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# STRESS MEASUREMENT BY SPECTRUM ANALYSES FOR ROUND BAR SUBJECTED TO TIME-VARYING LOAD †

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† Presented at the conference title, place, and date.

Published: date (leave it empty)

**Abstract:** In this research, a feasibility study to measure a magnitude and a cycle of a time-varying stress of a specimen by a natural frequency was carried out. An experiment was conducted. We used a specimen of a round bar with 8 mm diameter and 290 mm span length which was fixed at both ends. A sinusoidal axial stress was applied to the bar. A deflection of the bar in a free vibration was measured by a laser beam displacement device. To collect much information on the deflection, a device which hit the bar periodically was made up. A fast Fourier transform method, a short-time Fourier transform method and a wavelet analysis were applied to the deflection. The methods give us relations among time, a frequency and a magnitude of a signal with complicated representations. Applying the analyses to the experimental data, we tried to evaluate a magnitude and a cycle of a time-varying load.

**Keywords:** Spectrum analyses; Time-varying stress measurement; Periodic hitting device

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## 1. Introduction

Members in a structure or a machine are generally subjected to a dynamic and periodic load. Being fabricated into a machine, they are expected to cooperate to suspend heavy burden. If we could evaluate with ease a magnitude and a cycle of time-varying load, it would bring us a fair evaluation of their integrity.

Strain gauges have been used to measure stress of a structure or a machine. The method is simple and reliable. New kinds of strain gauges like a semiconductor or a piezoelectric element have been developed. Fiber Bragg Grating sensor was employed for health monitoring for a structure or a composite material [1]. In order to apply the method to a structure or a machine, we need many gauges. The measurement by strain gauges takes a lot of labor and accompanies troublesome works, and is unsuitable to measure working stress over a long period.

Another kind of methods to estimate strain distribution in an area have been developed. They are a digital image correlation method [2] or an acoustic-elastic method [3]. These methods need to compare a picture or a property before and after the deformation of a structure or a specimen. The procedure is still complicated and is not easily employed on site.

We have applied a fast Fourier transform (FFT) method to a sound brought about by an impact to a round bar under a stationary axial load. We investigated a relation between a natural frequency and an axial stress of the bar with fix-supported ends or with simply-supported ends. Theoretical relations and experimental results agreed. The agreement enables us to measure an axial stress of a bar under these ends by a natural frequency [4].

In this research, we tried to evaluate a magnitude and a cycle of a non-stationary load submitted to a bar specimen by a natural frequency. An experiment was conducted. We applied a sinusoidal load to the bar and measured a deflection of the bar in a free vibration. Time-frequency analyses were applied to the deflection variation.

## 2. Theories and experiments

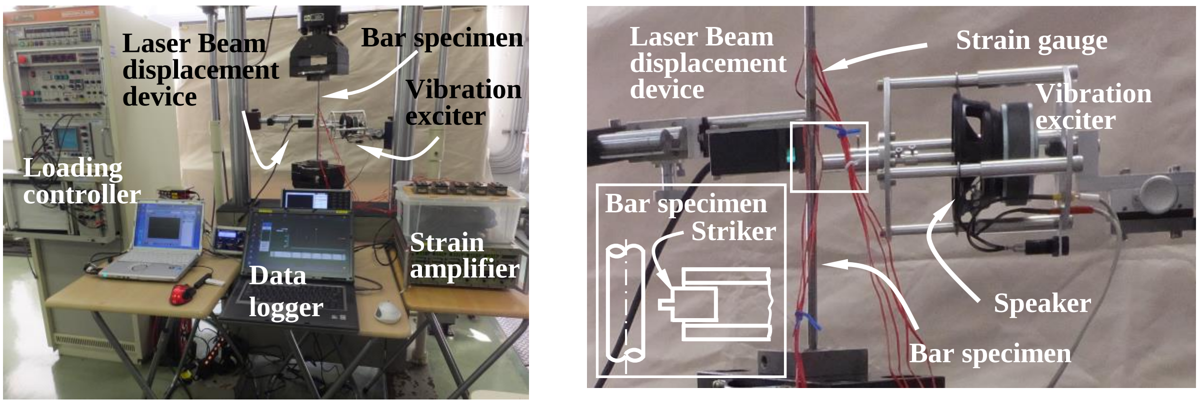
### *2.1. Experiment system*

A round bar specimen with 8 mm diameter, 290 mm span length and fix-supported ends was used in an experiment. We measured a deflection of the bar in a free vibration to evaluate a natural frequency. But the deflection dies away in an instant. To collect much information on the deflection, we made an exciter to give impacts to the bar periodically. We measured a deflection of the bar by a laser beam displacement device and an axial stress by strain gauges attached to the bar. The loading and measurement systems were shown in Figure 1.

