

Proceeding

# Comparison of Fracture Resistance of the Normal and High Strength Concrete Evaluated by Brazilian Disc Test<sup>†</sup>

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**Abstract:** Nowadays, high performance concrete is used more frequently because of the many advantages compared to traditional concrete. The higher mechanical properties (e.g. compressive strength, flexural strength, and Young's modulus) allow for larger spans and slender cross-sections. Despite the use of advanced material, standards for structural design do not fully use materials' potential. This can be minimized by using fracture mechanical properties in structural analysis. The fracture mechanical properties help to perform advanced structural analysis, especially when some of the structural elements have a crack. The load presence on the structure can be divided into tensile - mode I, shear - mode II, and combination of tension and shear - mixed mode I/II load. Therefore, it is necessary to perform test, which covers mixed mode loading conditions. One of the tests usually used for the evaluation of fracture resistance of concrete is Brazilian disc test. This contribution compares fracture resistance of two types of structural concrete (normal and high strength) under the mixed mode I/II. The generalized maximum tangential stress (GMTS) criterion was used for the evaluation of the fracture resistance.

**Keywords:** Brazilian disc test; fracture mechanics; GMTS; high strength concrete; mixed mode.

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## 1. Introduction

The design of concrete structural elements used in civil engineering is optimized to reduce material consumption and to improve structural behavior. However, the traditional concrete does not satisfy increasing demands on the structural and material performance. Therefore, new materials are developed with focus on mechanical performance such as compressive strength, flexural strength, and Young's modulus. The use of concrete with high compressive strength (HSC) [1] in structural design can provide slenderer cross-section, which leads into reducing total material consumption with retaining similar mechanical performance of the structure.

The advanced structural analysis uses fracture mechanical properties as an input parameter to predict total structural service life time and fracture resistance. The structural elements after certain time can show minor surface damage or shrinkage can create micro-cracks. These defects are zones of weakness, where the crack can initiate. The load presence on the structural element can be characterized by tensile mode I and shear mode II. In reality, some cracks are loaded by combination of tension and shear - mixed mode I/II load. Hence, it is necessary to test material under the mixed

mode loading conditions [2-4]. One of the tests usually used for evaluation of fracture resistance of concrete is Brazilian disc [5,6].

The aim of this contribution is to evaluate and compare the fracture resistance of two concrete types used for precast structural elements under the mixed mode load. The first one is traditional concrete with grade C 50/60 and the second type is HSC concrete with compressive strength around 100 MPa. The assessment of the fracture resistance of both studied materials, is evaluated from the experimental results by employing fracture resistance curve calculated from the generalized maximum tangential stress (GMTS) criterion, which is based on two-parameter linear elastic fracture mechanics. The results are compared and discussed.

## 2. Theoretical Background

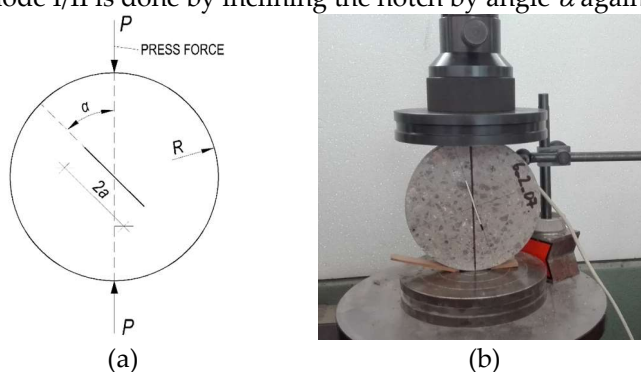
This contribution is based on a linear elastic fracture mechanics. The linear elastic fracture mechanics concept uses the stress field in the close vicinity of the crack tip described by Williams expansion [7]. This expansion is an infinite power series originally derived for a homogenous elastic isotropic cracked body. The stress field for mode I and mode II can be described by a following equation:

$$\sigma_{i,j} = \frac{K_I}{\sqrt{2\pi r}} f_{i,j}^I(\theta) + \frac{K_{II}}{\sqrt{2\pi r}} f_{i,j}^{II}(\theta) + T + O_{i,j}(r, \theta), \quad (1)$$

where  $\sigma_{ij}$  represents the stress tensor components,  $K_I$ ,  $K_{II}$  are the stress intensity factors (SIF) for mode I and mode II, respectively,  $f_{i,j}^I(\theta)$ ,  $f_{i,j}^{II}(\theta)$ , are known shape functions for mode I and mode II usually written as  $Y_I$  and  $Y_{II}$ ,  $T$  (or  $T$ -stress) represents the second term independent on  $r$ ,  $O_{ij}$  represents higher order terms, and  $r$ ,  $\theta$  are the polar coordinates (with origin at the crack tip; crack faces lie along the  $x$ -axis).

