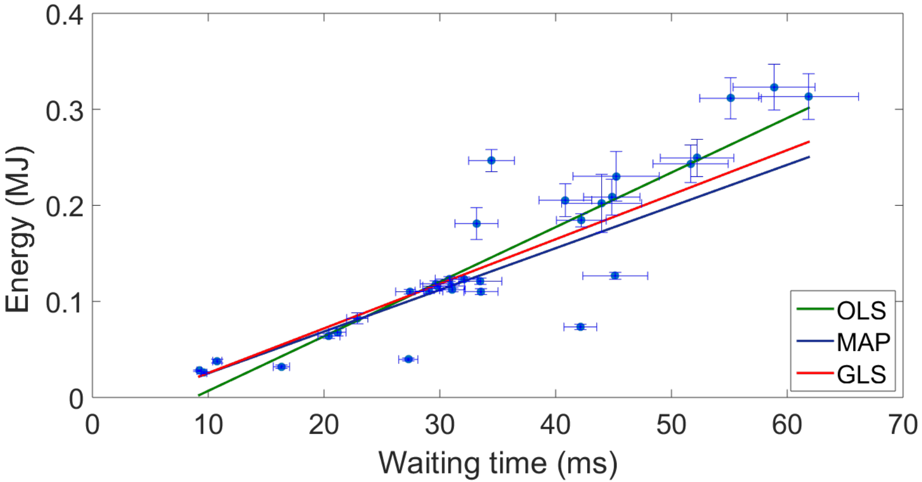
Error bars on averages

- Standard deviation / $\sqrt{n} \rightarrow$ error bars

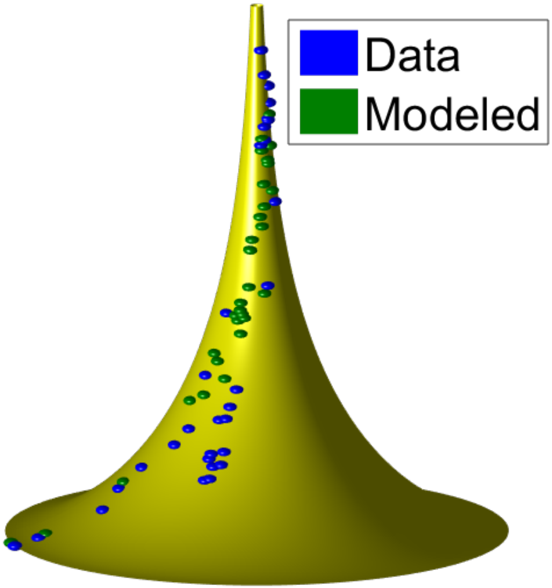


Regression on averages

$$E_{\mathrm{ELM}} = \beta_0 + \beta_1 \Delta t_{\mathrm{ELM}}, \qquad \sigma_{E,\mathrm{obs}} \propto \mu_{E,\mathrm{obs}}$$

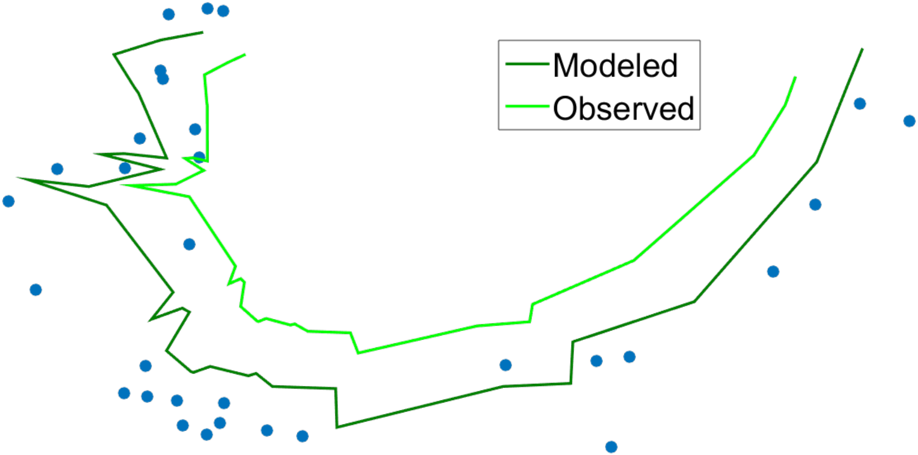


Regression results on pseudosphere



Projected regression results

Multidimensional scaling:



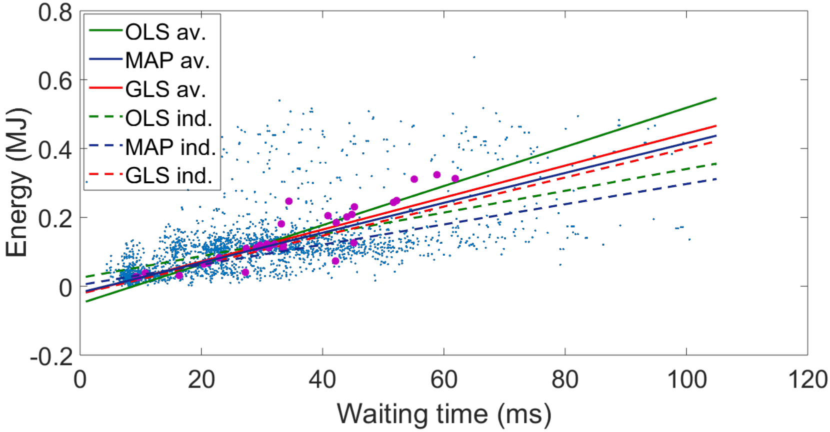
Average vs. collective trend

Average

Method	β_0 (MJ)	β_1 (MJ/s)
OLS	-0.050	5.7
GLS	-0.021	4.6

Individual

Method	β_0 (MJ)	β_1 (MJ/s)
OLS	0.024	3.2
GLS	-0.022	4.2



Overview

Baryonic Tully-Fisher Relation (BTFR)

- Simple, tight relation for disk galaxies:

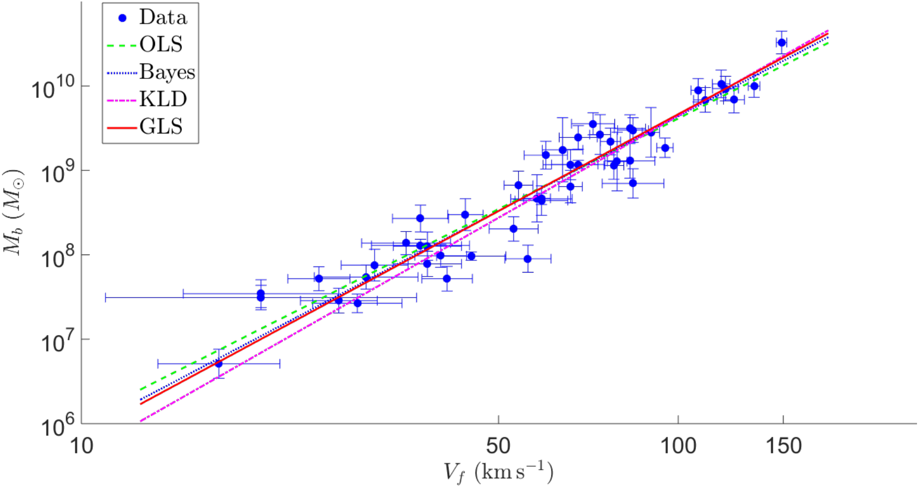
$$M_b = \beta_0 V_f^{\beta_1} \qquad \begin{cases} M_b = \text{total (stellar + gaseous) baryonic mass } (M_\odot) \\ V_f = \text{rotational velocity (km s}^{-1}) \end{cases}$$

- Various purposes:
  - Distance indicator
  - Constraints on galaxy formation models
  - Test for alternatives to ΛCDM cosmological model (slope and scatter)

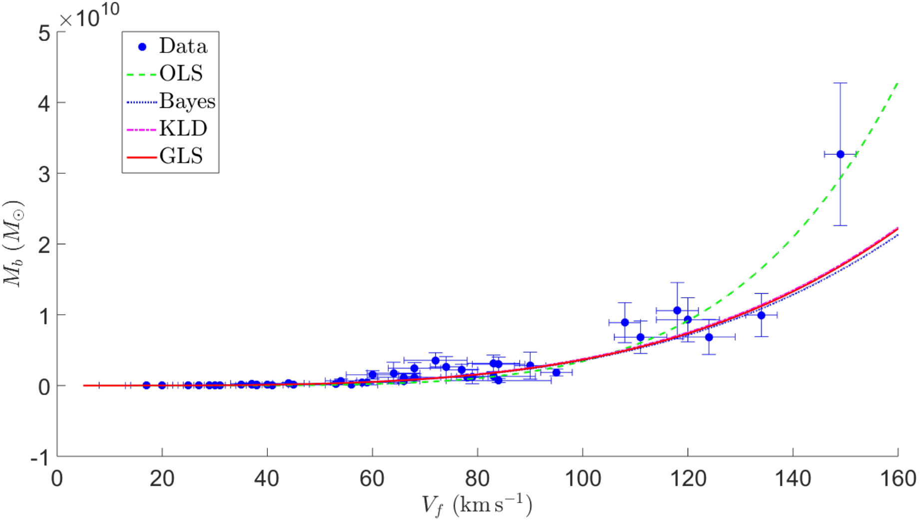
Experiments

- 47 gas-rich galaxies (McGaugh, Astron. J. **143**, 40, 2012)
- Loglinear ($\sigma_{\text{obs},i} \equiv s_{\text{obs}}$) and nonlinear ($\sigma_{\text{obs},i} = r_{\text{obs}}\, M_b$)
- Benchmarking:
  - Ordinary least squares (OLS)
  - Bayesian: errors in all variables, marginalized standard deviations (Bayes)
  - Geodesic least squares (GLS)
  - Kullback-Leibler least squares (KLS)