We evaluate an axial stress of a bar by a natural frequency. In the evaluation, we need material properties such as an elastic modulus and a density. The material is a rolled carbon steel for cold-finished steel bars. We obtained its density as 7.84 g/cm$^3$. We measured its elastic modulus employing the relation given by the formula (1) between an elastic modulus and a natural frequency of the specimen suspended in the air. For the measurement, we used a specimen with 8 mm diameter and 100 mm length cut out from the same lot with the bar specimen. The elastic modulus was 210 GPa.

$$f_{p\text{-th}} = \frac{1}{2\pi L^2}\sqrt{\frac{EI}{\rho A}}(\lambda_{p-\text{th}} L)^2 \quad , \tag{1}$$

here $f_{p\text{-th}}$ is a natural frequency, $L$ is a specimen length, $A$ is an area, $\rho$ is a density, $E$ is an elastic modulus, $I$ is a second moment of area and $(\lambda_{p\text{-th}} L)$ is an eigen value.




(**a**) (**b**)

**Figure 1.** Experiment system: (**a**) Whole arrangement; (**b**) Detailed configuration.

### *2.2.Time-frequency analyses*

We employed an FFT method, a short-time Fourier transform (STFT) method and a wavelet analysis. These methods and the analysis are now employed for various fields and explained in detail elsewhere[5,6]. We leave out their explanations. An FFT gives us a clear information on a frequency, but does not provide any information on time. For a non-stationary load, a frequency-domain representation changes over time. A STFT method and a wavelet analysis are expected to give us a relation among time, a natural frequency and a magnitude of a signal.

### *2.3.Simulation model*

An angular natural frequency of the *p*-th natural mode of a bar in a free vibration, $\omega_{p\text{-th}}$ is a function of an applied load, $\sigma_{\text{Load}}$. The relation is expressed as the function (2). If you brought a mind a guitar tuning, you would understand this situation. If you pulled a string and gave it larger tension, the string would sound with higher tone.

$$\omega_{p\text{-th}} = \omega_{p\text{-th}}\,(\sigma_{\text{Load}}) \quad . \tag{2}$$

We applied a sinusoidal load to the bar described by the expression (3).

$$\sigma_{\text{Load}}(t,\sigma_{\text{Mean}},\sigma_{\text{Amp}},\,f_{\text{Load}}) = \sigma_{\text{Mean}} + \sigma_{\text{Amp}} \cdot \sin(2\pi\, f_{\text{Load}}\, t) \;, \tag{3}$$

here $\sigma_{\text{Mean}}$ is an average stress, $\sigma_{\text{Amp}}$ is an amplitude of the stress and $f_{\text{Load}}$ is a cycle.

In an analysis, we usually postulate the deflection of the *p*-th mode natural vibration of a bar, $w_{p\text{-th}}(x,t)$ as the next expression

$$w_{p\text{-th}}(x,t) = X_{p\text{-th}}(x) \cdot \exp\,(j\,\omega_{p\text{-th}}\,t) \quad , \tag{4}$$

here $X_{p\text{-th}}$ is the *p*-th natural mode shape and *j* is an imaginary unit.

The deflection of a free vibration decays. We usually refer the effect by the next expression.

$$w_{p\text{-th}}^{\text{damp}}(x,t) = w_{p\text{-th}}(x,t) \cdot \exp(-\kappa \cdot t) \quad , \tag{5}$$

here $\kappa$ is a damping coefficient.

The deflection of a bar in a free vibration is composed of many natural mode deflections. We express it as the expression (6).

$$w = w(t,\sigma_{\text{Mean}},\sigma_{\text{Amp}},\,f_{\text{Load}}) = \sum_{x-\text{th}=1}^{\infty} w_{x\text{-th}}^{\text{damp}}(x,t) \cdot \chi_{x-\text{th}} \;, \tag{6}$$

here $\chi_{p\text{-th}}$ is a ratio of a contribution of each mode to a whole deflection. We measure this deflection.

Other variables such as a bar length, a sectional area are also included in the expression (6), but they do not vary in our experiment. They are excluded in the expression as variables. In this research, we evaluate, $\sigma_{\text{Mean}}$, $\sigma_{\text{Amp}}$ and $f_{\text{Load}}$ by a signal analysis taking up the 1st mode natural vibration. We employed FlexPro [7] for a data analysis of signals.

