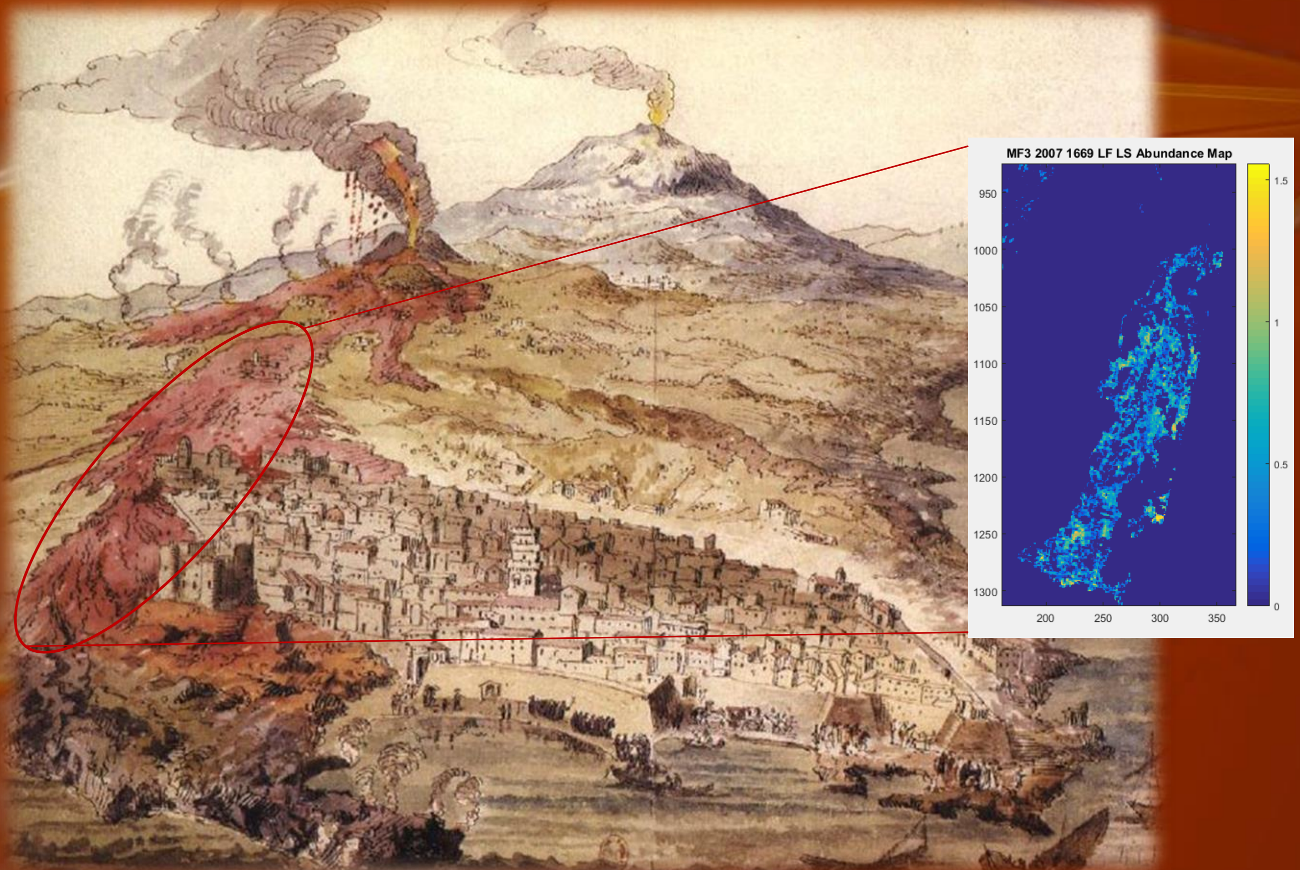


1

2 UNIVERSITY OF CRETE

Title:

Application of Spectral Unmixing on Hyperspectral data of the Historic volcanic products of Mt. Etna (Italy)

Vasiliki Daskalopoulou [1,2,*], Olga Sykioti [1], Catherine Karagiannopoulou [1], Konstantinos Koutroumbas [1], Athanasios Rontogiannis [1]

Outline

- A. Geological Setting
- B. Problem Definition
- C. Data
- D. Methodology
- E. Results
- F. Discussion

Research Goals

- Delineate volcanic products with the use of Unmixing
- Accurate estimation of Abundances of deposited volcanic products
- Test different Signal Transformations to achieve optimum unmixing results
- Determine the degree of correlation between LFs
- Qualitative overview of the volcanic surface complexity
- Extract underlying information of sub-pixel analysis, wrt to ground truth
- Paradigm shift → future extension to younger, more correlated, Lavas

A. GEOLOGICAL SETTING

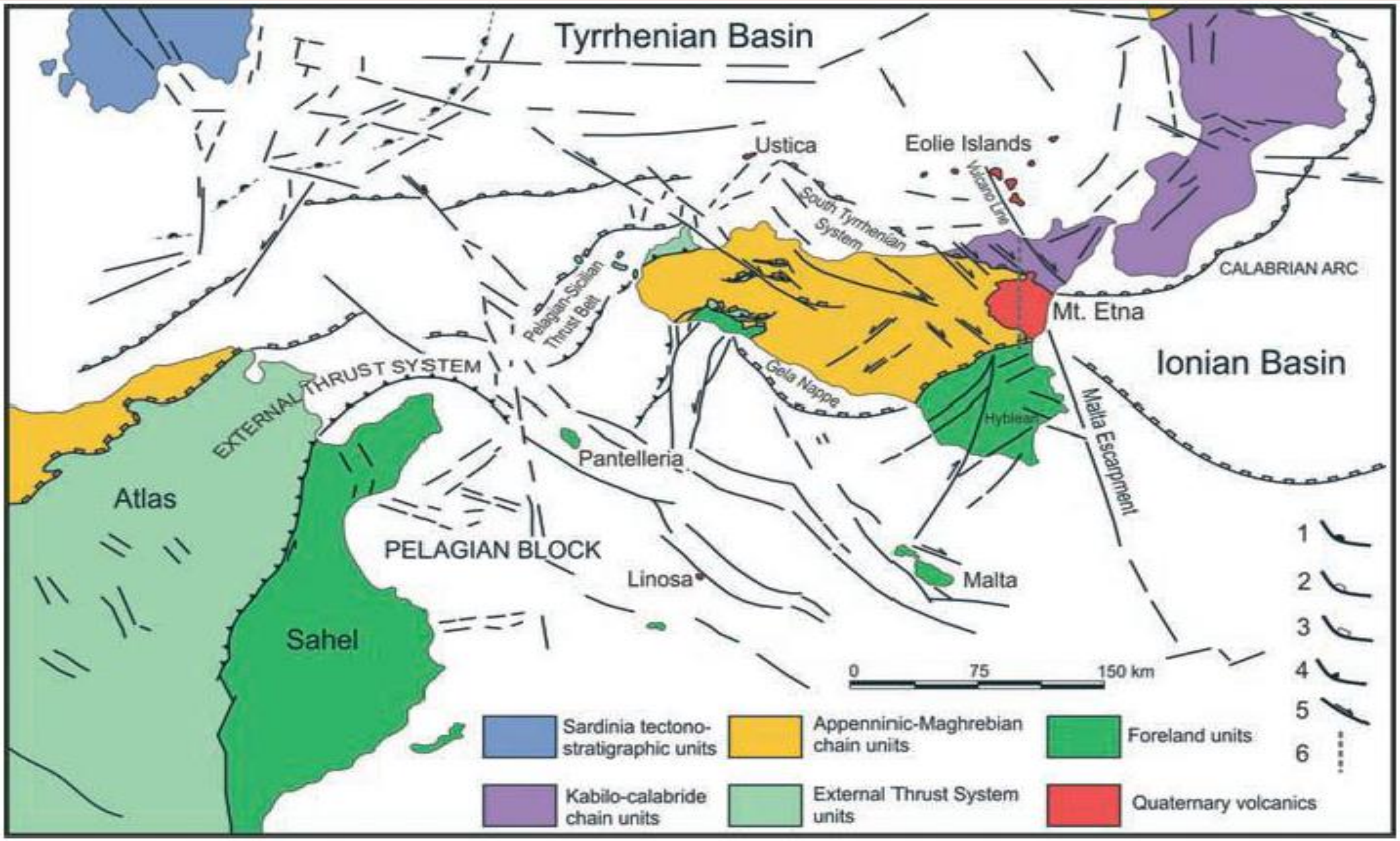
Mt. Etna(1)

Volcanoes are manifestations of the physical/chemical processes happening on the Earth's interior

- **Mt. Etna** Eruptive Region: active supersynthem stratovolcano, producing continuous eruptive events, complex stratigraphy
- Located on the eastern coast of Sicily, Italy (37°44'3"N, 15°0'16"E)
- Summit elevation ~ **3.329m** , Base Diameter ~ **1.178** $\boldsymbol{Km^2}$
- Borders with the orogenic belt of **Hyblean Foreland** to the south, the **Apenine Chain** to the north and **foredeep deposits** in between
- Lies on the verge of the **Eolie Islands** magmatic arc
- Predominant assumption on Etna's volcanism and seismicity origin:

Subduction of the **Ionian** lithospheric slab beneath the **Aeolian slab**

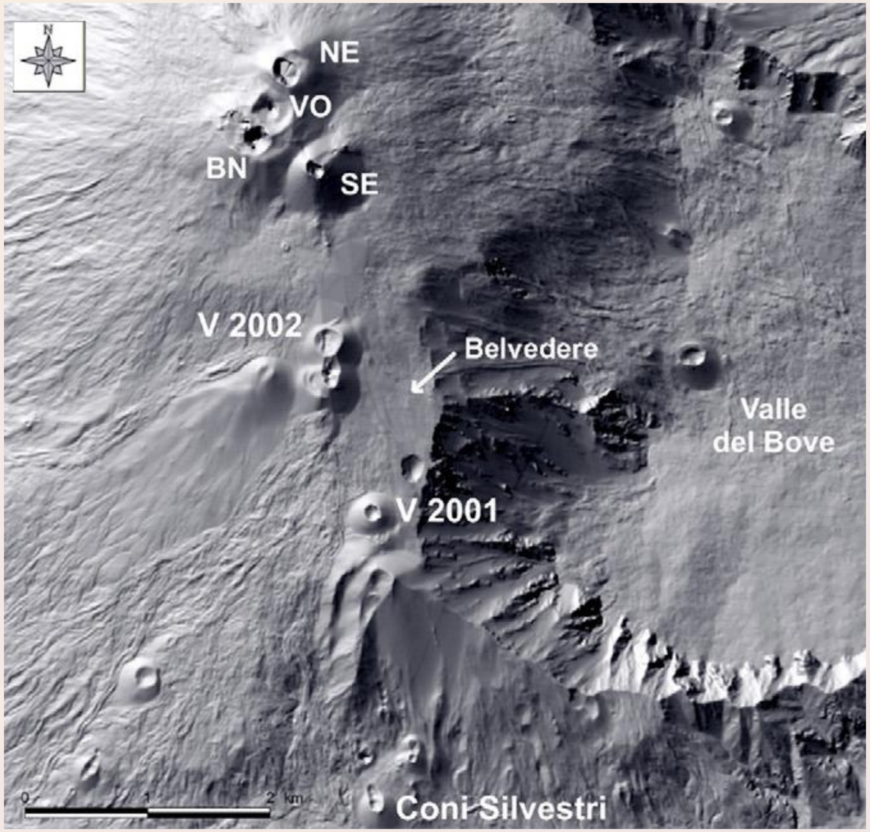
Mt. Etna(2)



Tectonic edifice of the Central Mediterranean Sea, the location of Mt. Etna is highlighted with red (from *Branca et al, 2011*).

Mt. Etna(3)

- **4** major craters → Northeast Crater (NEC), Voragine, Bocca Nuova and the Southeast Crater (SEC)
- Produce: voluminous *Summit eruptions, Paroxysmal events, Lava flows, Lava fountains*
- Most eruptive: SEC
- **> 300** secondary flank craters
- Flank eruptions: historically more hazardous for populated regions



(modified by Spinetti et al., 2009)

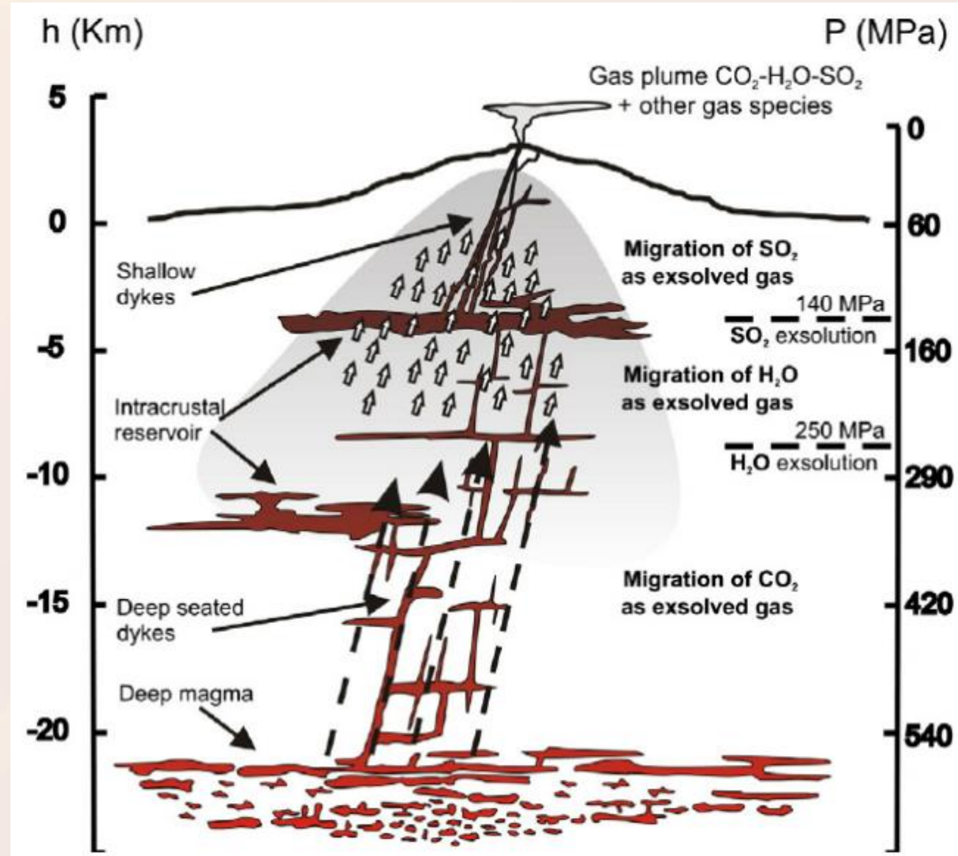
Volcano's Plumbing System

Subduction forces the deep magmatic material to resurface.

Central conduit system is located west of the most active volcanic region, **Valle del Bove** (VdB) and is:

~ *7 Km* x *5 Km* wide and *1000 m* deep

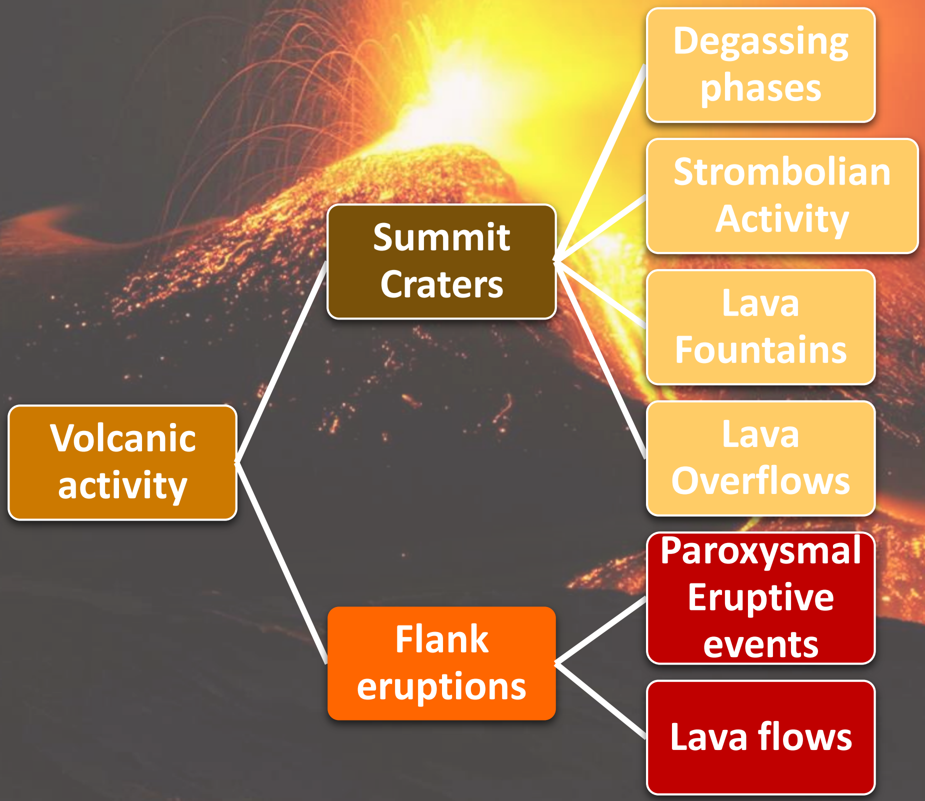
Consists of: Intracrustal reservoirs and levels of exsolution for various gases.



(modified by Ferlito et al., 2013)

Volcanic Activity

- Volcanism started ~ **500 ka** ago
- Etna's historical record of eruptive activity is well documented, with many attempts to identify systematic trends.
- The post 1600 AD eruptive period is subdivided into **4** cycles.