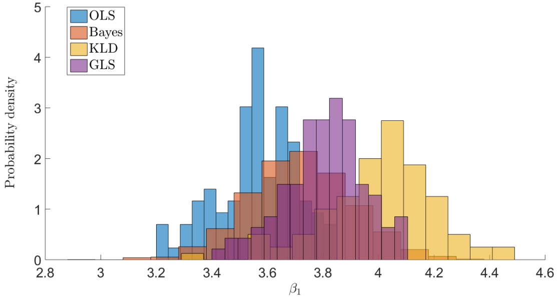
Loglinear regression

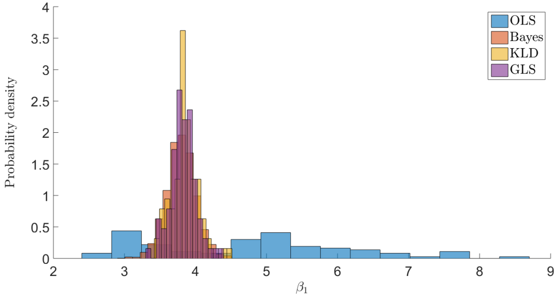


Nonlinear regression

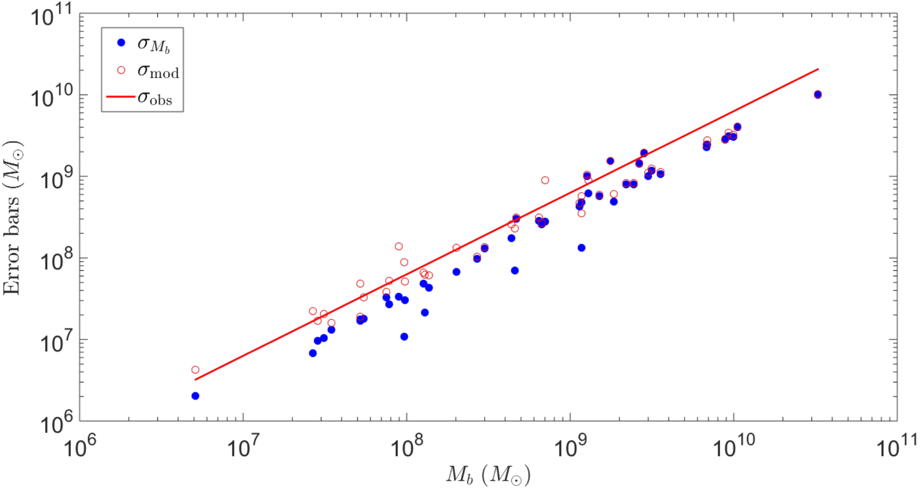


Parameter distributions



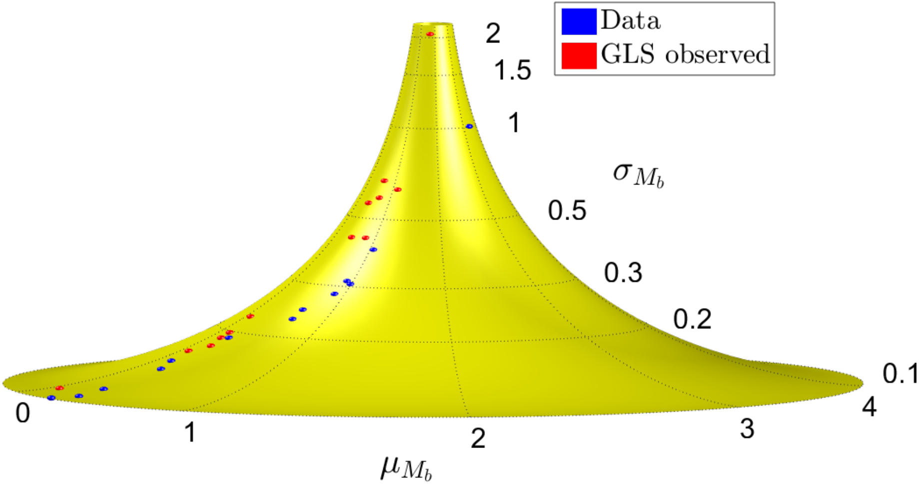


GLS uncertainty estimates



$r_{M_b} \approx 38\%$, $r_{\mathrm{obs}} \approx 63\%$

Interpretation on pseudosphere



Overview

Conclusions

- Probabilistic modeling of stochastic system properties
- Information geometry: distance measure, geometrical intuition
- Pattern recognition in probabilistic spaces
- More information, more flexibility
- Geodesic least squares regression: ***flexible*** and ***robust***
- ***Easy*** to use, ***fast*** optimization