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2 **Application of Spectral Unmixing on Hyperspectral**
3 **data of the Historic volcanic products of Mt. Etna**
4 **(Italy)**

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14 **Abstract:** Considering that ground areas, with intense compositional variability appear as mixed
15 pixels on hyperspectral data, focus on the imposed mixing problem of various volcanic products
16 found in the vicinity of Mt. Etna's volcanic craters. Mt. Etna, as being one of the most active
17 volcanoes globally, is itself a generator of diverse mineralogical environments. Therefore, the
18 inherent abundant information of hyperspectral imagery above the volcanic edifice, calls for the use
19 of time-efficient and accurate spectral unmixing methods in order to unravel them. Lava Flows (LFs)
20 and related products of the historical 1536-1669 era were selected in terms of distinct spatial
21 distribution and lava field segregation. Based on the selection of appropriate pixel representatives,
22 distinct optimizing signal transformations were implemented, with the most dominant being the
23 Fourier Transform, in order to be used in Linear Least Squares Unmixing (LLSU) and Bilinear
24 Unmixing (BLU). We, thus, report the results of the Historic Lava Flow characterization and
25 respective Abundance analysis, qualitatively and quantitatively evaluated through the Structural
26 Similarity Index (SSIM) for each method. Ultimately, method intercomparison gives the optimum
27 selection for the volcanic products segregation.

28 **Keywords:** hyperspectral data; Etna; lava flow characterization; linear spectral unmixing; bilinear
29 spectral unmixing; fast fourier transform; structural similarity index
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31 **1. Introduction**

32 As part of the plethora of technologies exploited in Remote Sensing, Hyperspectral Imaging
33 (HSI) sensors provide information on hundreds of distinct and contiguous channels of the
34 electromagnetic spectrum, thus enabling the identification of multiple ground objects through their
35 detailed spectral profiles. However, restrictions on the spatial resolution of HSI data, multiple
36 scattering of the incident light between objects and microscopic material mixing pose the mixed pixel
37 problem. Pixels are identified as mixed when composed of spectral signatures of more than one
38 ground objects. Therefore, we adopt Linear and Non-Linear Spectral Mixing techniques [1,2], which
39 model the pixel spectra as a combination of pure components (endmembers) weighted by the
40 fractions (abundances) which contribute to the total reflectance of the mixed pixel. Ideally, each

41 selected endmember from the HSI under study, has the maximum possible abundance of a single
42 physical material present and minimum abundance for the rest of the physical materials. Spectral
43 Unmixing (SU) consists of two main substages: a. Endmember Extraction, b. Abundance Estimation
44 [3]. In addition, certain signal transformations are adopted prior to the performance of SU [4,5]. The
45 determination of the number of endmembers is critical, since underestimation may result in poor
46 representation of the mixed HSI pixels, whereas overestimation in an overly segregated area. Clearly,
47 the ultimate success of unmixing depends heavily on the appropriate selection of endmembers. In
48 addition, since only a small number of the available materials' spectra are expected to be present in
49 a single pixel, the abundance vectors are often sparse in non-zero entries.

50 In mineralogy, the identification of the mineral constituents of major rock types are, typically,
51 approached with unmixing methods [1]. The sicilian volcano, Mt. Etna, is an intriguingly diverse case
52 study, since it is one of the most active stratovolcanoes globally. Its intense volcanic activity [6],
53 geomorphological complexity [7] and well-documented flank eruptions [8], perplex the remote
54 sensing monitoring of the bulk volcanic edifice. Various spectroscopy studies [9,10] over the volcanic
55 area, examine the mineralogical composition of the extensive lava fields. Nonetheless, there are no
56 findings concerning Etna's Lava Flow (LF) delineation through unmixing, which further intensifies
57 the need for multitemporal analysis over the volcano.

58 In this paper, we are focusing on the accurate estimation of fractional abundances of the
59 deposited volcanic products on older lava fields. For this purpose, a NASA EO-1 Hyperion HSI image
60 of Etna is selected and for the sub-pixel analysis we propose **four** signal transformations that are
61 assimilated in Linear Least Squares Unmixing (LLSU). Our approach is narrowed to the part of Etnian
62 Historical Eruptions on the sensor's FOV, namely between 1536–1669 AD, where deposited products
63 present lower correlation coefficients than those of younger lava flows. The latest geological map of
64 Etna is used as ground truth [7]. The resulting abundances from each method, are quantitatively
65 evaluated via the Structural Similarity Index (SSIM) [11] and lastly, a comparative analysis of the
66 selected methods is discussed.