First Period: **1600 – 1669 AD**
Second Period: **1669 – 1763 AD**
Third Period: **1755 – 1865 AD**
Fourth Period: **1865 – Present**

Volcanic Products

Emerging lava: **‘a’a** type → basaltic, rough surface, broken lava blocks & sharp texture

< 10% **pahoehoe** type → ropy, smoother surface. Flows downwards → cools and may change to **‘a’a**

- Volcanic products :

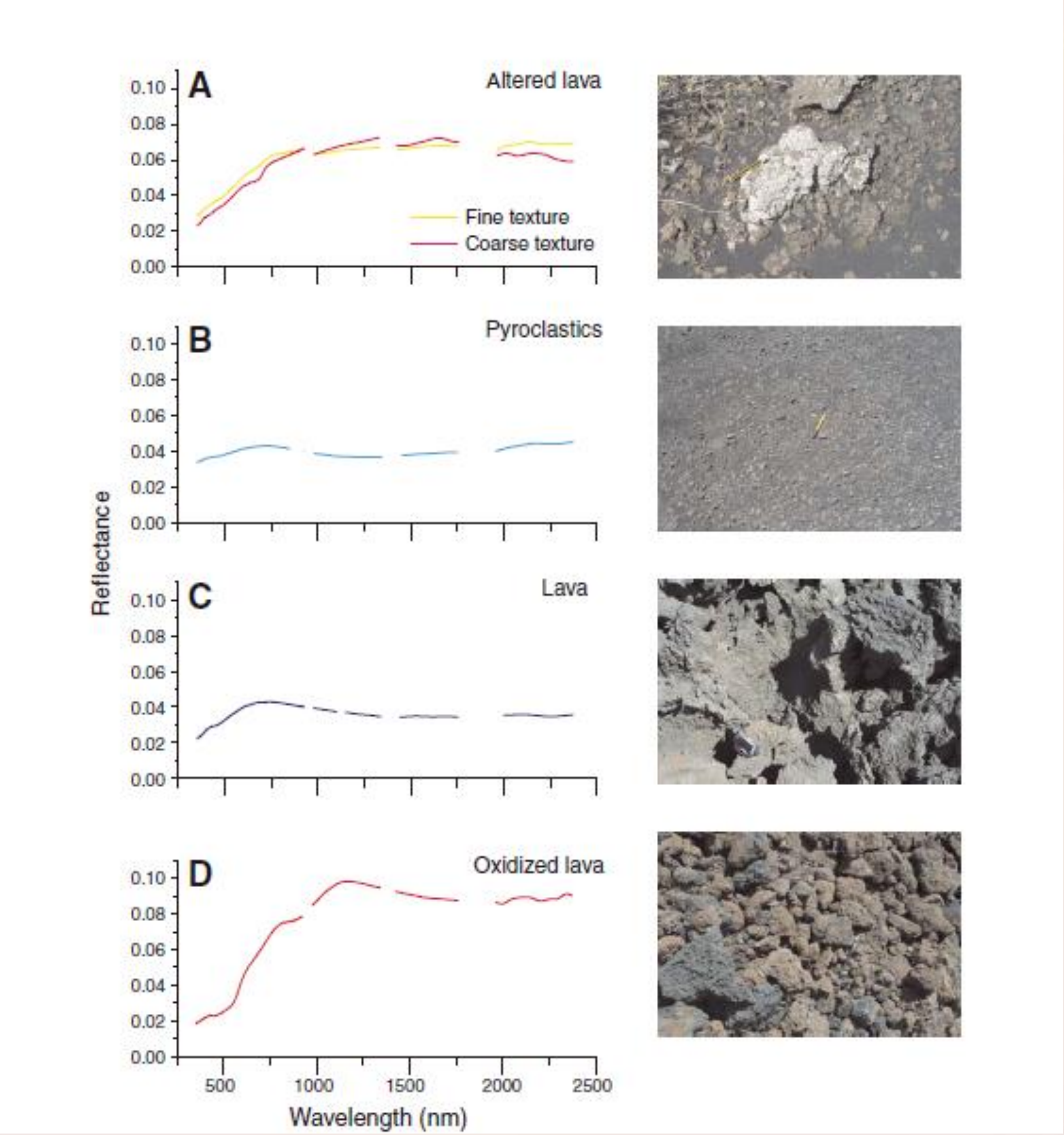
I. **Lava Flows (LFs)** occur from both summit and flank activity, subject to weathering, alteration and vegetation cover ⇒ **morphology**

II. **Pyroclastics** are violent movement of gaseous compounds enriched with volcanic material, deposited on top of lava fields.

III. **Surface ash** and **scoriae !**

Spectral Signatures of Volcanic Products

(from Sgavetti et al., 2006)



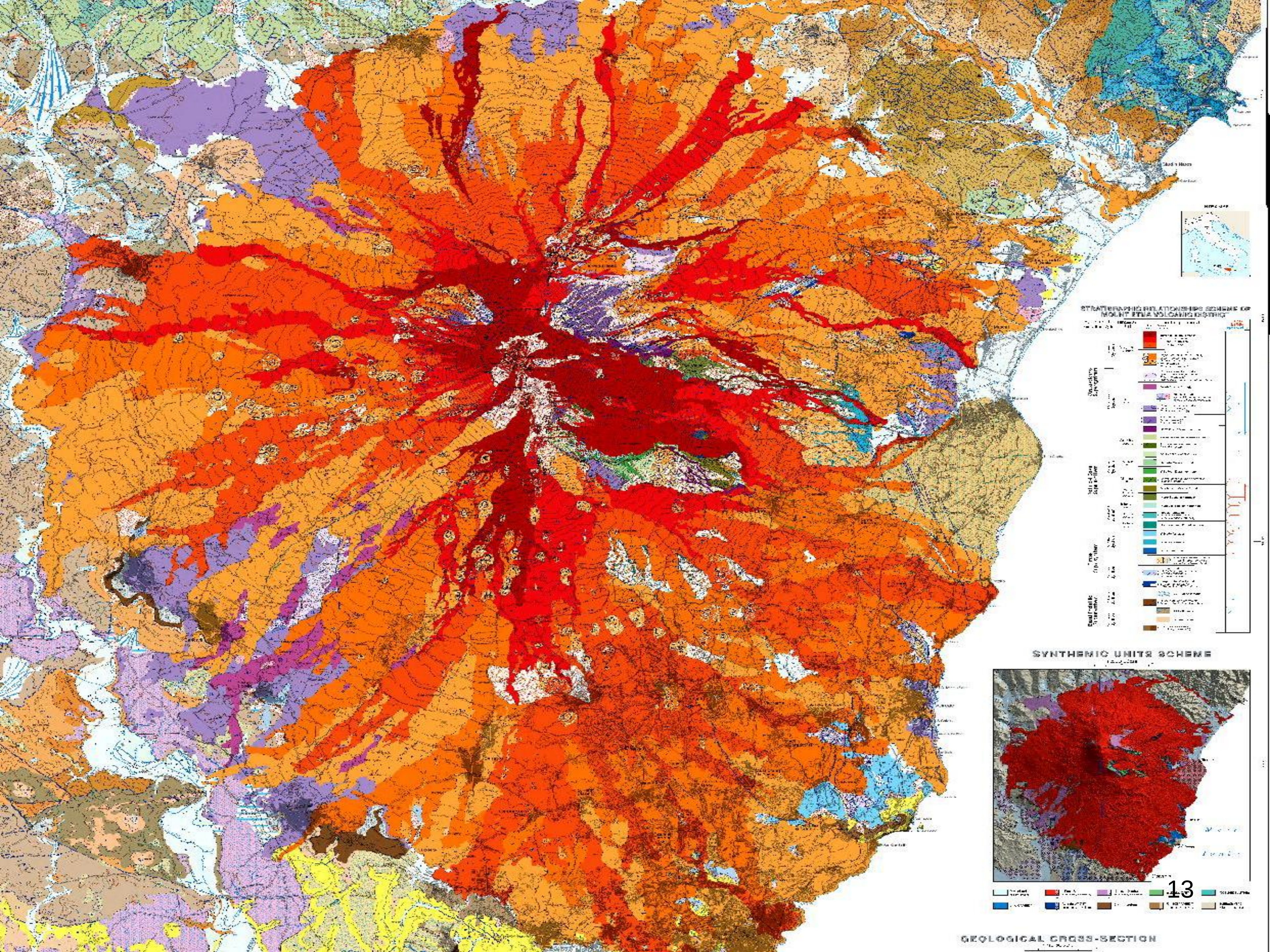
Major Volcanic Formations

- 1:50.000 scale Geological Map of Etna (*Branca et. al, 2011*) → **Ground Truth**

- **5** geological stages: Pre – Etnian activity, Fusion of four major stratovolcanoes: *Trifoglietto I, Trifoglietto II, Calanna and Mongibello*

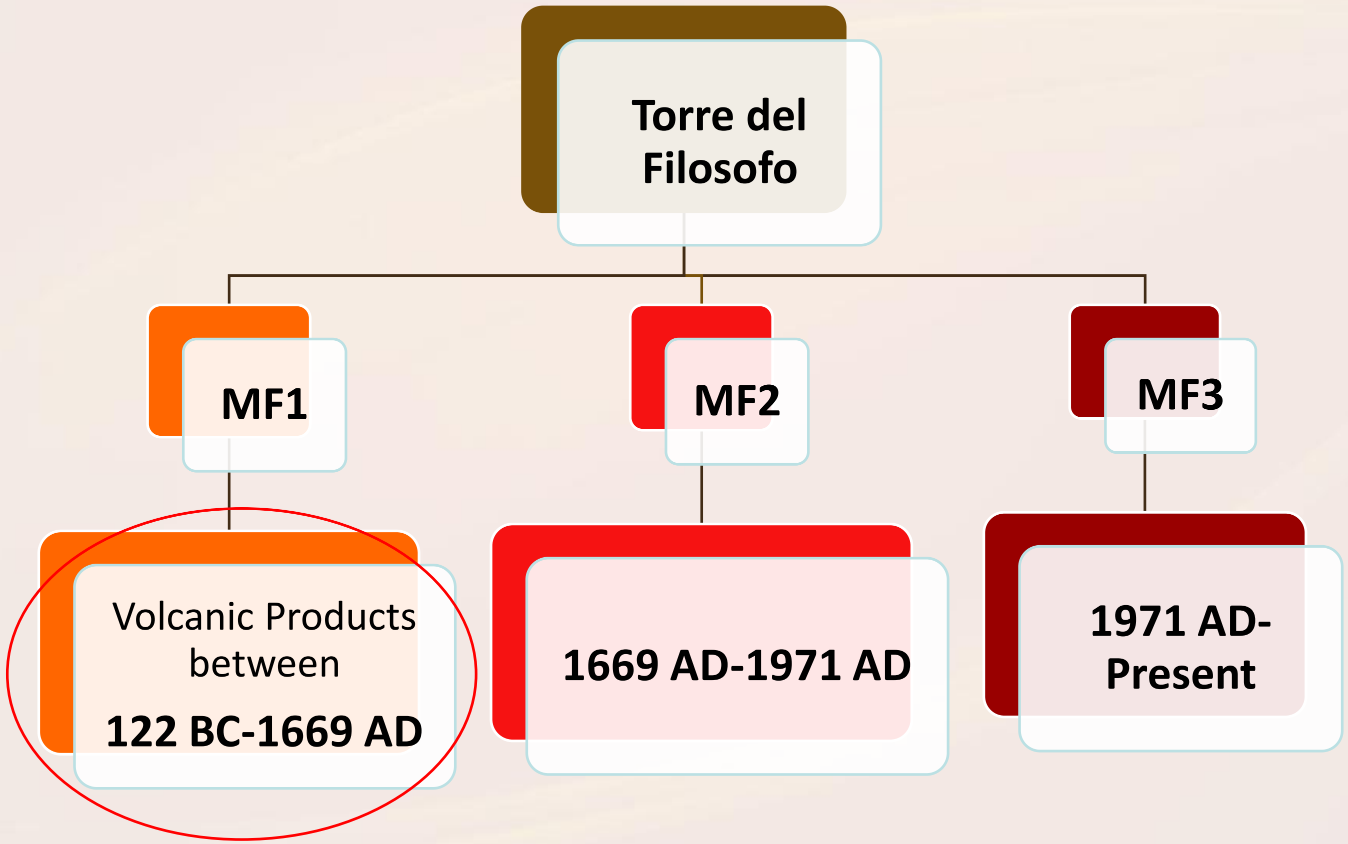
Recent edifice bulk comprised of 2 volcanoes:

- **Ellitico Volcano:** distinguishable, steep slopes, mainly summit portion, flanks reach the Alcantara river on the north.
- **Mongibello Volcano:** formed during the last 15 ka, covers ~ 85% of previous landforms, VdB dominates the Eastern side. 122 BC eruption revealed the *Torre del Filosofo* formation.



STRATIGRAPHIC RELATIONSHIPS SCHEME OF MOUNT ETNA VOLCANIC DISTRICT
SYNTHEMIC UNITS SCHEME
GEOLOGICAL CROSS-SECTION

Major Volcanic Formations(2)



- Selected Formation MF1, 1536-1669 AD

MF1 Historic Lava Flows (1)

- Flank eruptions spanning 1536-1669 AD.

- **1536 AD**

Overflown summit at NW, NE and flank eruption at ~ 2200-1500 m. Extensive damage.

- **1537 AD**

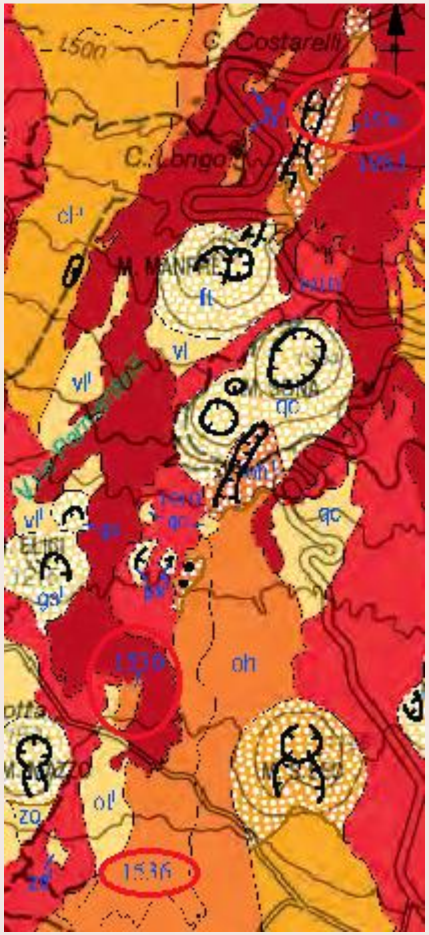
S flank vents at ~ 1900-1700 m,
destroyed Nicolosi, total length of 15 Km,
largely buried under 1892 lava.

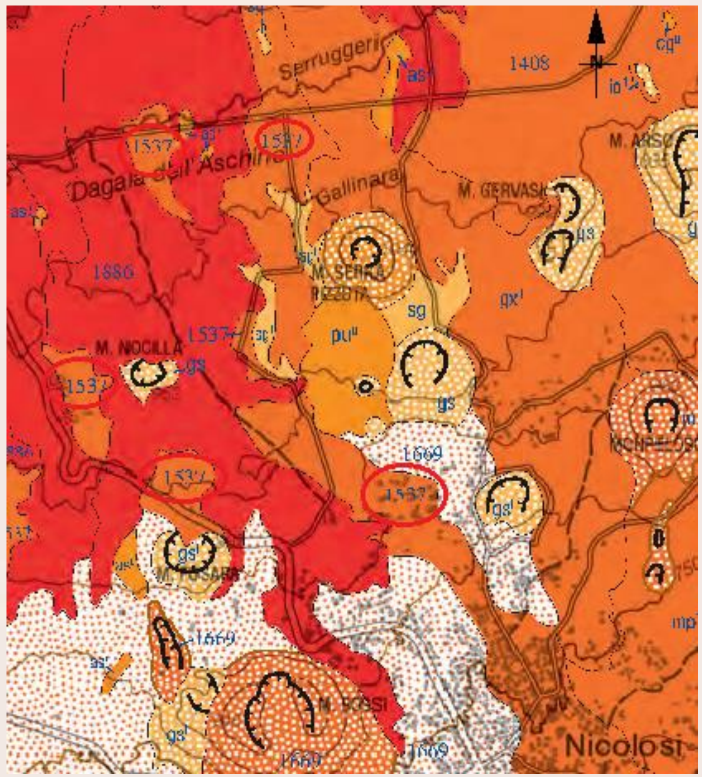
- **1566 AD**

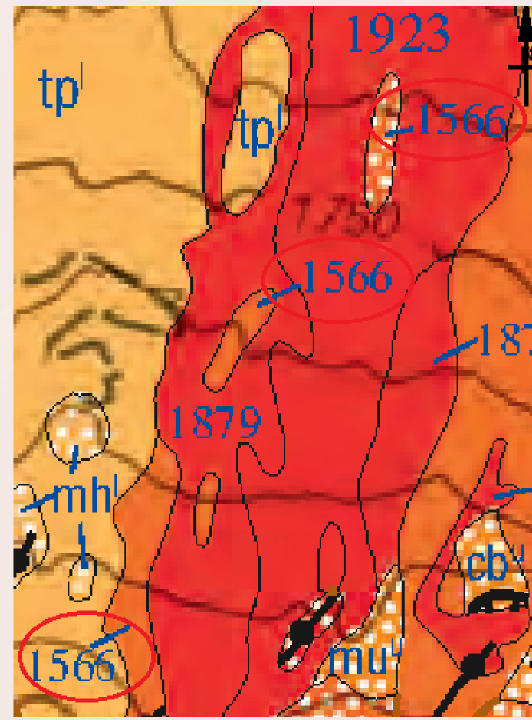
NE flank eruption, multiple fissures.
Largely covered.

MF1 Historic Lava Flows (2)

- Flank eruptions spanning **1536**, **1537** & **1566** AD (from left to right)







MF1 Historic Lava Flows(3)

- **1610 AD**

SW flank vent at 2350-1950m, 2 fissures. Destroyed cultivated vineyards

Total lava vol. = $120\ x\ 10^6 m^3$

- **1614-24 AD**

NW-W flanks at 2500-2000m, voluminous.

Mostly **pahoehoe**. No damages reported.

- **1634-36 AD**

S-SSE flank, short fissure at 2090-1975m, damage across.