## 3. Results

We measured a deflection of the bar at the middle position and stresses at the location 20 mm up from the middle by strain gauges. We compare the stress with one obtained by spectrum analyses.

Hereafter four kinds of figures are presented. They are one for time-stress variation, the vertical axis of which is a stress of the bar measured by a strain gauge, one for time-deflection, the vertical axis of which is a deflection of the bar measured by a laser beam displacement device and one for a frequency-spectrum by an FFT method, the horizontal axis of which is a frequency and the vertical axis is a normalized amplitude spectrum. One for time-frequency variation by a STFT method uses the vertical axis as a frequency and a shade of color in the figure shows an intensity of a normalized amplitude spectrum, which is scaled by the indicator at a right-hand side of the figure.

### *3.1.Experimental relation between natural frequency and stationary axial stress*

By an experiment, we obtained the relation between a natural frequency of the 1st mode natural vibration and a stationary axial stress. The result is shown in Figure 2 which corresponds to the expression (2). Employing the relation, we can evaluate an axial stress by a natural frequency of the bar.

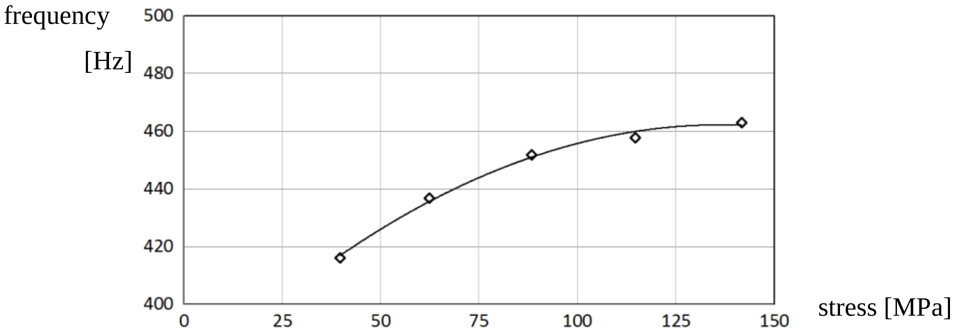


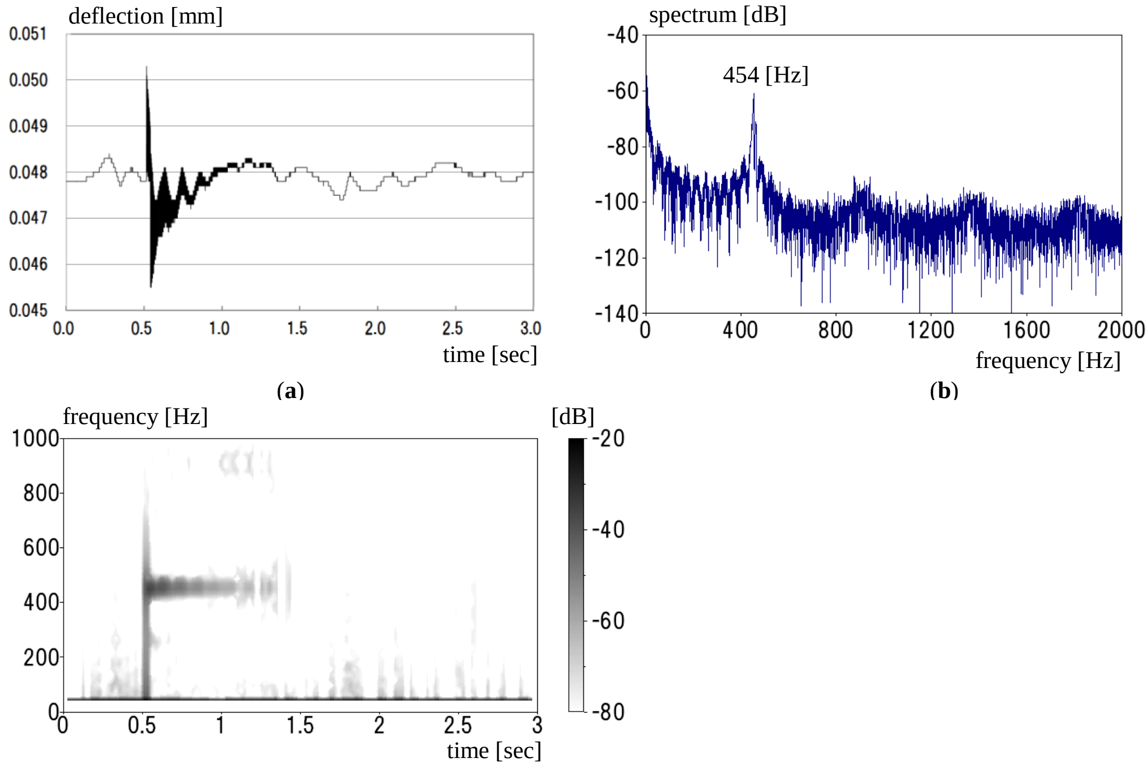

**Figure 2.** Relation between natural frequency and axial stress.

### *3.2. Relation between time, deflection and non-stationary axial stress by experiment and analysis*

After we gave a stationary load, 101 MPa to the bar, we hit the bar once to bring about a free vibration. Figure 3(a) shows the deflection variation of the bar. We see that the bar was impacted at around 0.5 sec and that the deflection decayed in a second.

Figure 3(b) shows an FFT result. We see the distinguished frequency, 454 Hz in the figure. When we evaluate a stress by this natural frequency through the relation in Figure 2, the stress was found to be 94.9 MPa. The stress value corresponds to one mentioned above.

Figure 3(c) shows a result by a STFT method. We used a Blackman window. In the figure, the frequency distributed as the blurred strip and faded away. Uncertainty principal between time and a natural frequency in a signal analysis is given by the equation, $\triangle t \cdot \triangle f \geqq 1/4\pi$. We are able to obtain definite frequency by an FFT analysis, but are not able to designate the time when the frequency occurred. In a time-frequency analysis, we seek information both on time and a frequency. As an outcome of compromise because of the principal, we are forced to obtain indistinct picture like one in Figure 3(c). But we see that the frequency around 450 Hz occurred at around 0.5 sec, continued in a second and that the vibration ended.