### 2.1. Brazilian Disc Test

Brazilian disc test with a central notch (BDC) is specimen with circular cross section, made from the cylinder with a notch in the middle of specimen (see Figure 1a) [8-10]. The test performed on the BDC specimen is carried out under relatively simple experimental conditions (see Figure 1b), using only the testing press with sufficient load capacity. The evaluation of the fracture parameters for modes I, II and mixed mode I/II is done by inclining the notch by angle  $\alpha$  against the load position.



**Figure 1.** Brazilian disc with central notch – principle of testing (a) and actual test setup (b).

The SIF for a finite specimen in shape of Brazilian disc and the polar angle  $\theta = 0^\circ$  can be calculated by following equations [11,12]:

$$K_I = \frac{P\sqrt{a}}{RB\sqrt{\pi}} \frac{1}{\sqrt{1-\frac{a}{R}}} Y_I(a/R, \alpha), \quad (2)$$

$$K_{II} = \frac{P\sqrt{a}}{RB\sqrt{\pi}} \frac{1}{\sqrt{1-\frac{a}{R}}} Y_{II}(a/R, \alpha), \quad (3)$$

where  $P$  is compressive load,  $a$  is a crack length,  $R$  is radius of the disc ( $D/2$ ),  $B$  is disc thickness,  $\alpha$  is inclination angle and  $Y_I(a/R, \alpha)$ ,  $Y_{II}(a/R, \alpha)$  are dimensionless shape functions for mode I and mode II, respectively. Geometry functions  $Y_I$  and  $Y_{II}$  used in Eqs. (2),(3) can be found in [8,9].

To calculate  $T$ -stress, a direct extrapolation method [13] is used, for polar angle  $\theta = 0^\circ$  the following equation is used:

$$T = \lim_{r \rightarrow 0} (\sigma_{xx} - \sigma_{yy}), \quad (4)$$

where  $\sigma_{xx}$  and  $\sigma_{yy}$  are the stress components in front of the crack tip in direction for  $\theta = 0^\circ$ .

## 2.2. GMTS Criterion

There are several criteria for predicting the onset of mixed mode fracture of brittle materials. The criteria which can be used on the BDC specimen the maximum tangential stress (MTS) criterion [14] and the minimum strain energy density (SED) criterion [15]. However, these criteria are not able to accurately predict onset of mix mode fracture. These disadvantages lead to the development of the generalized maximum tangential stress (GMTS) criterion [16]. The GMTS criterion has been recently used to the fracture resistance of the BDC specimen by Aliha et al.[17] for PMMA and Hou et. al [18] for mortar and concrete. All studies displayed an accurate prediction of fracture resistance.

According to the first hypothesis of the GMTS criterion, the onset of fracture is the angle of maximum tangential stress  $\theta$  and can be determined from:

$$\frac{\partial \sigma_{\theta\theta}}{\partial \theta} \Big|_{\theta=\theta_0} = 0 \quad \text{and} \quad \frac{\partial^2 \sigma_{\theta\theta}}{\partial^2 \theta} < 0, \quad (5)$$

Assumption mentioned in Eq. 5 leads into:

$$[K_I \sin \theta_0 + K_{II} (3 \cos \theta_0 - 1)] - \frac{16T}{3} \sqrt{2\pi r_c} \cos \theta_0 \sin \frac{\theta_0}{2} = 0, \quad (6)$$

Crack initiation angle  $\theta$  is then used for evaluation of beginning of mixed mode I/II on BDC specimen.

### 2.2.1. Application of the GMTS on Brazilian Disc Specimen

Pure mode I fracture initiation appears when  $K_I = K_{Ic}$ ,  $K_{II} = 0$ . and  $\theta = 0^\circ$ , this assumption leads into Eq. (7):

$$K_{Ic} = \cos \frac{\theta_0}{2} \left[ K_I \cos^2 \frac{\theta_0}{2} - \frac{3}{2} K_{II} \sin \theta_0 \right] + \sqrt{2\pi r_c} T \sin^2 \theta_0, \quad (7)$$

where  $K_{Ic}$  is materials' fracture toughness. Fracture resistance for both modes is expressed by ratio  $K_I/K_{Ic}$  and  $K_{II}/K_{Ic}$ . This ratio is obtained from Eq. (7) by dividing the whole expression by  $K_I$ ,  $K_{II}$ , respectively.