Total lava vol. = $150\ x\ 10^6 m^3$

- **1646-47 AD**

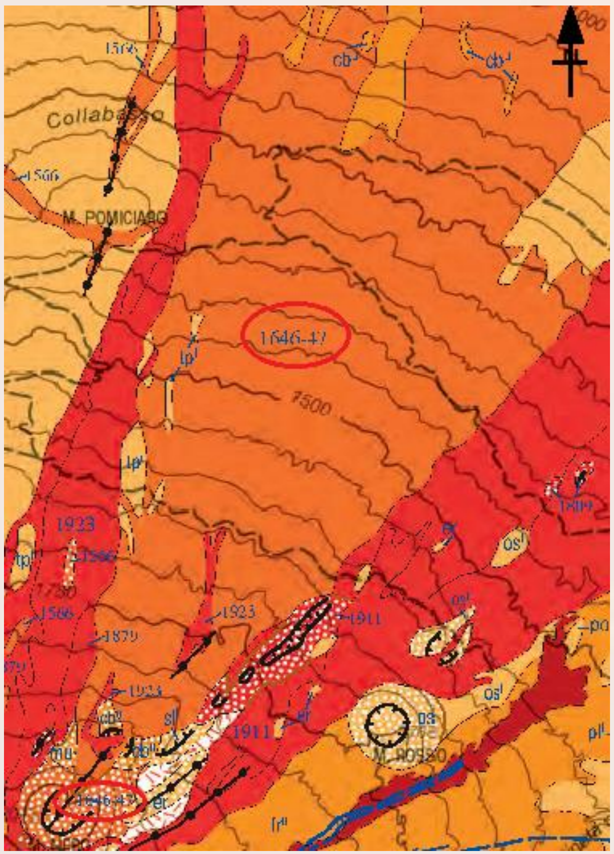
NNE flank at 1900m, several villages destroyed.

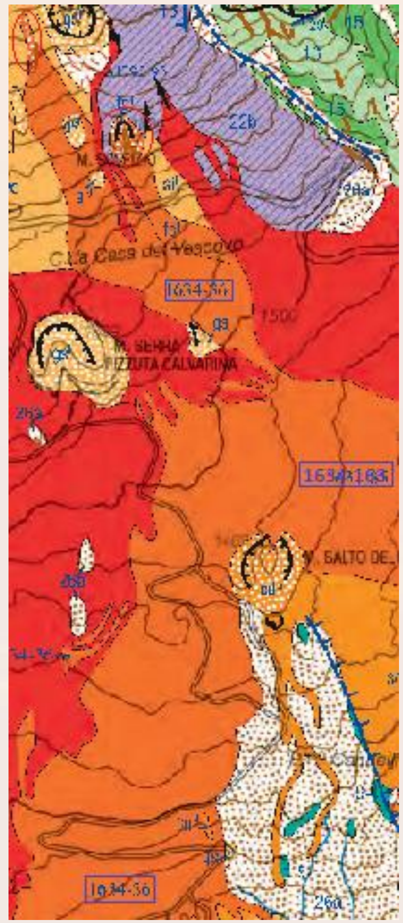
Prominent pyroclastic cone.

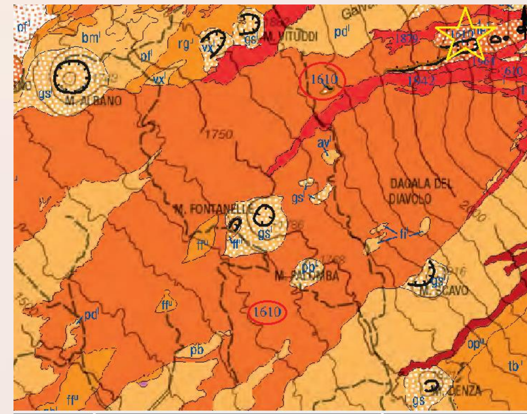
Total lava vol. = $190\ x\ 10^6 m^3$

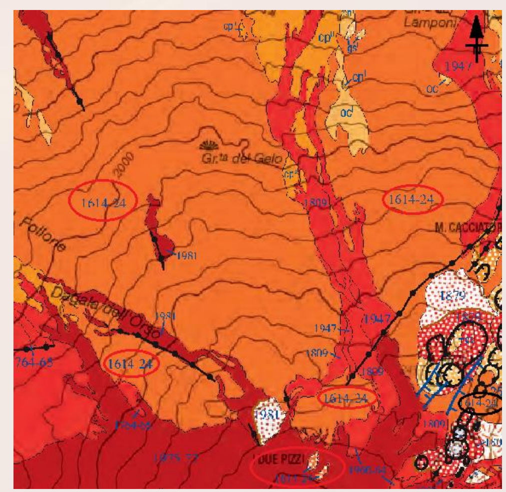
MF1 Historic Lava Flows(4)

Flank eruptions **1610**, **1614-24**, **1634-36** & **1646-47 AD**

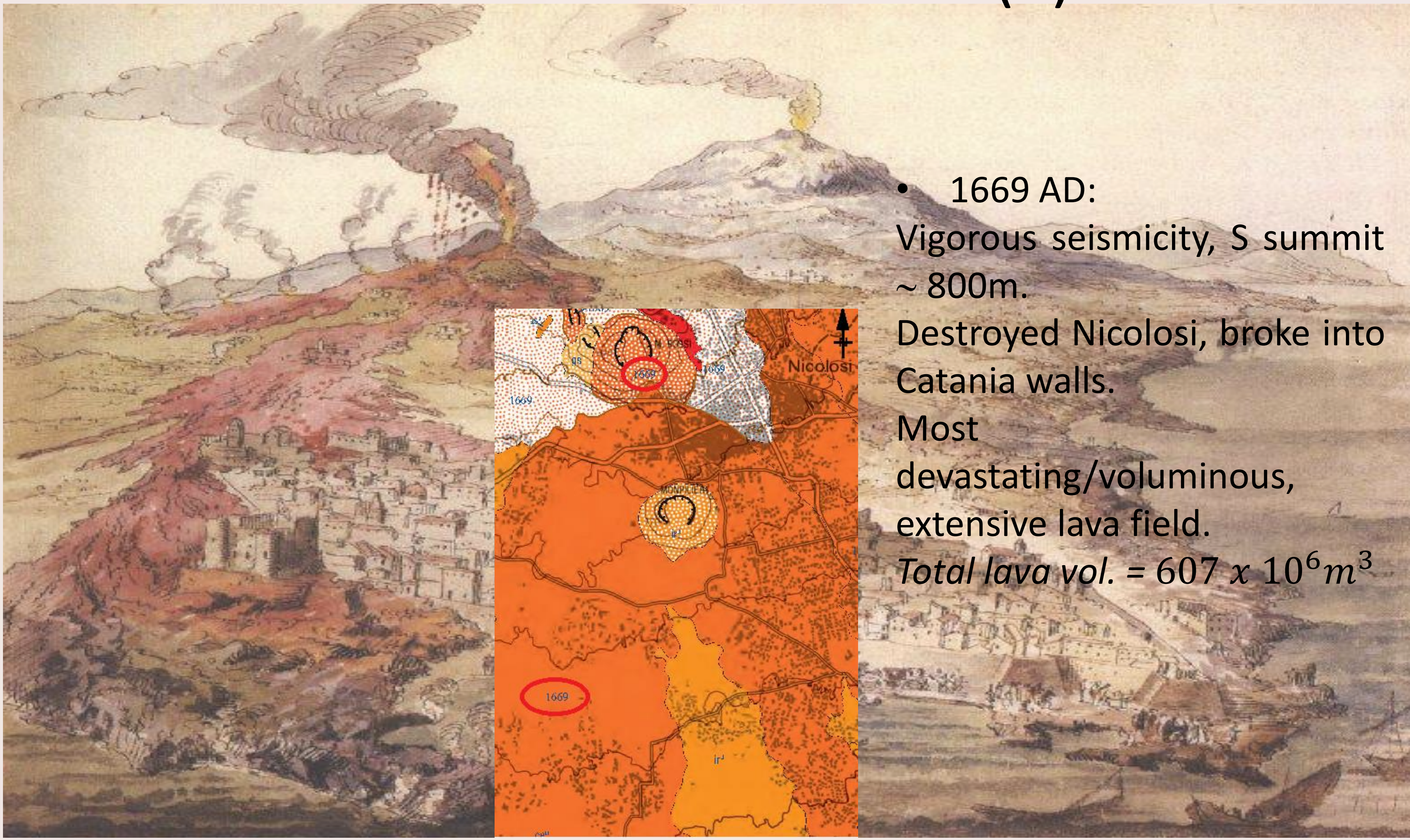








MF1 Historic Lava Flows(3)

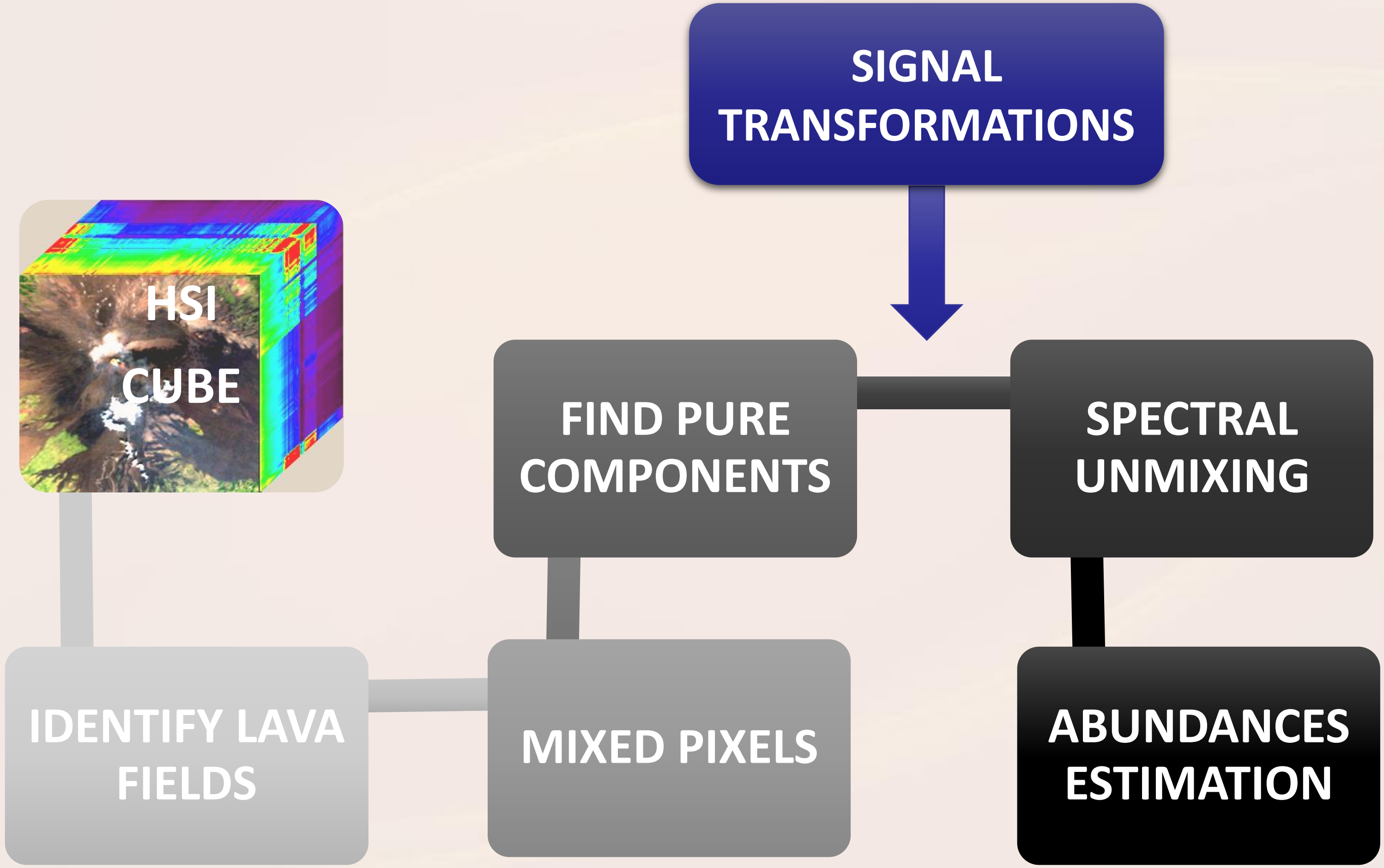


- 1669 AD:

Vigorous seismicity, S summit ~ 800m.
Destroyed Nicolosi, broke into Catania walls.
Most devastating/voluminous, extensive lava field.
Total lava vol. = $607\ x\ 10^6 m^3$

(Fresco from Catania Cathedral)

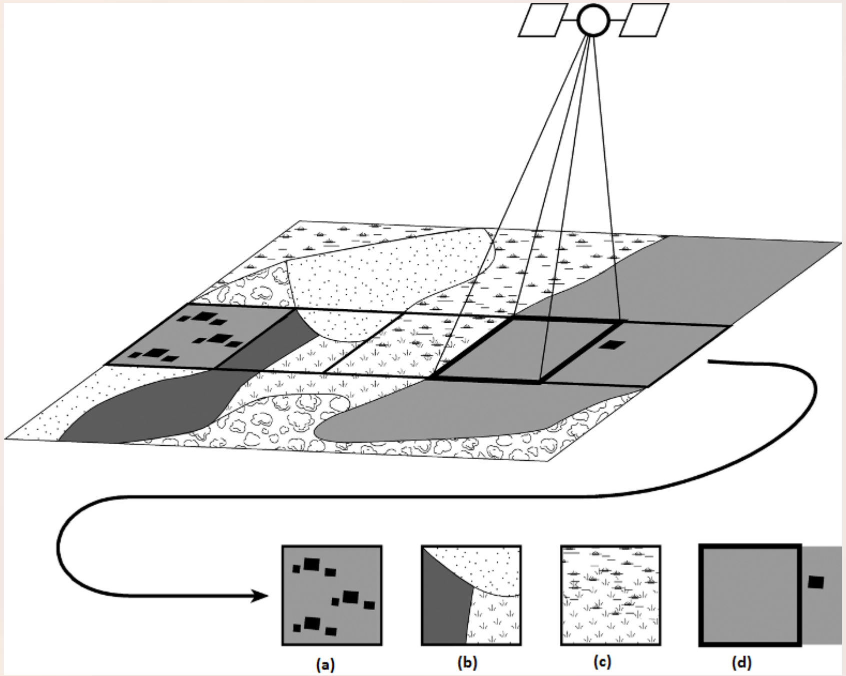
B. PROBLEM DEFINITION


SIGNAL TRANSFORMATIONS
HSI CUBE
FIND PURE COMPONENTS
SPECTRAL UNMIXING
IDENTIFY LAVA FIELDS
MIXED PIXELS
ABUNDANCES ESTIMATION

Spectral Mixture Models

Spectral Mixing: each image pixel may contain one or more LC components ∴ **mixed** spectral characteristics

- Several Mixture Models depending on the mixed pixel morphology
- Solution: *soft sub-pixel classification* techniques partition each pixel on different classes (UNMIXING)



Mixture Models
- Linear
- Bilinear

Linear Mixture Model (LMM)

✧ **Assumption**:

each endmember covers a defined region within the pixel area & multiple scattering is negligible → Pure components **Linearly** mixed

✧ Representation per pixel:

$$\mathbf{y} = \mathbf{M}\boldsymbol{a} + \mathbf{n}$$

N x *1* pixel signature vector

N x *K* matrix: contains K $m_r = [m_{1,r}, \ldots, m_{N,r}]^T$ endmember signatures

$[a_1, .., a_K]^T$ fractional abundance vector

additive white noise

- N: # of bands
- r=1,...K: # of endmembers,

✧ **Potentially** induced Constraint: $a \geq 0\ (NNC)$, for every image pixel

Bilinear Mixture Model (BMM)

✧ **Assumption**:

linear components as in LMM + endmember correlation terms

$$K^* = \frac{1}{2}K(K+1)$$

✧ Formula:

$$\mathbf{y} = \mathbf{M}\boldsymbol{a} + \sum_{\mathrm{i}=1}^{\mathrm{K}-1}\sum_{\mathrm{j}=\mathrm{i}+1}^{\mathrm{K}} a_{i,j}\mathbf{m_i} \odot \mathbf{m_j} + \mathbf{n} \Leftrightarrow \mathbf{y} = \sum_{\mathrm{k}=1}^{K^*} a_k{}^* \mathbf{m_k}{}^* + \mathbf{n}$$

where $\mathbf{m_i} \odot \mathbf{m_j}$ denotes the ith & jth endmember interaction.