67 2. Study Area and Methodology

68 In this section, we define the key characteristics of Etna's morphology and describe the
69 implemented methods for the linear and bilinear unmixing of the Historic Lava fields.

70 2.1. Study area

71 Mt. Etna is a large basaltic complex stratovolcano with a broad base diameter of 1.178 Km². Its
72 volcanism started approximately 500 ka ago, from a submarine fissure of the Gela-Catania basin and
73 is characterized by two major components: i) the central *summit craters* and ii) the *flank eruptions*.
74 Summit events are consistent for many years, while flank eruptions occur every few years and are
75 correlated with tectonic faults and shallow seismic activity [12]. Etna has the ability to change its land
76 field rapidly, vigorously and continuously. It is capable of producing both brief paroxysmal episodes
77 and long-standing, relatively milder eruptions. During the last 100 years Mt. Etna has produced on
78 average 10⁷ m³ of new lava per year [8], both from its summit craters and from its flank areas. The
79 youngest active bulks of Etna are *Ellitico* and *Mongibello*. We focus our research on the persistent
80 activity of the Mongibello volcano, which entrails the major volcanic formation of Torre del Filosofo.
81 It contains voluminous products younger than 122 BC, with continuous temporal succession and
82 similar lithologies. The formation is divided in *three* sub-formations formed in different time periods
83 [7]:

- 84 • **Mongibello Filosofo 1 (MF1)**: The first sub-formation with volcanic products from 122 BC –
85 1669 AD. The effusive products originate from intense summit activity and are combined with
86 intermediate flank eruptions. As expected, MF1 products follow the mountain morphology and
87 spread radially of the main volcanic volume.
- 88 • **Mongibello Filosofo 2 (MF2)**: It includes the time interval of the post 1669 AD eruption period
89 to pre 1971 AD eruption period.

90 • **Mongibello Filosofo 3 (MF3):** It contains the most recent and well documented summit/flank
91 eruptions and flows eastward to VdB.

92 MF1 is ideal for testing unmixing processes, since it contains mostly altered lavas with high
93 reflectance. The respective lava fields are distinct, voluminous and often do not mingle with younger
94 lavas. Considering the Hyperion's limited swath range and spatial resolution over Etna, lava fields
95 from 1536 – 1669 AD are contained in the data cube.

96 2.2. Data

97 The Hyperion HSI dataset was acquired on 09/07/2007, with low cloud coverage and $\pm 5^\circ$ looking
98 angle to avoid geometrical distortions. The retrieved L1T product is radiometrically and
99 geometrically corrected, used also for spectroscopy studies in Etna [13]. Extensive pre-processing was
100 applied in order to obtain the optimum ground reflectance dataset and ultimately 140 spectral bands
101 were retained, with central wavelengths at 477.7-884.7 nm, 973-1336.2 nm, 1477.4-1790.2 nm and
102 2062.5-2355.2 nm. Preprocessing included: *i.* Atmospheric correction, *ii.* Dimensionality and noise
103 reduction via Minimum Noise Fraction analysis, *iii.* Vegetation masking using NDVI values, *iv.*
104 Active areas segregation: Valle del Bove intense current eruptive activity was masked from the initial
105 image and lastly, *v.* Formation masking: digitization of the MF1 formation within the sensor's FOV,
106 using the formation's boundaries as depicted in the geological map.

107 2.3. Endmember extraction

108 Endmembers were retrieved through Regions of Interest (ROIs) that were strictly constrained
109 within denser lava deposition fields and close to the eruption vents. Borderline regions with potential
110 vegetation growth are excluded and ROIs with limited pixels are merged. Also, spectral profile
111 examination ensures minimum variability within a certain ROI. Since the broader region of the
112 volcano contains small urban and semi-urban areas, ROIs were selected for these classes also. We
113 extracted ROI pixel values manually and retained spectral information within 25%-75% of the normal
114 distribution of the reflectance values per ROI and per spectral band. Therefore, outliers and non
115 characteristic signatures are omitted. Then, we calculate the mean value per band for the total number
116 of pixels defining thus the associated *endmember vector*. We retrieved, in total, 13 endmembers
117 corresponding to *eight* LFs (1536, 1537, 1566, 1610, 1614-24, 1634-36, 1646-47 & 1669 AD), *two* scoria
118 cones (1610 & 1646-47 AD) and *three* artificial materials (industrial, semi-urban and tile rooftop areas).