Eq. (6) shows that the angle  $\theta$  for any combination of modes I and II depends on  $K_I$ ,  $K_{II}$ ,  $T$ , and  $r_c$ . Critical distance  $r_c$  can be evaluated from Eqs. (8) and (9) for plane stress and plane strain respectively [11].

$$r_c = \frac{1}{2\pi} \left( \frac{K_{Ic}}{\sigma_t} \right)^2, \quad (8)$$

$$r_c = \frac{1}{6\pi} \left( \frac{K_{Ic}}{\sigma_t} \right)^2. \quad (9)$$

## 3. Materials

### 3.1. Normal Strength Concrete

The C 50/60 concrete type was chosen for the study because it is typically used for the pre-stressed precast elements which are produced nowadays. The studied concrete contains 450 kg of CEM I 42.5 R, the water to cement ratio  $c/w$  is 0.40. Fine aggregate was natural sand 0/4 mm and crushed aggregates 4/8 mm and 8/16 mm from high quality granite was used as well as drinking water. The concrete was mixed in a volume of 1 m<sup>3</sup> and poured immediately into molds. A polycarboxylates-based superplasticizer was used to reach good workability [10].

### 3.2. High Strength Concrete

High strength concrete was designed with intent to produce subtle elements. The maximum size of aggregate was chosen 8 mm. The aggregates were composed from natural sand 0/4 mm and crushed high quality granite 4/8 mm. Portland cement CEM I 42.5 R was used with three mineral admixtures. The first, was metakaolin, with strong pozzolanic properties. The second and third admixture were chosen to reach synergy in ternary binders [19] based on experiments, see for example [20]. Generally, binder consist 81 % of CEM I 42.5 R, 9.5 % of metakaolin, 7.5 % of GBFS and 2.5 % of limestone. Water / binder ratio was 0.22. A polycarboxylate based superplasticizer was selected based on its compatibility with cement. The concrete was mixed in volume 0.7 m<sup>3</sup> and poured into molds.

### 4. Experimental Measurement

The machine for tests has a maximum loading capacity 200 kN, the speed of the induced displacement of the upper support was equal to 0.025 mm/s. BDC specimens with relative notch length  $a/R = 0.4$  were inclined against loading positions under the selected angles. **Table 1** and **Table 2** give overview of the mean values of the specimen dimensions for C 50/60 and HSC, respectively.

**Table 1.** Dimensions of BDC specimens made from C 50/60

Specimen nmr.	Inclination angle $\alpha$ [°]	Diameter $D$ [mm]	Thickness $B$ [mm]	Notch length $2a$ [mm]	$a/R$ [-]
6_03	0	149.200	31.380	60.210	0.404
6_09	0	149.182	29.927	60.170	0.403
6_01	0	149.155	31.440	60.920	0.408
6_01	5	149.162	31.440	60.920	0.408
6_05	10	149.143	30.580	60.140	0.403
6_04	10	149.162	30.973	60.040	0.403
6_04	15	149.208	30.970	60.040	0.402
6_06	15	149.214	32.390	60.150	0.403
6_02	25.2	149.180	30.770	60.110	0.403
6_07	25.2	149.205	31.173	59.940	0.402

**Table 2.** Dimensions of BDC specimens made from HSC.

Specimen nmr.	Inclination angle $\alpha$ [°]	Diameter $D$ [mm]	Thickness $B$ [mm]	Notch length $2a$ [mm]	$a/R$ [-]
6_2_02	0	149.09	29.43	59.70	0.400
6_2_01	0	149.15	29.99	59.44	0.399
6_2_05	5	149.23	28.35	59.91	0.401
6_2_10	10	149.32	28.48	59.27	0.396
6_2_11	10	149.01	27.57	60.13	0.403
6_2_08	15	149.18	28.09	60.06	0.403
6_2_09	15	149.28	28.70	59.96	0.402
6_2_06	20	149.21	28.33	60.01	0.402
6_2_07	20	149.12	28.45	60.03	0.403
6_2_03	25.2	149.18	28.45	59.81	0.400
6_2_04	25.2	149.23	28.96	59.93	0.402

### 5. Results and Discussion

Fracture mechanical properties (SIFs) of investigated materials were evaluated by using Eq. (2) and (3). From Figure 2(a) it can be seen, that the fracture of HSC material is done under higher fracture force than for the C 50/60 material. The Figure 2(b) show fracture resistance of studied materials.