✧ **Potentially** induced Constraint: $a_k{}^* \geq 0$

C. DATA

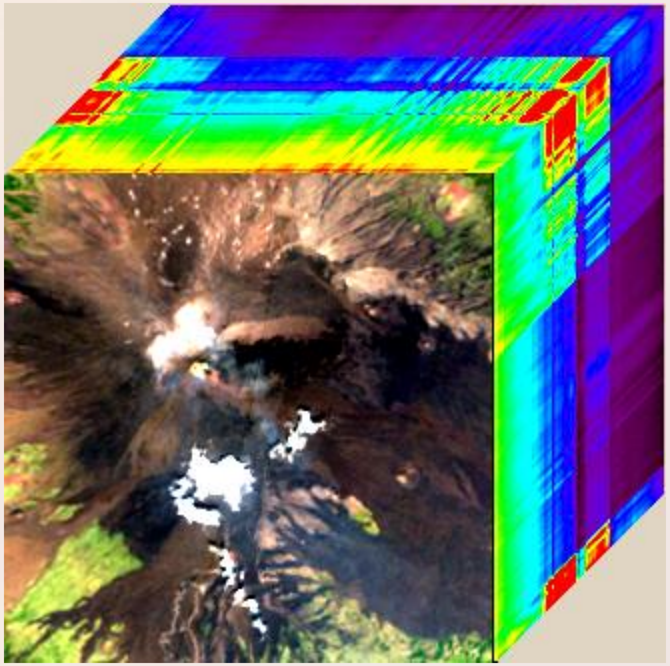
C. DATA

Data

- *Used*: Hyperspectral image cube over Eastern Sicily, **09/07/2007**

👌 Big Data manipulation

- From: NASA EO-1 **HYPERION** sensor
  - 220 calibrated spectral bands (out of 242)
  - 10 nm spectral res. from 0.4 to 2.5 microns
  - 30 m spatial res. over a 7.7 Km swath
  - Highest SNR on Vis-VNIR
  - Level 1T radiometric & geometrically corrected product



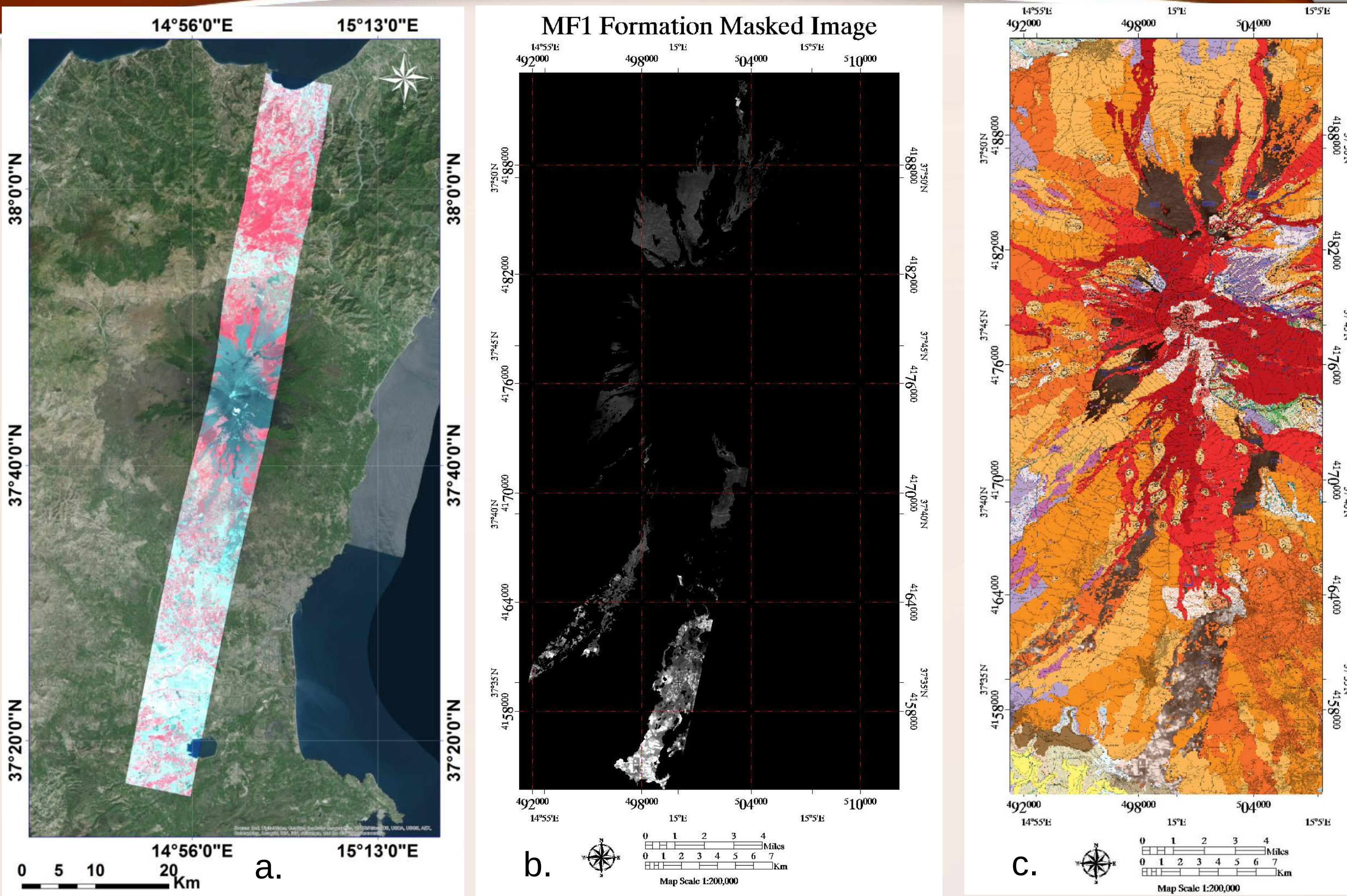
Total # of bands : **140** after bad or noisy band exclusion

Data Preprocessing

The followed preprocessing steps:

- *Atmospheric Correction* (via FLAASH algorithm): Radiance ⟶ Reflectance[1]
- *Water vapor and Cloud Masking*
- *Dimensionality / Noise Reduction* via PCA: first 4 PCs ⟶ Inverse PCA
- *Vegetation Masking* via NDVI: vegetated areas threshold on **0.41**
- *Active Areas Segregation:* VdB omitted as a separate ROI
- *Formation Masking:* MF1 manually digitized, masked initial image

[1] not prerequisite step for unmixing analysis



a) from K. Karagiannopoulou, (2017). *"Use of Hyperion spectral signatures and Sentinel-1 Polarimetric backscatter for lava flow differentiation in Mt. Etna, Sicily".*

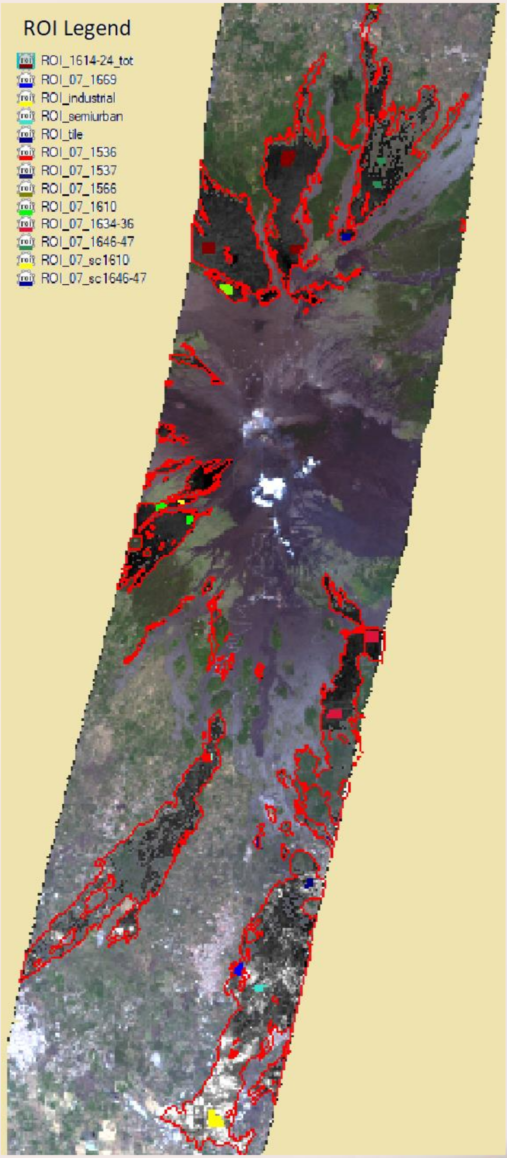
D. METHODOLOGY

Endmember Extraction(1)

✧ Criteria of ROI selection:

- Dense **Lava** deposits, close to geological map dates
- **Avoid** Borderline regions
- ROI > **30 pxls**, otherwise merging
- Spectral profile inspection → **minimum variability**
- Include **populated** environments

👌 # of ROIs = **13**, for 9 *LFs*, 2 *scoria cones* and *industrial, semi-urban, tile rooftops*.



Endmember Extraction(2)

✧ Endmembers:

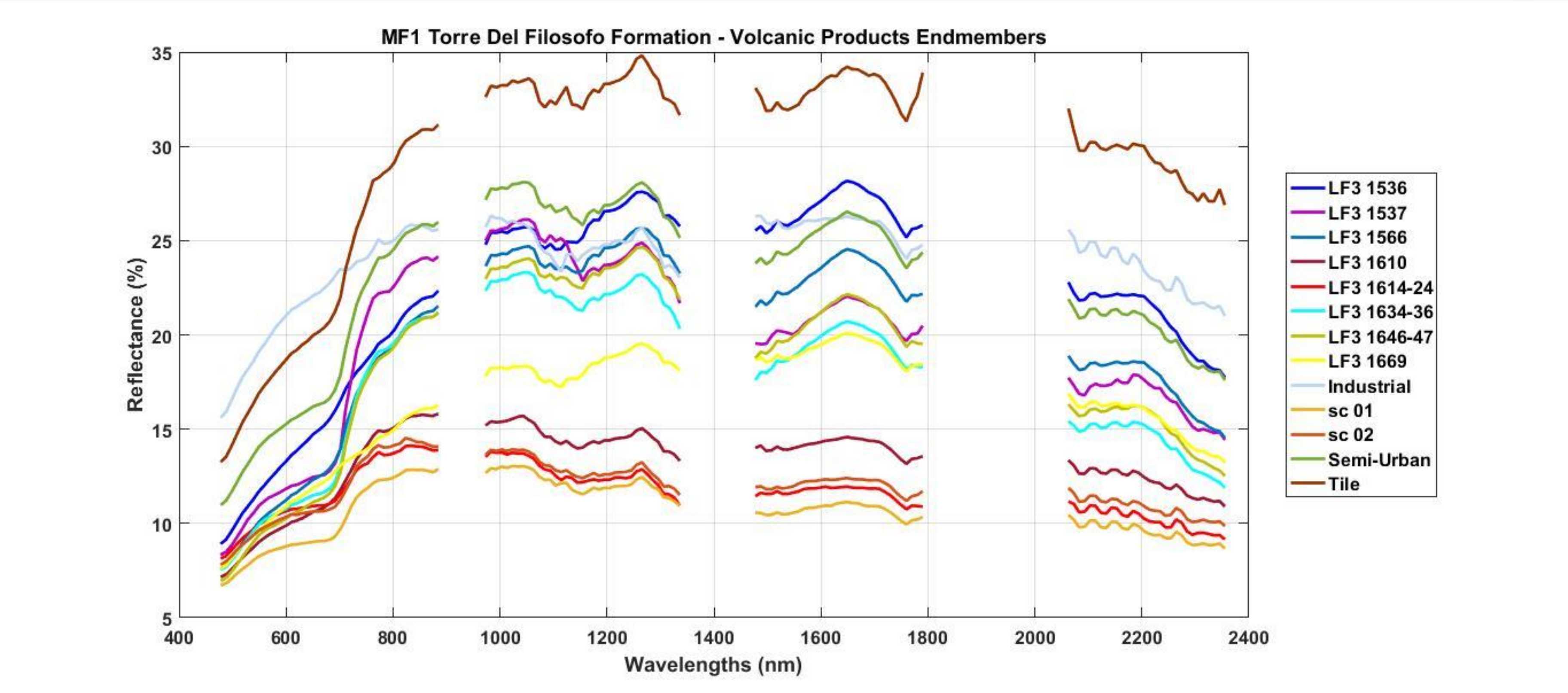
- Pixel values of the same ROI follow a **Normal Distribution**
- Assumed the majority of spectral information is between **±25%** from the mean value → resize each ROI

✧ HOW ?

- Find the Gaussian borders → **exclude the outliers**
- Calculate new ROI average
- Mean value represents the 1x140 Endmember vector
- Not physical image pixel

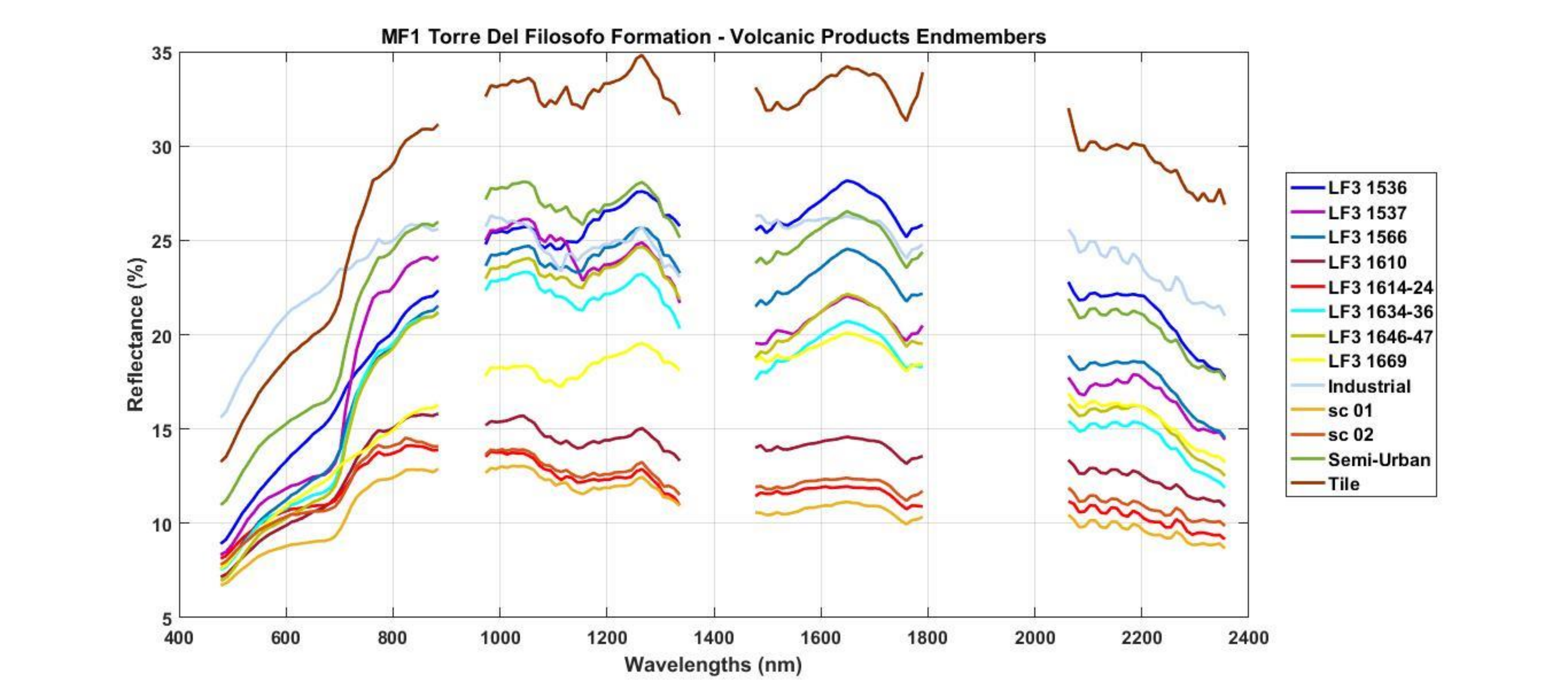
👌 More efficient than simple averaging or median value

Volcanic Products Spectra

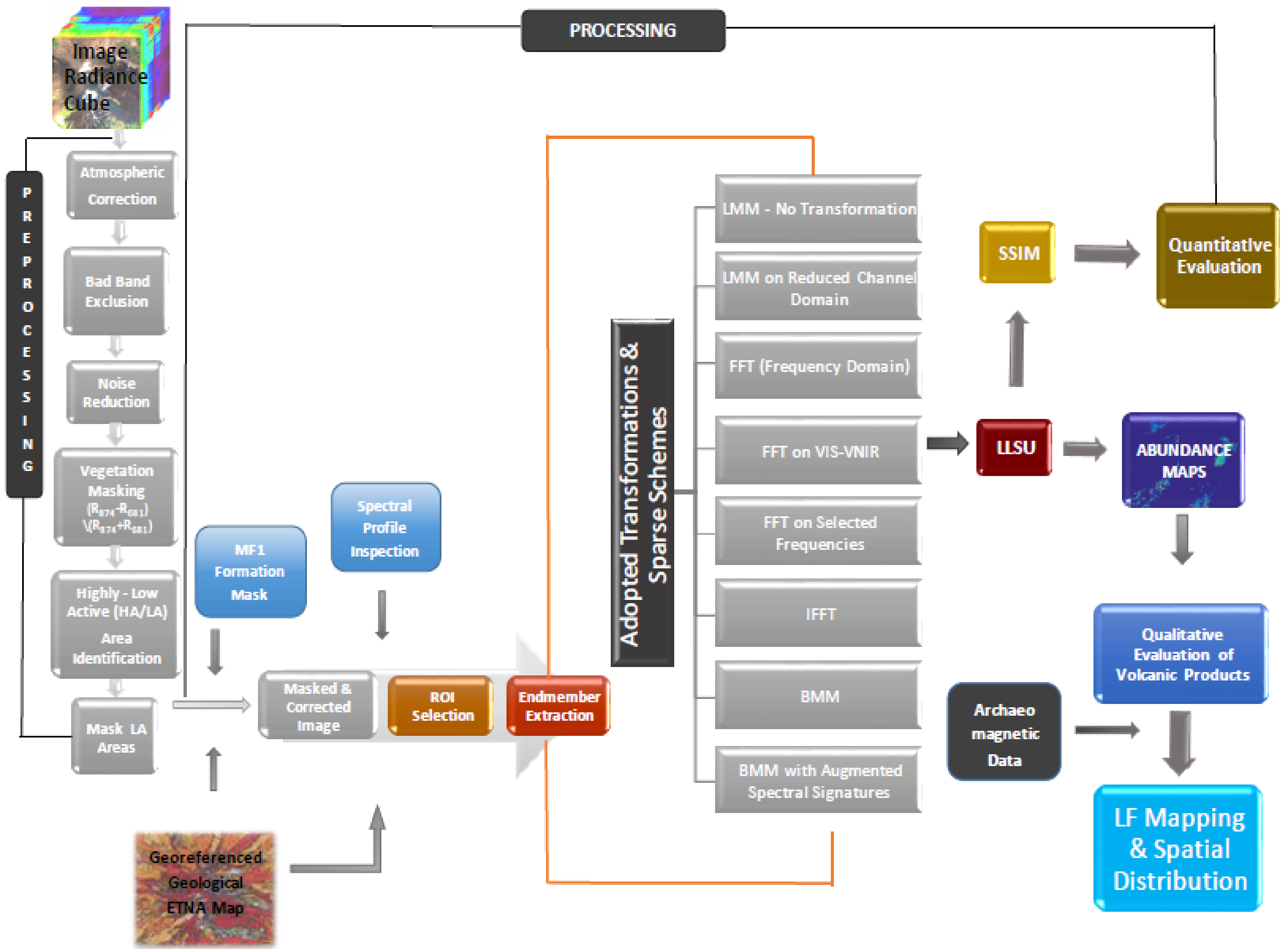


- Flat | slightly convex profile in 750-2500nm range
- Potential olivine presence: absorption feature on ~ 900nm, need continuum removal
- Higher overall refl.: *Tile rooftop* buildings, Lowest overall refl.: *1610* scoria cone
- Older LFs have higher reflectance (**consistent**)

Volcanic Products Spectra



- ∃ vegetation absorption: all products **except** industrial areas and 1536, 1669 LFs
- Alteration on: 1536, 1537, 1566, 1610, 1634-36, 1646-47 LFs
- Band reduction excludes crucial bands for compositional analysis


PROCESSING
Image Radiance Cube
PREPROCESSING
Atmospheric Correction
Bad Band Exclusion
Noise Reduction
Vegetation Masking
Highly - Low Active (HA/LA) Area Identification
Mask LA Areas
MF1 Formation Mask
Spectral Profile Inspection
Masked & Corrected Image
ROI Selection
Endmember Extraction
Georeferenced Geological ETNA Map
Adopted Transformations & Sparse Schemes
LMM - No Transformation
LMM on Reduced Channel Domain
FFT (Frequency Domain)
FFT on VIS-VNIR
FFT on Selected Frequencies
IFFT
BMM
BMM with Augmented Spectral Signatures
SSIM
Quantitative Evaluation
LLSU
ABUNDANCE MAPS
Archaeo magnetic Data
Qualitative Evaluation of Volcanic Products
LF Mapping & Spatial Distribution

Linear Least Squares Unmixing (LLSU)

- Solution to mixed pixels problem: **Linear Least Squares Unmixing**
  - Quadratic optimization problem + linear inequality constraints
  - Seeks the Optimum Abundance vector

- **Minimize Least Squares Error (LSE):**

$$LSE = (\mathbf{M}a - \mathbf{y})^{\mathrm{T}}(\mathbf{M}a - \mathbf{y}), \qquad subject\ to\ a \geq 0$$

Constrained abundance vector estimation:

$$a_{NCLS} = \left(\mathbf{M}^{\mathrm{T}}\mathbf{M}\right)^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{y} - \left(\mathbf{M}^{\mathrm{T}}\mathbf{M}\right)^{-1}$$

- Non-negativity constrained LS on Matlab via *lsqnonneg*[2]

[2] STOC not included

Signal Transformations (1)

✧ **Methods** used on **LLSU**:

1. *Raw Data - Channel Domain Representation*
    - ❖ Simple and robust
2. *Reduced Channel Domain*
    - ❖ Exploit the HSI spectral redundancy
    - ❖ Dimensionality reduction via Feature Selection (FS)* → **22** optimum bands
    - ❖ Majority of information in R-VNIR !
3. *FFT transformed Image Spectra*
    - ❖ DFT from Channel Domain → Frequency Domain
    - ❖ *LLSU Frequency Domain* ∝ *LLSU Channel Domain*

* Special Thanks to Dr. Kostas Themelis and Dr. Irida Xenaki for providing their scripts.