119 2.4. Methods and Adopted Transformations

120 In this section, we will describe, in detail, the adopted signal transformations and unmixing
121 models implemented for the LF segregation. The implicit fractional abundances are, very often,
122 constrained to be non-negative (NNC) and to sum-to-one (STOC) [14].

123 **Method 1: Linear Least Squares Unmixing (LLSU) on the original image**

124 This method is based on the Linear Mixture Model, Eq.1 for the original reflectance dataset,
125 assuming that each endmember covers a defined region inside the pixel area and multiple scattering
126 is negligible [14].

$$\mathbf{y} = \mathbf{M}\boldsymbol{\alpha} + \mathbf{n} \quad (1)$$

127 where \mathbf{y} : spectral signature column vector of a certain pixel, \mathbf{M} : matrix containing the m_i , $i=1,\dots,K$,
128 endmembers in his columns, $\boldsymbol{\alpha}$: abundance vector and \mathbf{n} : additive white noise. As for the unmixing
129 method, we use a Linear Least Squares Non Negative approach, where the estimated abundances
130 [15], derived from a suitable defined Lagrangian function, are given by Eq.2 :

$$\boldsymbol{\alpha}_{NCLS} = (\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T\mathbf{y} - (\mathbf{M}^T\mathbf{M})^{-1}\boldsymbol{\lambda} \quad (2)$$

131 **Method 2: LLSU on the Reduced Channel Domain**

132 Since, the removal of multi-collinearity improves the performance of spectral unmixing and
133 based on the assumption that LFs of the same eruptive cycle can be distinguished by the dataset's

134 intrinsic dimensionality, we remove the redundant spectral information. For this a Feature Selection
135 (FS) code is used that exploits the sparse unmixing algorithm Bi-ICE [16]. Consequently, total number
136 of endmembers was defined considering a sparsity promoting scheme, hence the extracted
137 endmember matrix is tall by definition. The output of the FS is a set of estimated vectors, the first of
138 which contains a spectral band index with decreasing significance order, while the second contains
139 the ranks denoting the level of significance for each of these bands. We retain only the first 22 bands
140 with higher significance, reduce the initial image dimension accordingly and then perform again an
141 LLSU scheme.

142 **Method 3: LLSU with Fast Fourier Transform (FFT) on selected frequencies**

143 For the implementation of this method, the FFT was performed on each of the pixel vectors of
144 the original dataset and on each signature vector of the endmember matrix. Again, the adopted
145 transformation is integrated in an LLSU model. In this case, we implemented the real number input
146 arguments as a linear combination of the squared imaginary and real parts of the image/endmember
147 matrices. Subsequently, the image is dimensionally reduced in a subspace of 20 frequencies, in the
148 frequency domain, that preserve the total energy content. The advantage of this approach is that it
149 counteracts the computationally intensive and time-costly FS.

150 **Method 4: Bilinear Unmixing (BLU) with the Enhanced Endmember Matrix**

151 Due to the complex structure of the deposited volcanic deposits, nonlinear effects still exist at
152 macroscopic scales. Therefore, linear representation of mixed pixels lacks the required detail between
153 secondary light interactions with the on-site materials. In this paper, the nonlinear representation of
154 mixed ground components is inserted through their correlation within the pixel area and the
155 implementation of interactions in a Bilinear Mixture model (Eq. 3) [17].

$$156 \quad \mathbf{y} = \mathbf{M}\mathbf{a} + \sum_{i=1}^{K-1} \sum_{j=i+1}^K a_{i,j} \mathbf{m}_i \odot \mathbf{m}_j + \mathbf{n} \Leftrightarrow \mathbf{y} = \sum_{k=1}^{K^*} a_k^* \mathbf{m}_k^* + \mathbf{n} \quad (3)$$