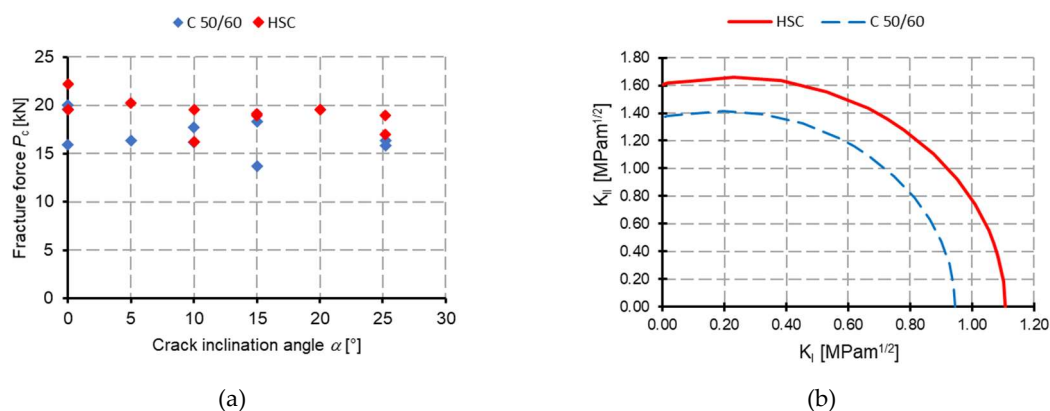


Figure 2. Comparison of measured forces (a) and values of SIFs (b) for C 50/60 and HSC

The comparison of experimental results is done by fracture resistance curves. Fracture resistance curves were calculated using Eq. (7) for each material and its critical distance  $r_c$ . From Figure 3 it can be noted, that the MTS criterion is very conservative for both materials. The GMTS criterion predict fracture resistance with great agreement especially for plane strain boundary conditions.

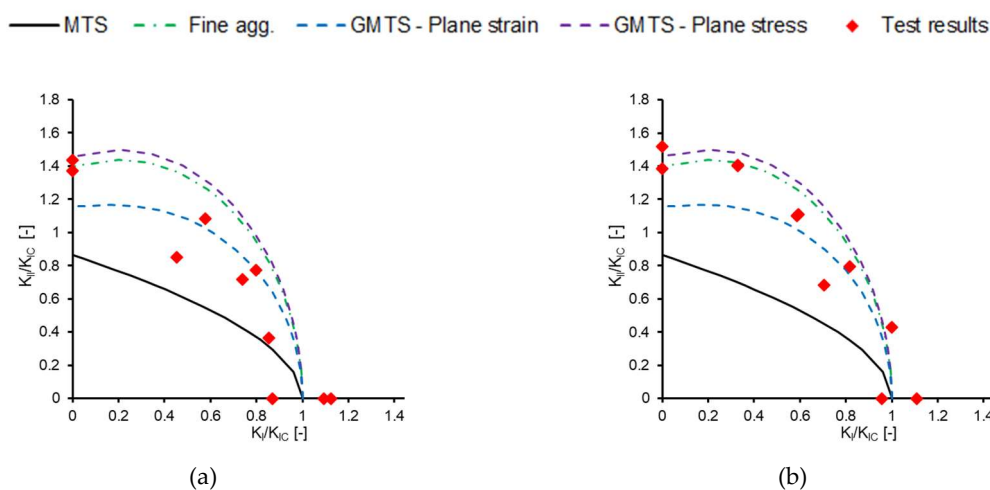


Figure 3. Mixed mode fracture toughness diagram for C 50/60 (a) and HSC (b) materials, using various critical distances  $r_c$ .

### 6. Conclusions

In this paper a fracture resistance of two concrete types C 50/60 and HSC was experimentally investigated by using Brazilian disc test. The mixed mode fracture resistance is evaluated by employing GMTS criterion. The following conclusions were found:

- The fracture toughness measured on the HSC material is higher in all investigated cases than for the traditional C 50/60 material.
- The experimental results done on the HSC material showed higher fracture resistance in mixed mode I/II than the traditional C 50/60 material.
- The fracture resistance of the C 50/60 material is characterized best by  $r_c$  for plain strain, yet for HSC, it is better to use value of  $r_c$  for fine aggregate.

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**Author Contributions:** Petr Miarka and Stanislav Seitl performed the experiments and analyzed the experimental data from BDC specimen; Vlastimil Bílek provides material's composition.

**Conflicts of Interest:** The authors declare no conflict of interest.

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