Signal Transformations(2)

✧ **Methods** used on **LLSU**:

4. *FFT on Visible-VNIR*

   - ❖ Subspace identification ⇒ selection of bands with low SNR → for the Hyperion dataset VIS-VNIR
   - ❖ High-pass filter that enhances spectral details

5. *FFT on Reduced Frequency Domain*

   - ❖ No significant variation over the 20th → **Dimensionality Reduction**

6. *IFFT transformed Image*

   - ❖ Keep Bands with significant energy content
   - ❖ Reduced Frequency Domain → Initial Channel Domain

Signal Transformations(3)

✧ **Methods** used on **Bilinear LSU**:

7. *Reduced Channel Domain*

   ❖ Dot product of each endmember with the rest as $m_i \odot m_j$ **= Endmember Correlation**

   ❖ Non-Linearity is induced by the enhanced Endmember matrix

8. *Augmented Spectral Signatures Domain*

   ❖ Dot product of each of the bands **= Band Correlation**

Image Reconstruction

HOW ?

Endmember Matrix **x** Extracted Abundance Matrix– Neglect additional noise

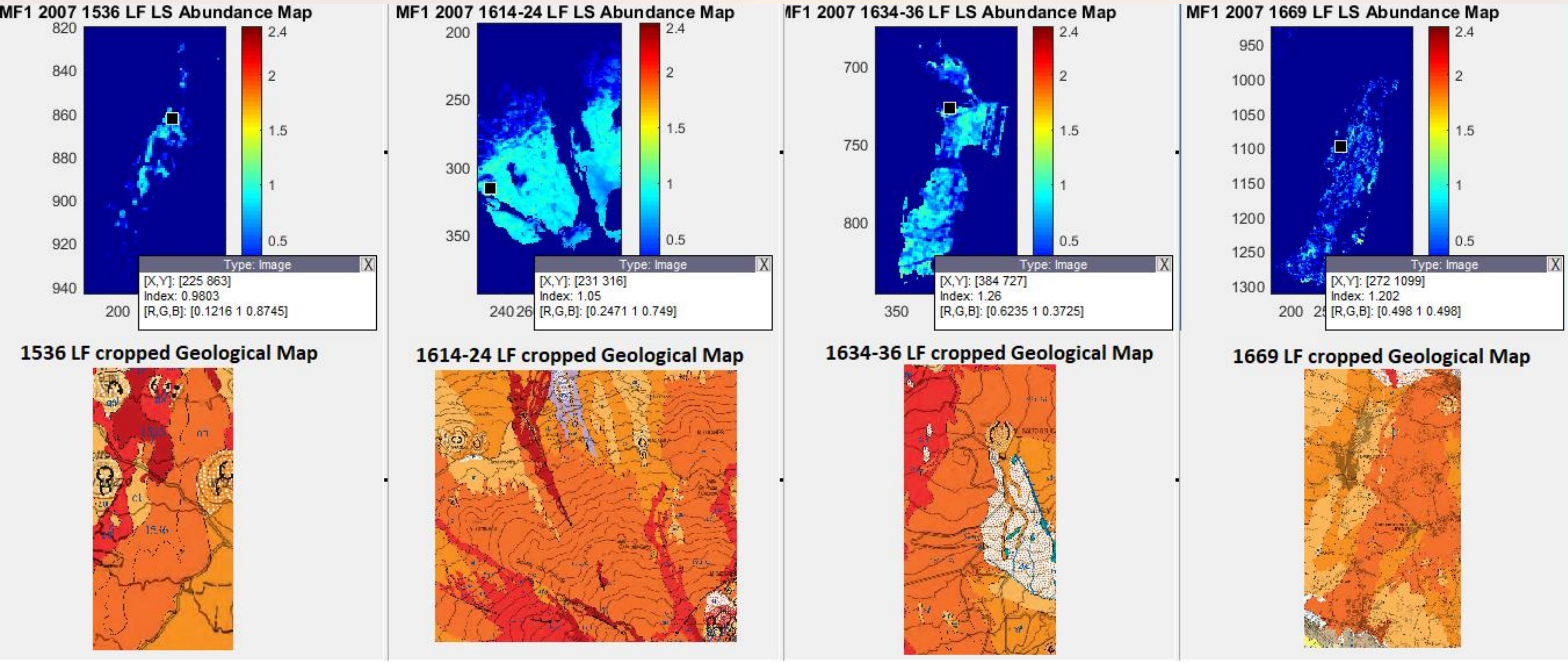
- *Unmixing Quality Assessment* → **Structural Similarity Index** (SSIM)
- Quantitative measure of the comparison between the *Initial image* and each *Reconstructed image*.
- Calculated on various windows of an image
- Formula based on three terms: luminance ($\boldsymbol{l}$), contrast ($\boldsymbol{c}$) & structure ($\boldsymbol{s}$)

HERE:

👌 Plot SSIM values for each of the 140 bands + mean SSIM values display.

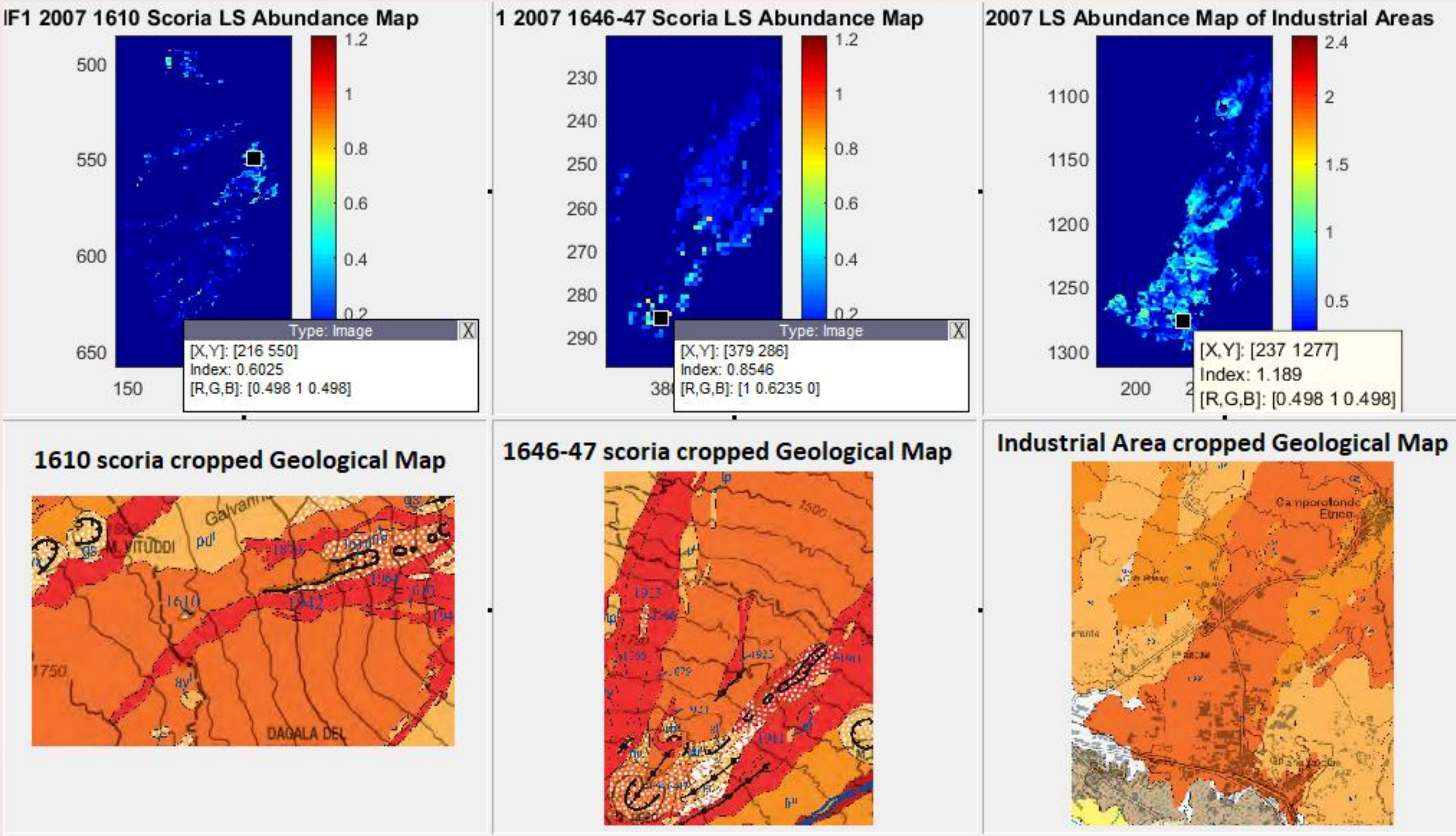
E. RESULTS

METH. 1 Abundance Maps(1)

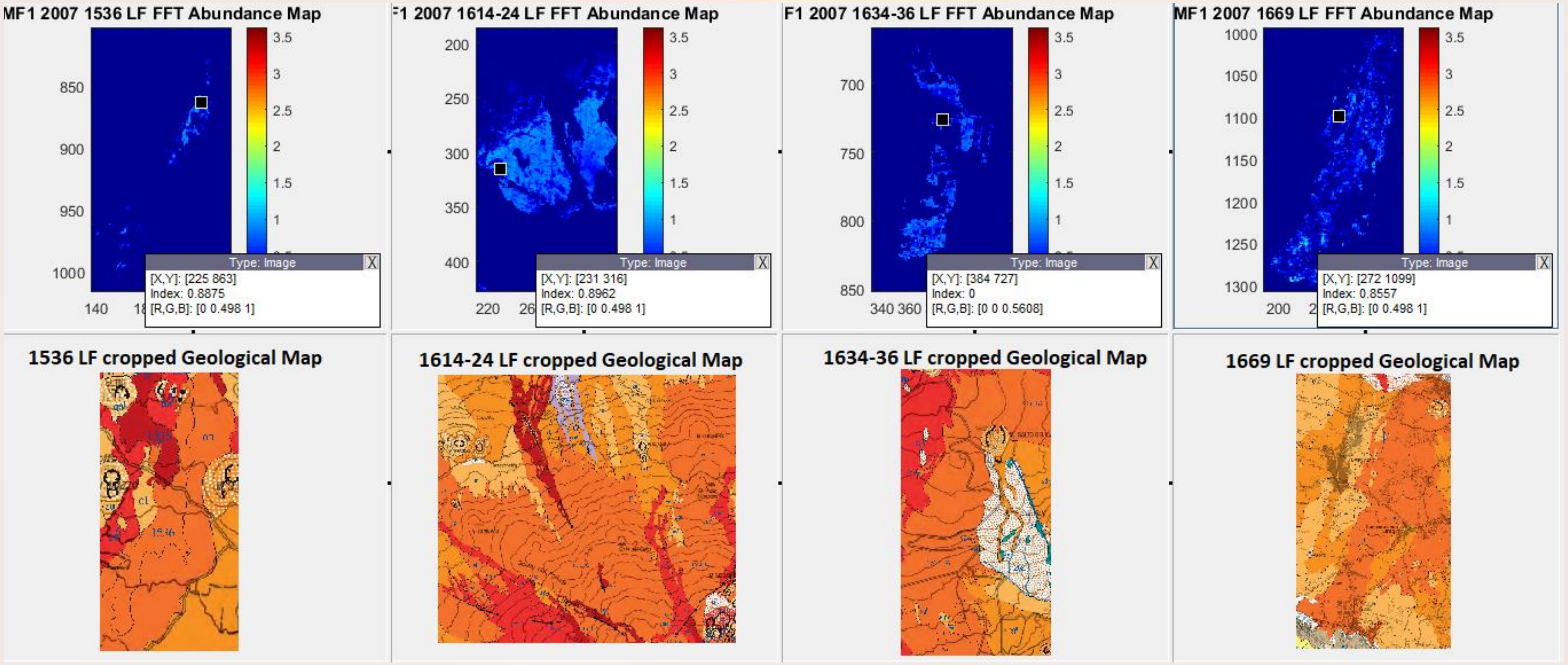


- LF colorbars are integrated on the **same maximum value**, unique for each method
- Generally: LFs are delineated
- Refined detail not provided by Geological Map
- High abundances on ROI areas, as expected

METH. 1 Abundance Maps(2)

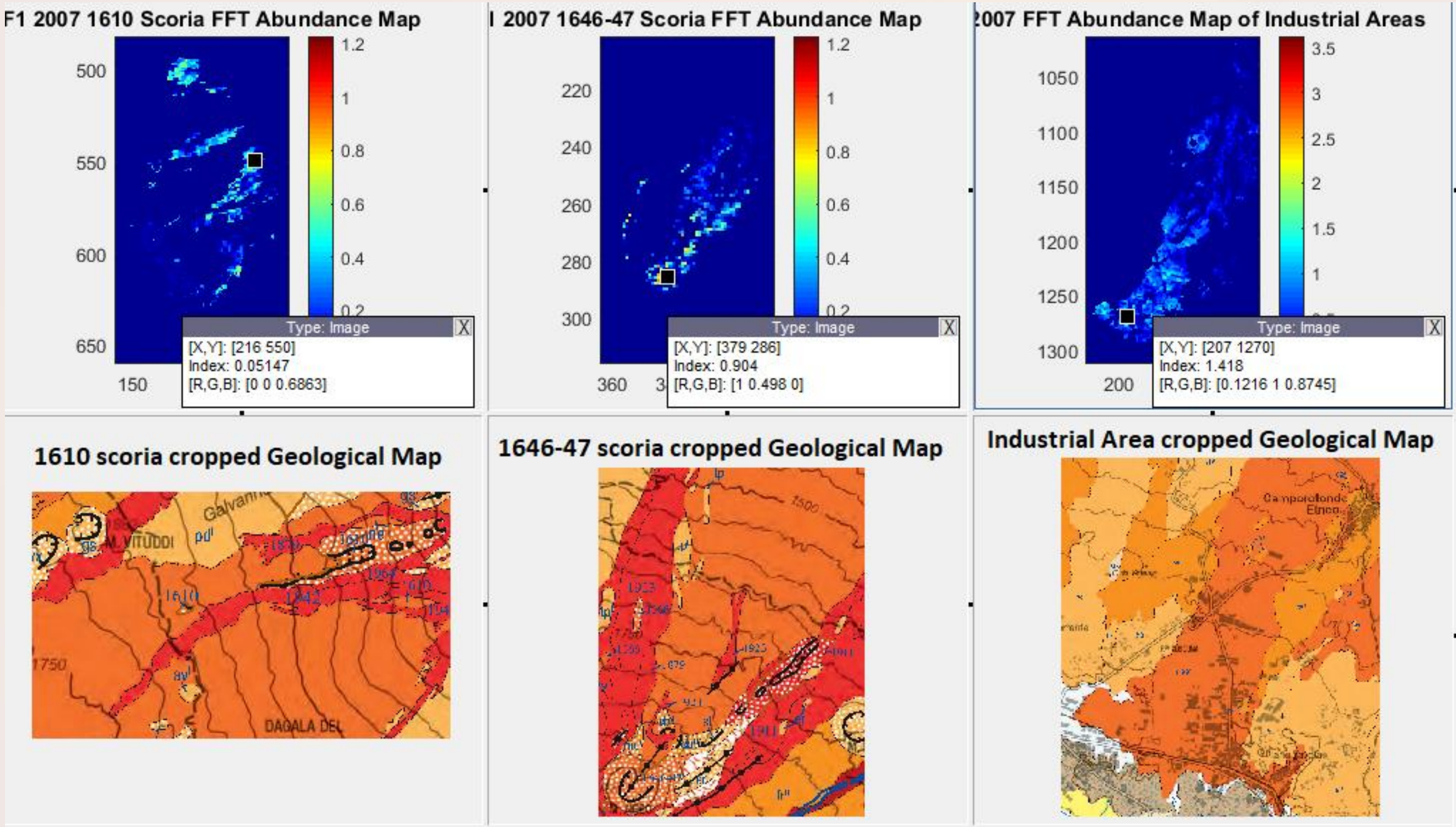


METH. 3 Abundance Maps(1)