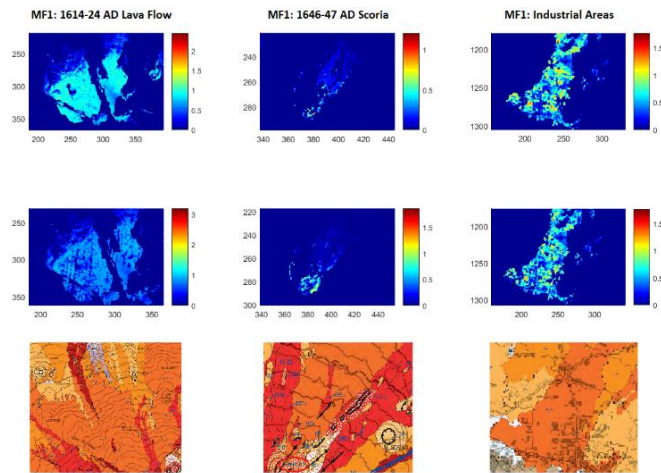
157 where the vector $\mathbf{m}_i \odot \mathbf{m}_j$ results from the elementwise product of $\mathbf{m}_i, \mathbf{m}_j, K^*=1/2K(K+1)$, constrained
158 for $\alpha_k^* \geq 0$. The bilinear approach is applied on the reduced image resulting as in the context of
159 method 2 and implemented in the LLSU scheme, where we insert the non-linearity with an enhanced
matrix, containing the dot product specified above.

160 **3. Results**

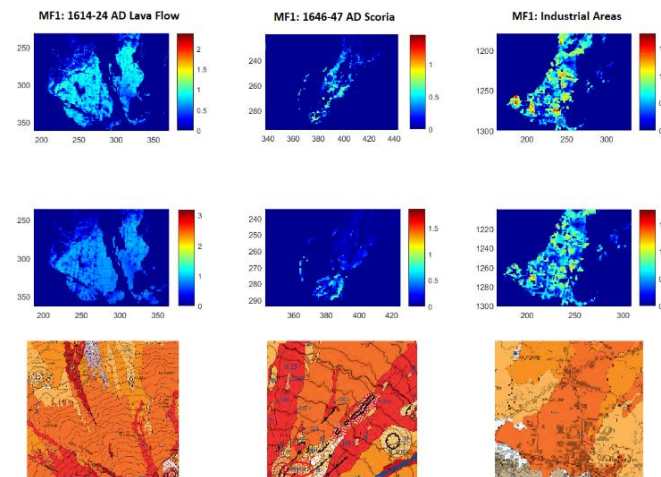
161 The tested unmixing methods give distinct abundance maps for each endmember which were
162 evaluated in terms of: *i.* Image reconstruction efficiency and *ii.* Method optimality.

163 *3.1. Endmember Abundances*

164 We indicatively present the abundances of **1614-24 LF, 1646-47 scoria, industrial areas** (Fig.1)
165 and the SSIM index values for each method (Fig.2). The SSIM index is the quantitative evaluation of
166 our results and indicates the degree of agreement between the initial image and the reconstructed
167 image with the estimated abundance vectors. Each of the SSIM plots exhibit different micro-trends,
168 while there are methods that under strict comparison produce similar results.



(a)

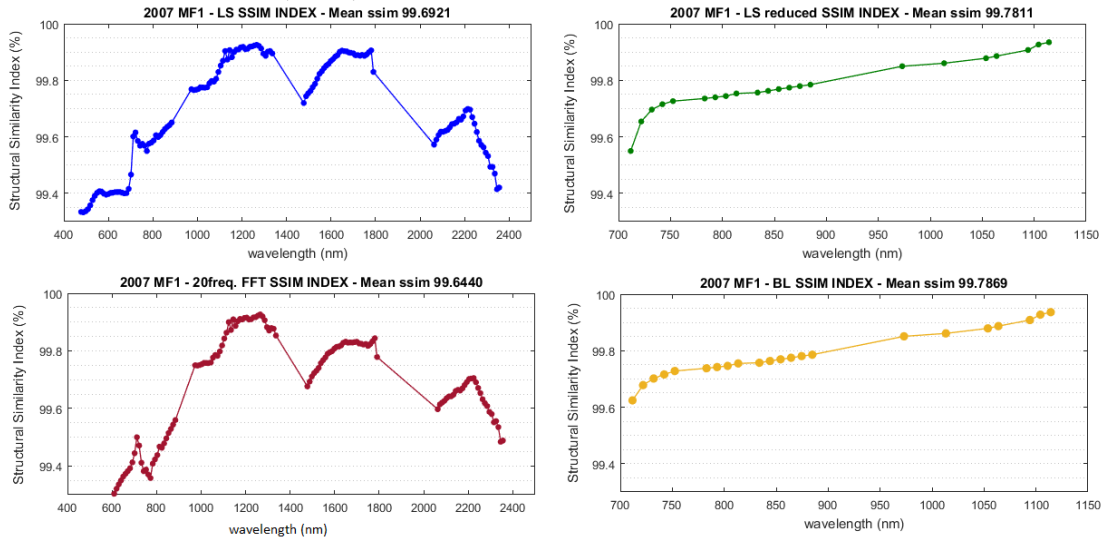


(b)