(c)

**Figure 3.** Experiment and analyzed data for stationary load by one impact: (**a**) Time-deflection variation; (**b**) Result by FFT method; (**c**) Result by a short-time Fourier transform method.

We gave a sinusoidal load defined by the expression (3) to the bar. Figure 4(a) shows the relation between time and an axial stress of the bar measured by two strain gauges. In the figure, we see sinusoidal stress variations. The gauges are attached to the bar just opposite. We see a slight difference between them. There occurred small bending stress. The parameters, $\sigma_{Mean}$, $\sigma_{Amp}$ and $f_{Load}$ in Figure 4(a) are averaged values of the two stresses.

Figure 4(b) shows the relation between time and a deflection of the bar by the sinusoidal load and by impacts. The $f_{Impact}$ in Figure 4(b) is an exciting frequency of impacts to bring about free vibrations to the bar by the striker shown in Figure 1(b). In Figure 4(b), we see small ripples on the main variation of the deflection. The number of the ripples coincides with the exciting frequency. The times of horizontal axes in Figure 4(a) and 4(b) are not synchronized. Their times are slightly off.

We analyzed the time-deflection data of Figure 4(b) by an FFT. The result is shown in Figure 4(c). We see that peaks densely occur with the frequency from 409 to 459 Hz. The bar loaded by the expression (3) with the parameters in Figure 4(a) is expected to vibrate with the frequency between 434 and 462 Hz by the relation in Figure 2.

The result by a STFT method is shown in Figure 4(d). We see that a strip of a frequency is varying sinusoidally with approximate 1 Hz. The same accounts are true for the axial stress. It is difficult to evaluate definitely the range of the frequency because of the uncertainty principal. We put broken lines into Figure 4(d), the frequency range obtained by the relation in Figure 2 employing the parameters in Figure 4(a). If we could draw a curve which followed each peak at each time in the blurred strip, the curve would show a sinusoidal variation between the broken lines. Similar result was obtained by a wavelet analysis as more blurred picture than one in Figure 4(d).