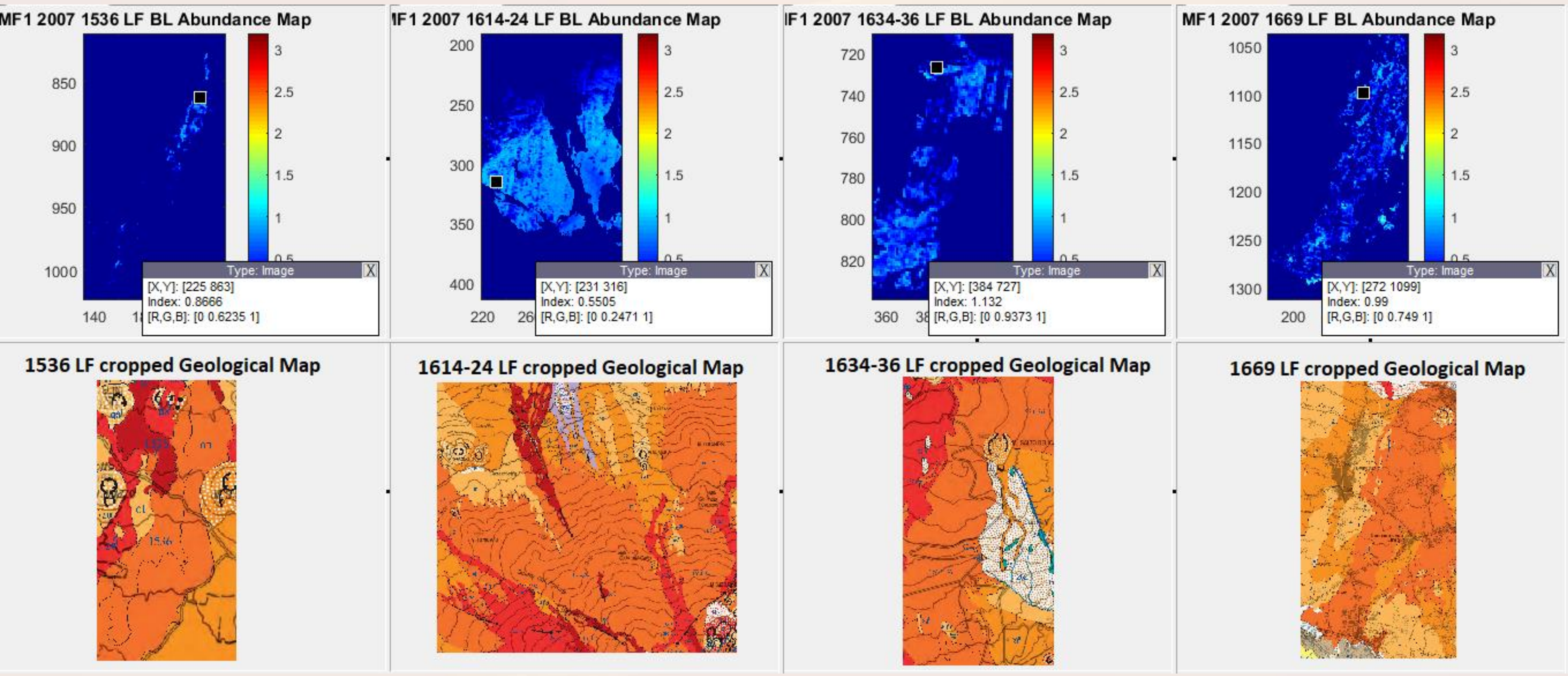


- Lava flows outline is maintained, lower abundances on same pixels
- Lava patterns are consistent
- Outline fades, 1634-36 LF → Gaussian noise reduction, stripping exists

METH. 3 Abundance Maps(2)

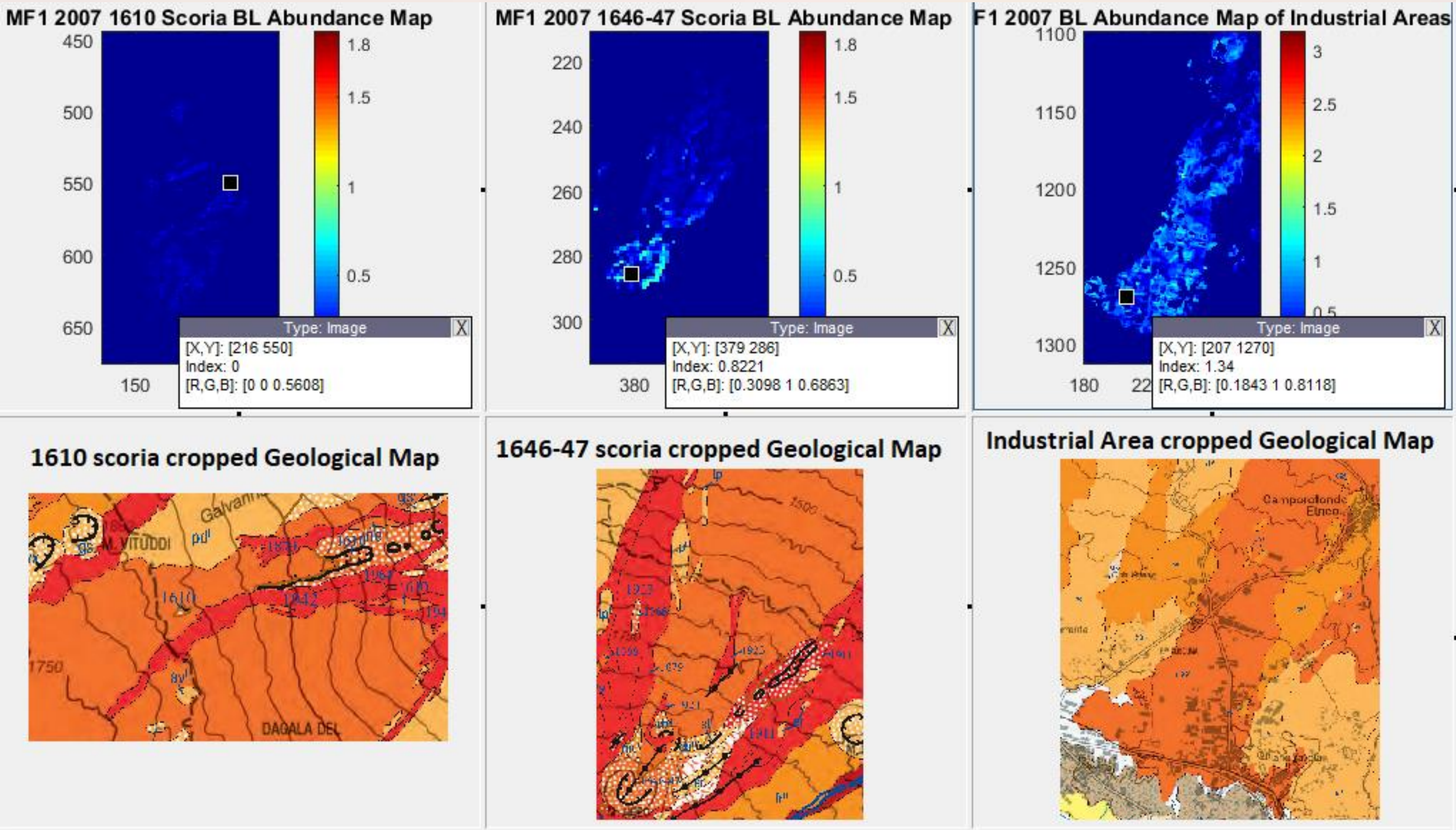


METH. 7 Abundance Maps(1)



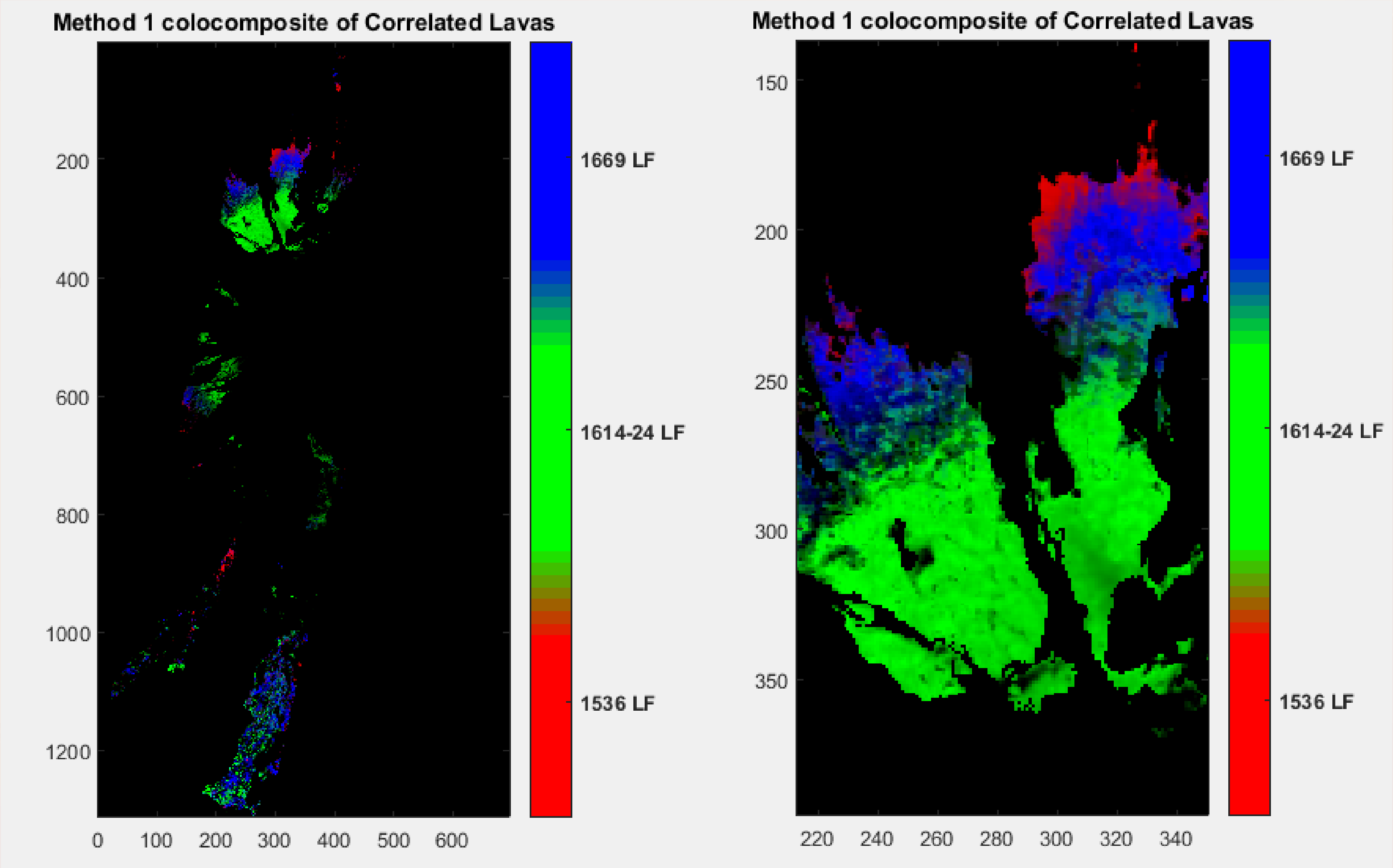
- Homogeneous LF distribution
- Delineated lavas
- Sharper features on volcanic surface due to high frequency spectra
- Comparable abundances with previous methods

METH. 7 Abundance Maps(2)

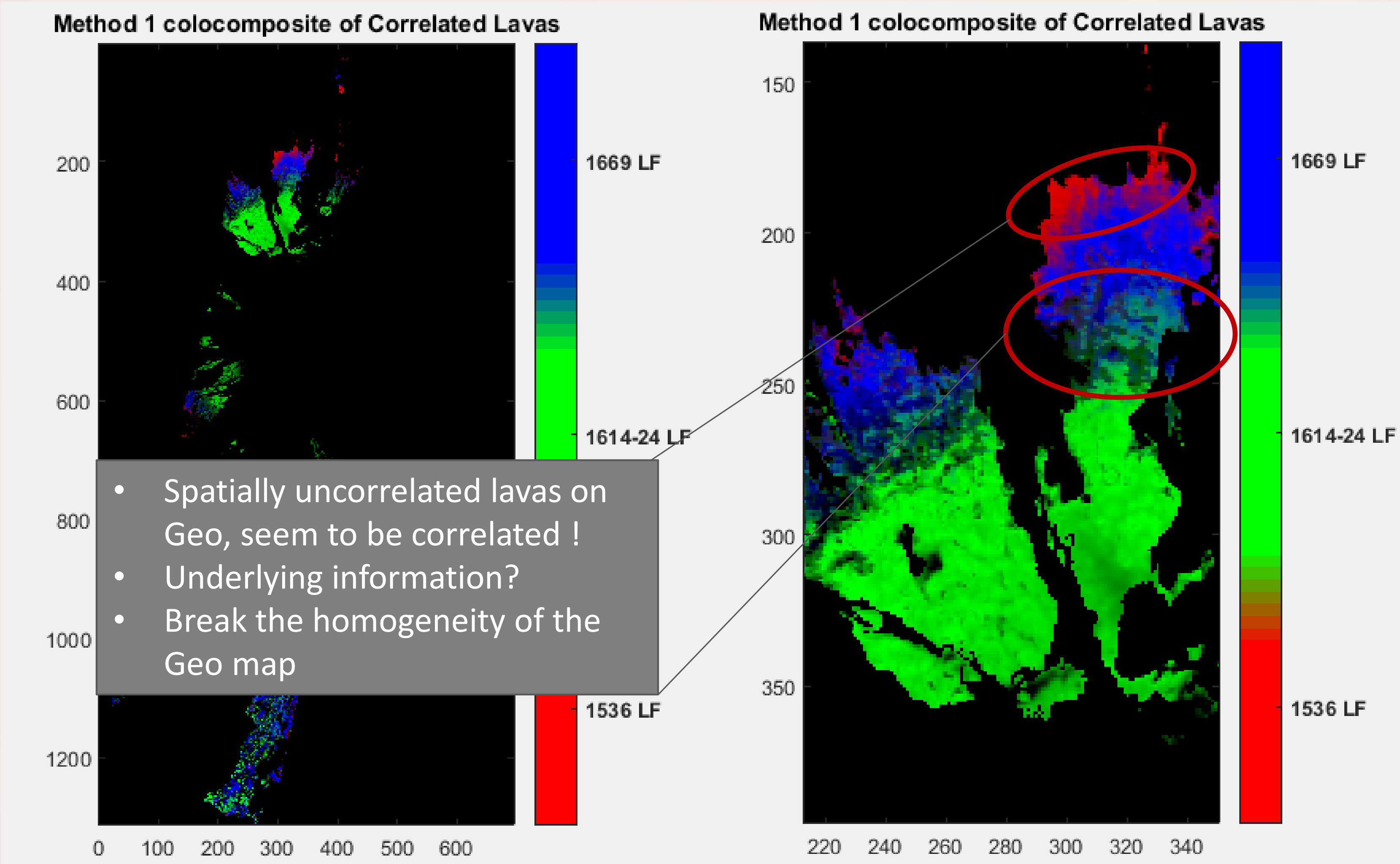


Faint 1610 scoria, low reflectance → spectrally correlated with other products

Comparative Analysis (1)

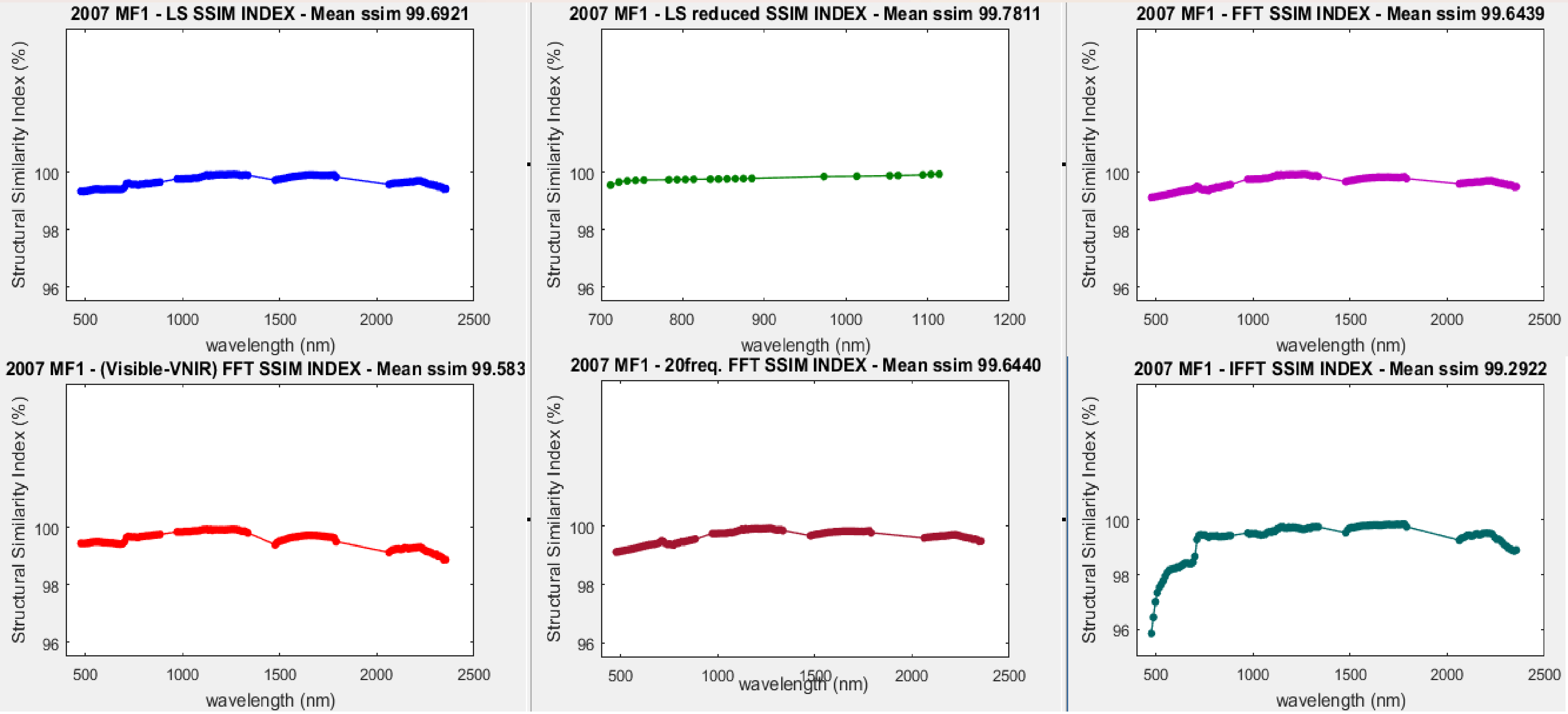


Comparative Analysis (2)