169 **Figure 1.** Abundance maps resulting from the linear and bilinear methods. (a) Top and middle row:
170 Abundances originating from methods 1 and 2, (b) Top and middle row: Abundances originating
171 from methods 3 and 4. Lower row: the respective region in the geological map. MF1: dark orange
172 areas, MF2: orange and MF3: red areas.

173 3.1.1. Method efficiency

174 Each method was timed during code performance with a simplistic tic-toc Matlab approach.
175 Table 1 gives the optimum method in terms of time efficiency and dimensionality reduction
176 possibilities. To be statistically accurate, the process is recorded five times per method, then the
177 average time duration is calculated along with the respective error. As inferred from the results,
178 unmixing of the FFT transformed image is the quickest approach, with also the lowest standard
179 deviation. This was expected considering the fast implementation of the Fourier Transform.



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Figure 2. SSIM calculated values for methods 1-4 from top left to bottom right. Methods 1 and 3 are on the entire channel domain and give similar results, while methods 2 and 4 are constrained in the reduced channel domain. Linear interpolation is performed between the resulting values and sharp changes are attributed to the lack of intermediate wavelengths due to pre-processing.

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Table 1. Timed operation of the unmixing code.

METHOD	Elapsed time (sec)	STDV	EFFICIENCY
	MEAN		
LLSU	15.094	±0.235	-
LLSU reduced	12.497	±0.161	Time
FFT 20Freq.	12.342	±0.149	Time & Dimensionality Reduction
BLSU	32.919	±0.343	-

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4. Discussion

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The evaluation of the resulting abundance maps, per endmember, carried out via two a stage process: *a.* Trace volcanic product abundances and inter-correlation on a per method comparison to ground truth and *b.* Delimit Lava Flows using different methods to extract comparative results, given the reconstruction efficiency for each method. Since the geological map is primarily used for the demarcation of the MF1 formation, we do not provide a definite LF mapping, but rather a spatial verification of each product's presence on the MF1. The above results show that the majority of LFs are accurately delineated by both linear and bilinear based unmixing approaches. As expected, higher abundance values are attributed on the ROI areas, consistently over the different lava fields. Manmade material areas are, also, clearly distinguished. Minor confusions are observed mainly on products with similar mineralogical composition and spectral features, such as the scoria cones. Specifically, the observed low abundances in the same areas, for different methods, can be attributed to the lack of low frequency information due to the dimensionality reduction of the reduced datasets. Generally, all adopted methods show a reconstruction efficiency greater than 99.5%, denoting successful unmixing implementations. Each of the SSIMs exhibit different micro-trends, while there are methods that under strict comparison produce the same reconstruction results. In terms of method efficiency, LLSU and LLSU with FFT on the selected frequencies exhibit the same SSIM trends, with respect to the wavelength content and slightly varying SSIM mean values.

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5. Conclusions

205 A quantitative analysis of potential spectral unmixing method on Etnian Lava fields is
206 presented. We conclude that the selection of appropriate regions of interest on such a diverse volcanic
207 environment, considering the particularity of the Hyperion dataset, is of great importance in order to
208 obtain accurate unmixing results. Considering the basic assumptions of the LLSU with Fast Fourier
209 Transform (FFT) on selected frequencies method and the subsequent reconstruction degree, we
210 conclude that this method obtains efficient Dimensionality Reduction of the initial abundance
211 spectral information, with high unmixing accuracy. To the best of our knowledge, volcanic products
212 of Etna are studied mostly from field measurements, complemented by satellite images, while there
213 are no references of Hyperspectral unmixing techniques on Etnian Lava Fields. Hence, the present
214 research is innovative since signal processing approaches are tailored on a multi-diverse volcanic
215 environment.

216 Abbreviations

217 HSI: Hyperspectral Imaging	LF: Lava Flow
218 LLSU: Linear Least Squares Unmixing	BLU: Bilinear Unmixing
219 SU: Spectra Unmixing	FFT: Fast Fourier Transform
220	
221 SSIM: Structural Similarity Index	FS: Feature Selection

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