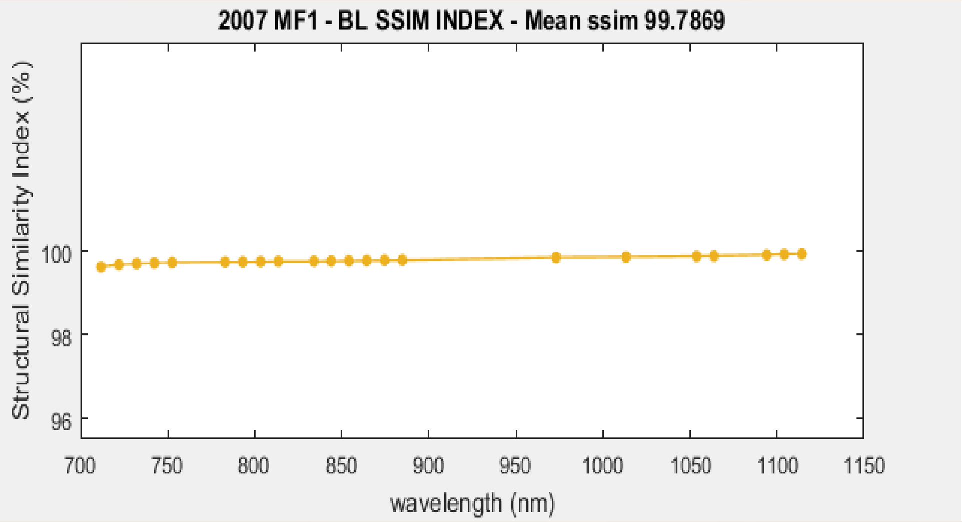
- Spatially uncorrelated lavas on Geo, seem to be correlated !
- Underlying information?
- Break the homogeneity of the Geo map

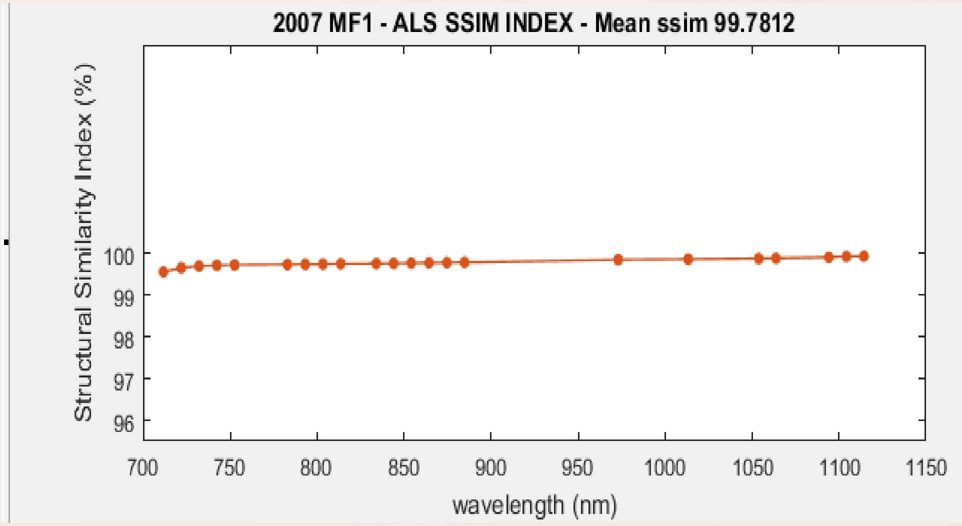
All Methods Image Reconstruction



- Reconstruction levels > **99.5%** → successful unmixing processes
- Ascending trend until 1350nm, generally descending on low frequencies
- IFFT lowest SSIM due to zero pudding, 20 → 140 bands
- Reduced FFT (20 freq.) gives the same results as FFT

All Methods Image Reconstruction





- *Bilinear Unmixing* on the enhanced domain → **Best overall** reconstruction accuracy
- Note: *Bilinear SSIM* **>** *LS SSIM* → bilinear model as a better representation

👌 Meth. 2, 7 & 8 → subspace, give equivalent results

Time efficiency and Noise Reduction

TIME EFFICIENCT

TIME EFFICIENT & NOISE REDUCTION

TIME EFFICIENT DIMENSIONALITY REDUCTION

Elapsed Time (sec)	MEAN	STDV
LS	15.094	± 0.235
LS Reduced	12.497	± 0.161
FFT	**14.168**	**± 0.034**
V_VNIR FFT	14.199	± 0.123
20freq	**12.342**	**± 0.149**
IFFT	15.325	± 0.314
BL	32.919	± 0.343
BL_aug	15.949	± 0.177

F. CONCLUSIONS & DISCUSSION

DISCUSSION

Conclusions

👌 To sum up:

- Mapping of the different volcanic products wrt to Geological map
- Extract abundances of LF components, determine their spatial distribution
- Qualitative overview of the volcanic surface complexity
- Added value on existing context, quantitative information
- Achieve time efficient and robust techniques with comparable unmixing results
- Perform Dimensionality Reduction with very low computational cost → Big Data efficient manipulation
- Created a Paradigm shift → Future extension

Discussion

👌 To the best of our knowledge:

- ❖ Volcanic products of Etna are studied mostly from field measurements, complimented by satellite images, in terms of mineralogical composition.
- ❖ There are no references of Hyperspectral Unmixing techniques on Etnian Lava Fields, potentially to no other volcanic edifice.
- ❖ Innovative work in terms of signal processing approaches tailored on a multidiverse volcanic environment.

Future Work

To compliment the HSI content:

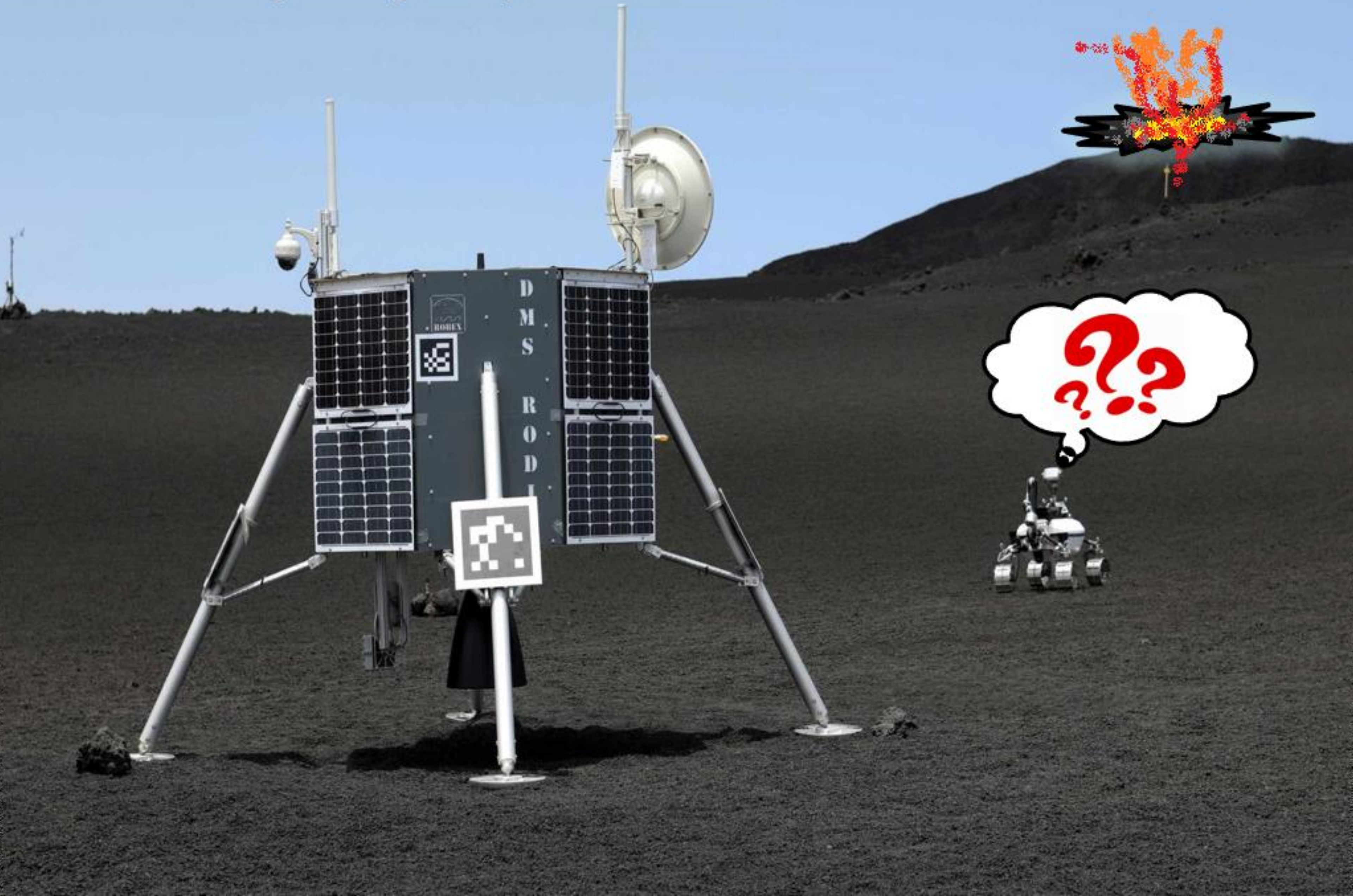
- Sentinel-2 data provide great potential on the field
- Constant coverage and high spatial resolution → Lava Discrimination
- Performed already similar techniques on Multispectral data
- Preliminary work on Etna with S2 data, that ultimately aims on:

A Spatio-temporal

characterization of Mt. Etna

volcanic products !

Thank you for your attention !

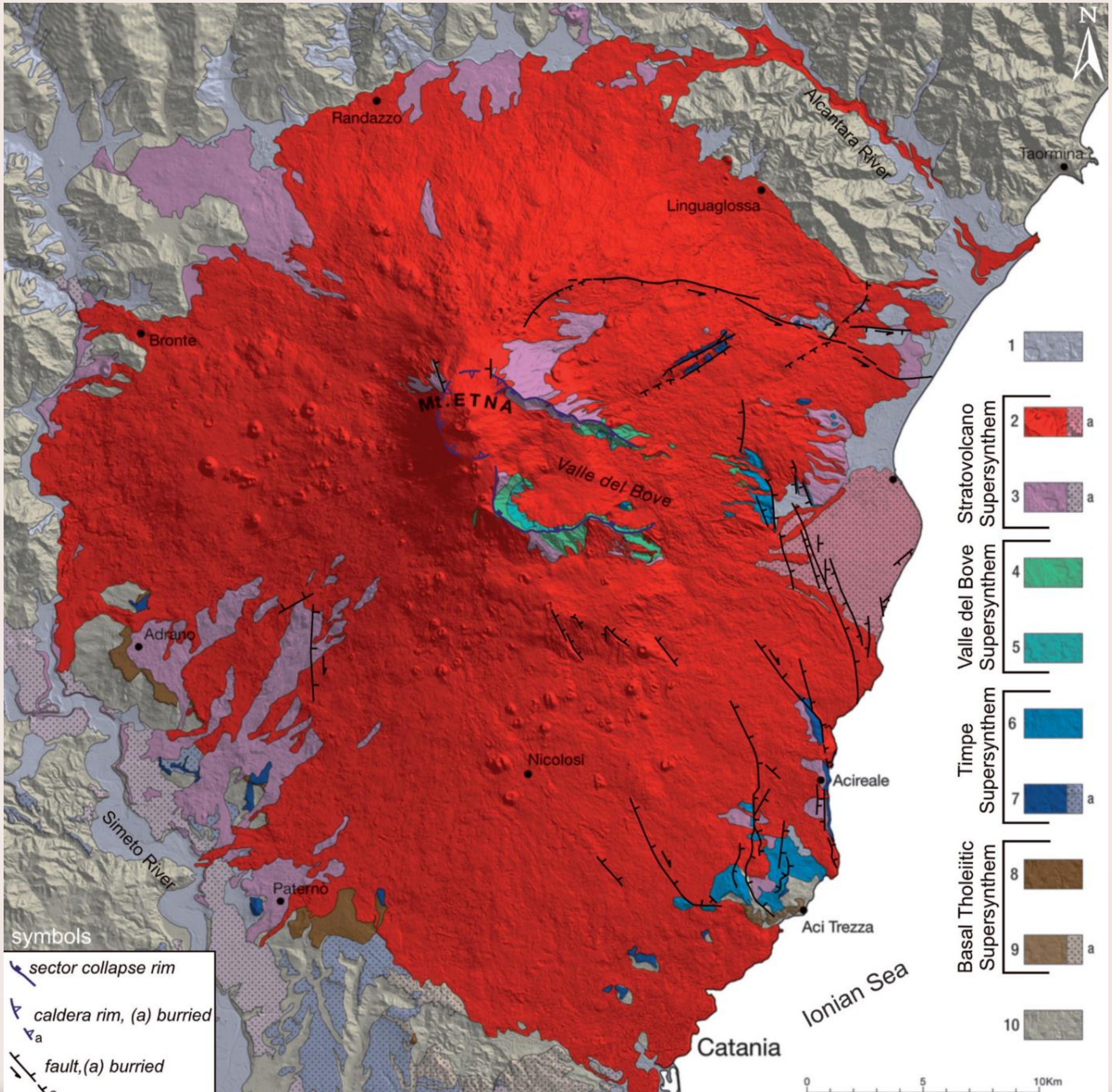


Appendix

Key Points

- RESEARCH GOALS
- STUDY AREA → "1536" to "1669" HISTORIC VOLCANIC PRODUCTS
- PROBLEM DEFINITION
- DATA PREPROCESSING
- MAIN DATA PROCESSING
- SPECTRAL MIXTURE MODELS: LINEAR & BILINEAR
- **METHODS** → SIGNAL TRANSFORMATIONS → LEAST SQUARES SPECTRAL UNMIXING → IMAGE RECONSTRUCTION
- RESULTS → ABUNDANCE MAPS/METHOD
- COMPARATIVE ANALYSIS
- CONCLUSIONS
- DISCUSSION
- FUTURE WORK

Etna’s Supersynthems



(from *Branca et al, 2011*).

Signal Transformations(2)

- **LLSU** on the:

1. *Untransformed Channel Domain* of the initial image

2. *Reduced Channel Domain*
   - Exploit the HSI spectral redundancy
   - Perform Dimensionality reduction via Feature Selection (FS)*
   - Feature Selection uses the sparsity induced Fast Bi-ICE algorithm*, implemented in NOA
   - Output: vectors **most significant Bands** & **Ranks**
   - Optimum Band number: **22** Bands
   - 1^{st} FS band (41) = 973 nm (0.1918), 22^{nd} FS band (31) = 783 nm (0.1648)
   - Channel Domain reduced to corresponding 22 bands.
   - Majority of information in R-VNIR !

* Special Thanks to Dr. Kostas Themelis and Dr. Irida Xenaki for providing their scripts.

Signal Transformations(3)

- **LLSU** on the:

1. *Untransformed Channel Domain* of the initial image

2. *Reduced Channel Domain*

   - Exploit the HSI spectral redundancy
   - Perform Dimensionality reduction via Feature Selection (FS)*
   - Feature Selection uses the sparsity induced Fast Bi-ICE algorithm*, implemented in NOA
   - Output: vectors **most significant Bands** & **Ranks**
   - Optimum Band number: **22** Bands
   - 1^{st} FS band (41) = 973 nm (0.1918), 22^{nd} FS band (31) = 783 nm (0.1648)
   - Channel Domain reduced to corresponding 22 bands.
   - Majority of information in R-VNIR !

TIME EFFICIENCY

* Special Thanks to Dr. Kostas Themelis and Dr. Irida Xenaki for providing their scripts.

Signal Transformations(4)

- LLSU on the:

3. *FFT transformed entire Image*
   - DFT from Channel Domain → Frequency Domain
   - Reduce complexity to $\boldsymbol{O}(\boldsymbol{nlogn})$
   - Perform FFT on the endmember matrix too
   - LLSU on the abs amplitude values of the Image - Endmember vectors
   - *LLSU Frequency Domain* ∝ *LLSU Untransformed*
   - Does it give quantitatively the same results?

4. *FFT on the Visible-VNIR* part of the spectrum
   - # of endmembers is related to the dimension of the subspace occupied by measurements
   - Subspace identification ⇒ selection of bands with low SNR → for the Hyperion dataset VIS-VNIR

Signal Transformations(5)

- LLSU on the:

3. *FFT transformed entire Image*
   - DFT from Channel Domain → Frequency Domain
   - Reduce complexity to $\boldsymbol{O(nlogn)}$
   - Perform FFT on the endmember matrix too
   - LLSU on the abs amplitude values of the Image - Endmember vectors
   - *LLSU Frequency Domain* $\propto$ *LLSU Untransformed*
   - Does it give quantitatively the same results?
4. *FFT on the Visible-VNIR* part of the spectrum
   - # of endmembers is related to the dimension of the subspace occupied by measurements
   - Subspace identification ⇒ selection of bands with low SNR → for the Hyperion dataset VIS-VNIR (a priori known)

Signal Transformations(6)

- LLSU on the:
  - 76 first bands
  - Essentially a High-pass filter that enhances spectral details
  - FFT on reduced bands computationally low-cost
  - Reducing Gaussian Image noise
  - Stripping effect remains (High Frequency noise)

👌 TIME EFFICIENT
NOISE REDUCTION

5. *Reduced Frequency Domain via FFT*
   - Plot the FFT frequencies of the Endmember Matrix
   - **20 First** → signal's major energy content
   - No significant variation over the 20^{th} → **Dimensionality Reduction**
   - Comparable results with the FFT on entire image

Signal Transformations(7)

- LLSU on the:
  - 76 first bands
  - Essentially a High-pass filter that enhances spectral details
  - FFT on reduced bands computationally low-cost
  - Reducing Gaussian Image noise
  - Stripping effect remains (High Frequency noise)

5. *Reduced Frequency Domain via FFT*

  - Plot the FFT frequencies of the Endmember Matrix
  - **20 First** → signal's major energy content
  - No significant variation over the 20th → **Dimensionality Reduction**
  - Comparable results with the FFT on entire image

Signal Transformations(8)

- LLSU on the:

6. *IFFT transformed Image*

    - From the diminished 20 freq. domain → *140 Channel domain*
    - Return to the initial image through zero padding the FFT transformed image
    - Keep only the energy dominant bands

👌 TIME EFFICIENT
DIMENSIONALITY REDUCTION

- **Bilinear LSU** on the:

7. *Reduced Channel Domain*
    - Exploit the FS reduced domain
    - Compute the Enhanced Endmember Matrix
    - Dot product of each endmember with the rest as Xi.*Xj = **Endmember Correlation**
    - Non-Linearity is induced by the enhanced Endmember matrix
    - Flexible model for multi-correlated LFs

Signal Transformations(9)

- LLSU on the:

6. *IFFT transformed Image*
   - From the diminished 20 freq. domain → *140 Channel domain*
   - Return to the initial image through zero padding the FFT transformed image
   - Keep only the energy dominant bands

- **Bilinear LSU** on the:

7. *Reduced Channel Domain*
   - Exploit the FS reduced domain
   - Compute the Enhanced Endmember Matrix
   - Dot product of each endmember with the rest as Xi.*Xj = **Endmember Correlation**

Signal Transformations(10)

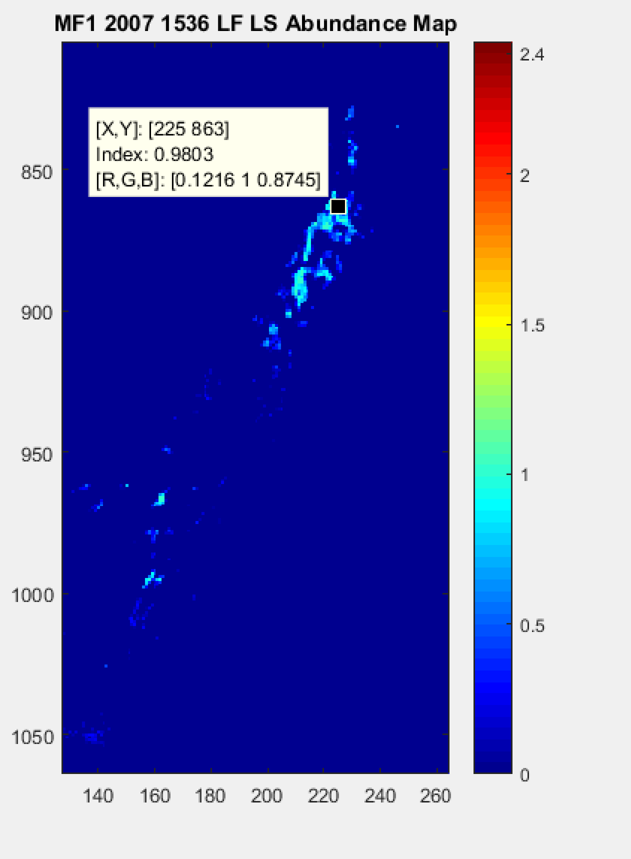
- ❖ Non-Linearity is induced by the enhanced Endmember matrix
- ❖ Flexible model for multi-correlated LFs

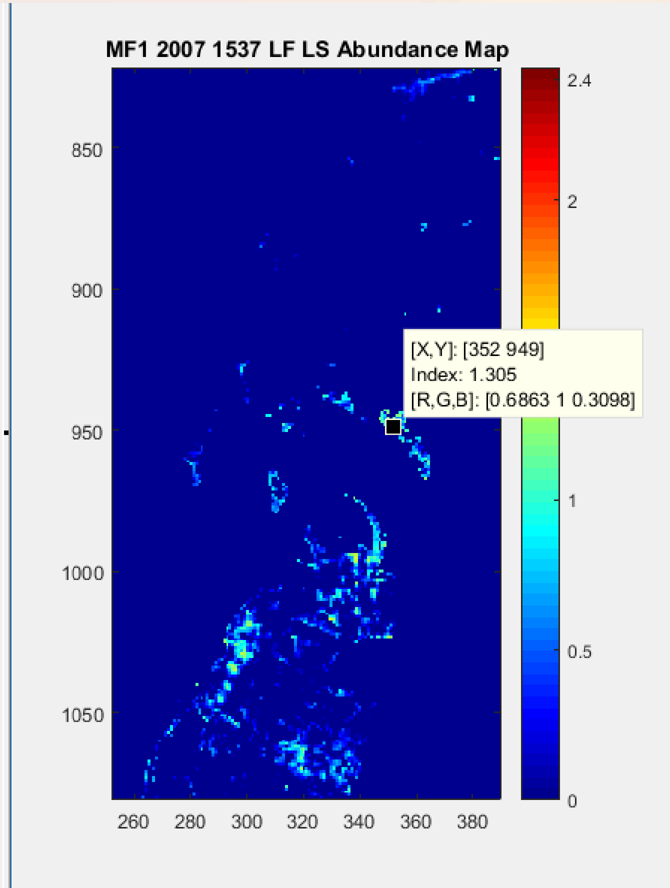
✧ **Bilinear LSU** on the:

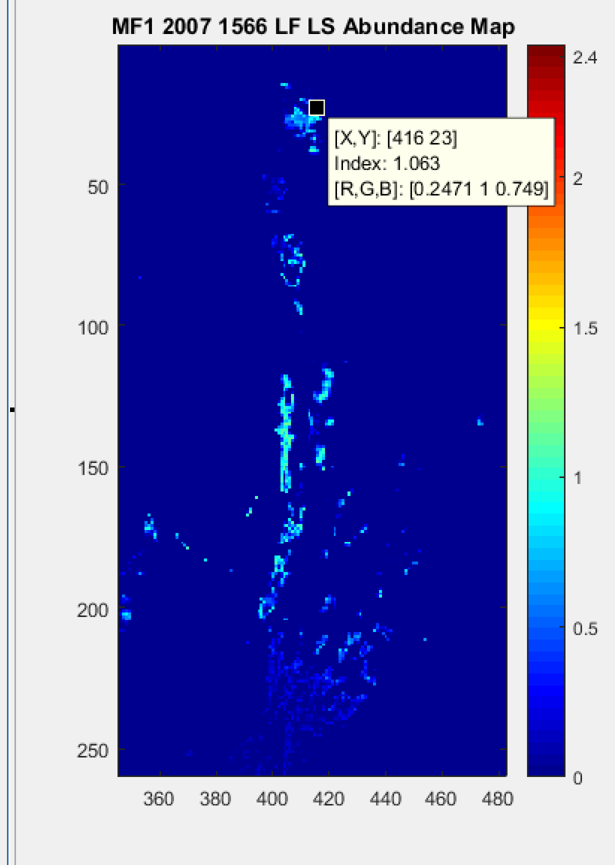
8. *Augmented Spectral Signatures Domain*

- ❖ Endmember matrix and Image spectral enhancement
- ❖ Again compute the dot product of each of the bands = **Band Correlation**
- ❖ Non-Linearity is induced by the augmented Endmember matrix

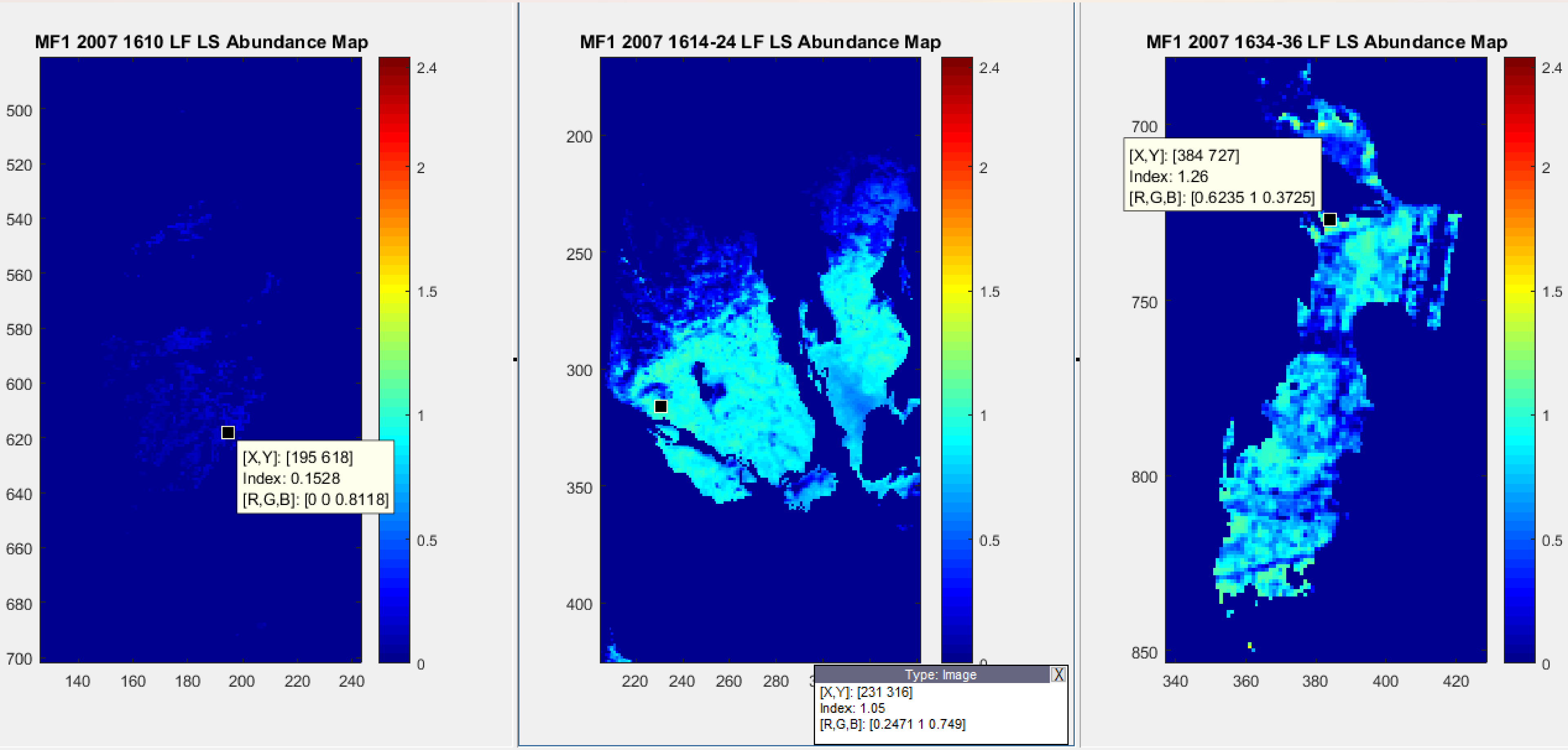
LLSU Untransformed Abundance Maps



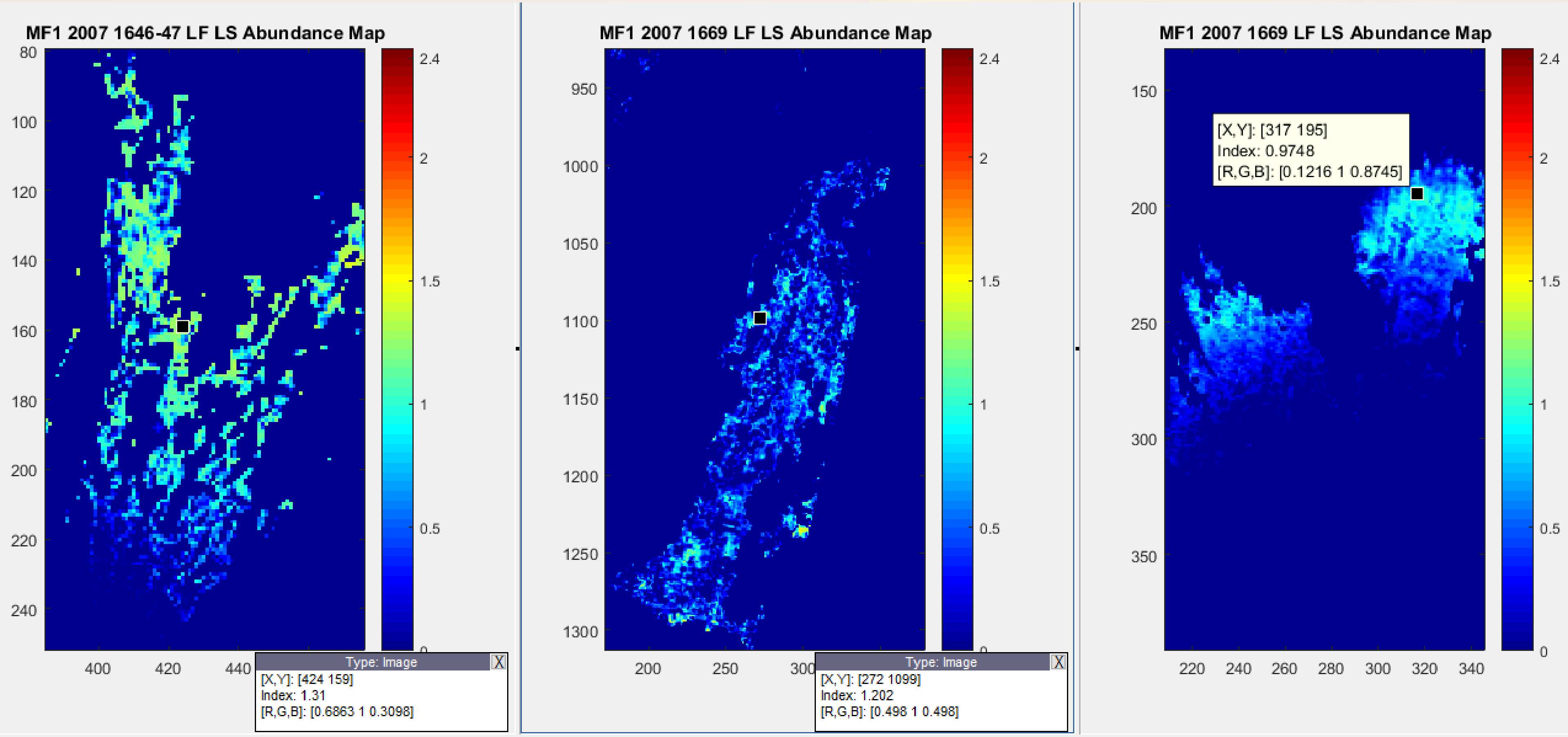




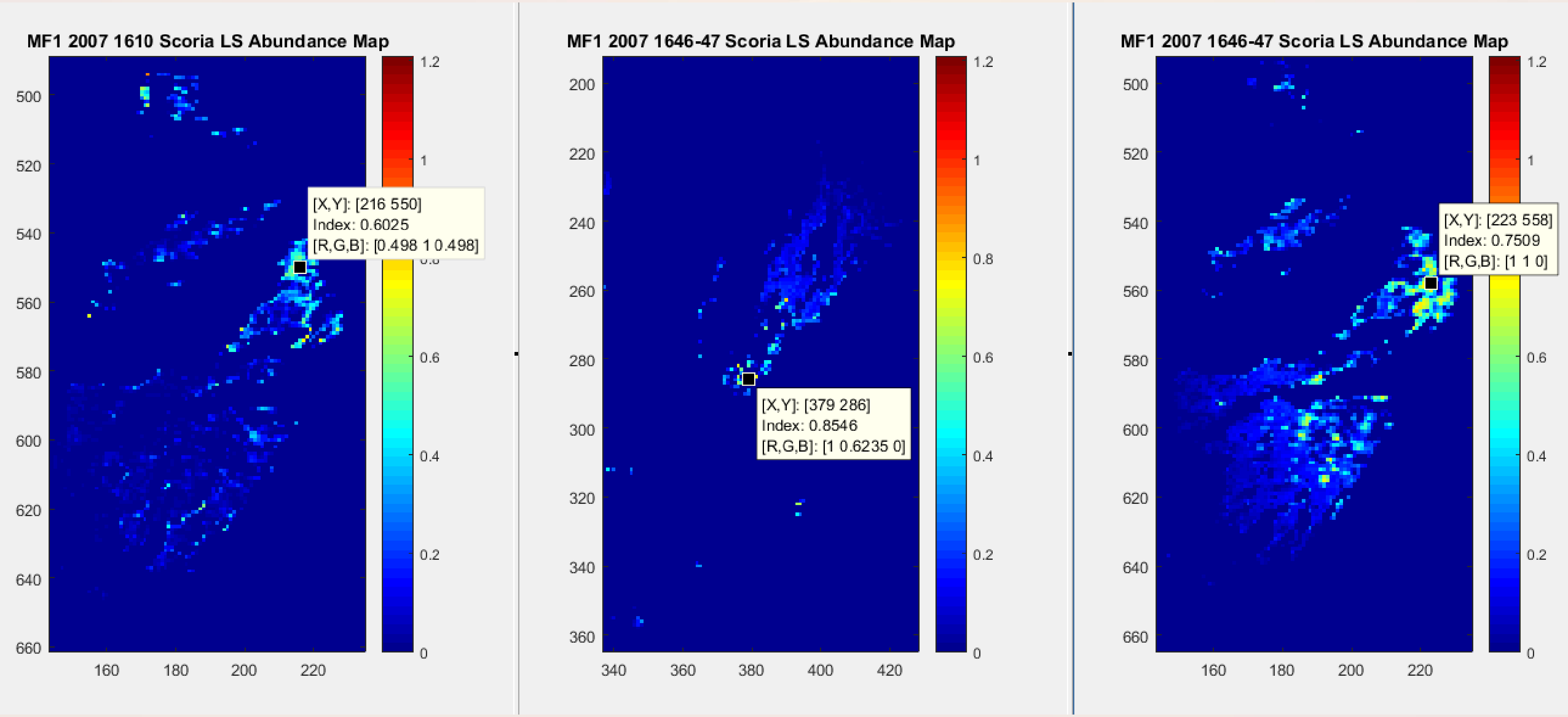
LLSU Untransformed Abundance Maps



LLSU Untransformed Abundance Maps



LLSU Untransformed Abundance Maps



LLSU Untransformed Abundance Maps