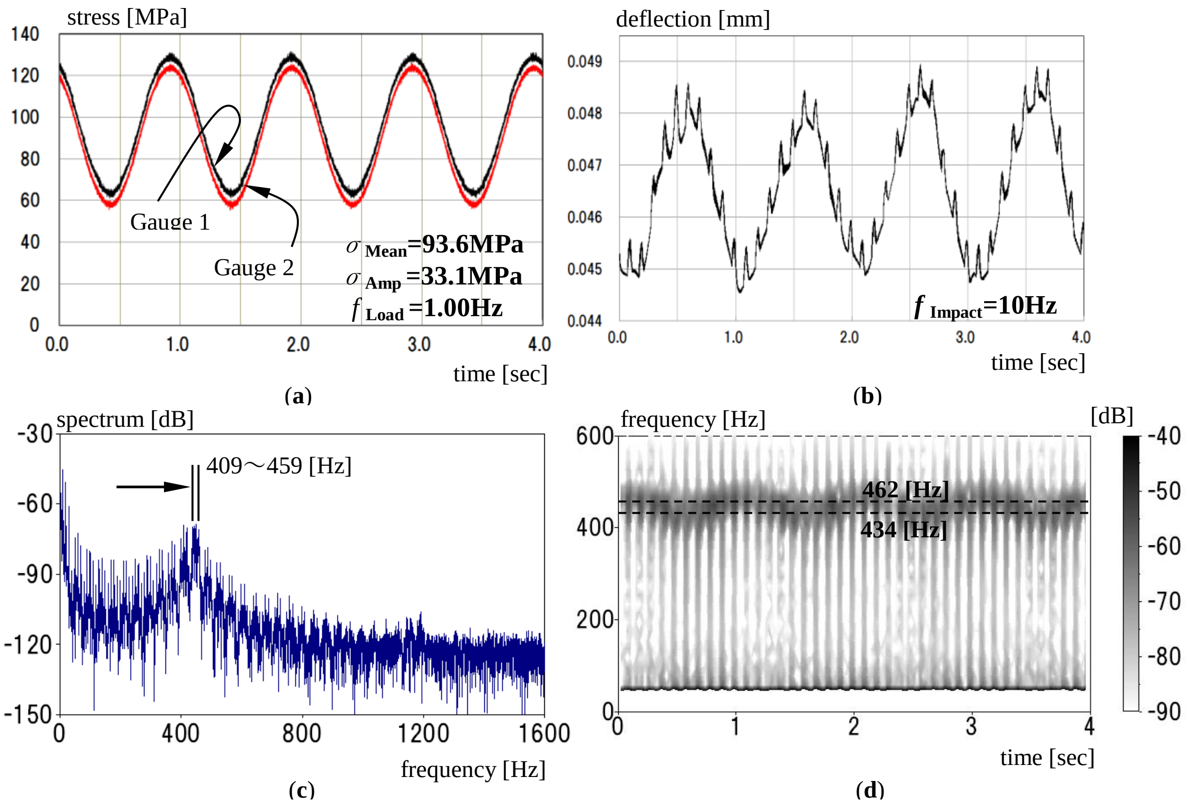



**Figure 4.** Experiment and analyzed data for sinusoidal load: (**a**) Time-axial stress variation; (**b**) Time-deflection variation; (**c**) Result by FFT method; (**d**) Result by STFT method.

To the bar, we gave a time-varying load, the mean stress of which was made to vary arbitrarily. The loading frequency and the stress amplitude were unchanged. The time-stress variation is given in Figure 5(a). The variation of the stress shows a saw-tooth variation. Figure 5(b) shows a result by a STFT method. The time-frequency variation is shown by a blurred strip also for this case. But, a curve of frequency peaks at each time seems to reflect the stress variation. In the next step, we are required to specify definite quantification of the time-frequency variation.

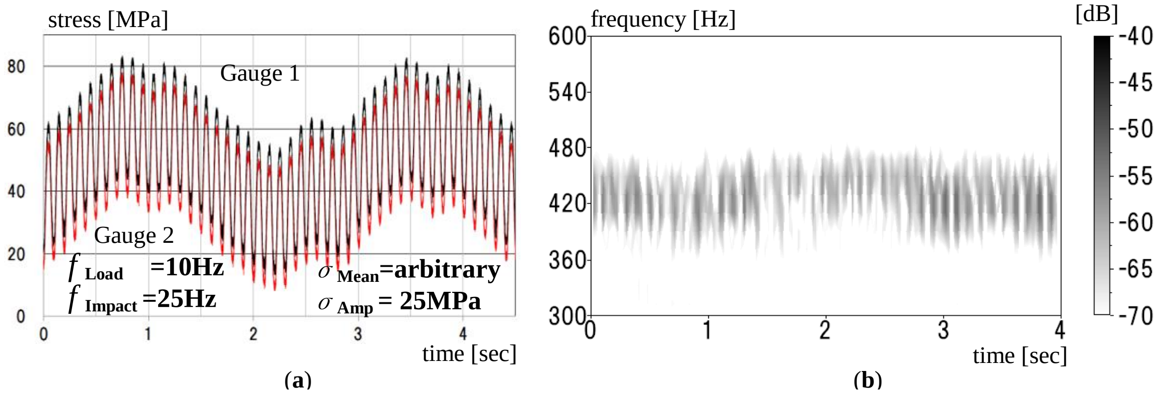



**Figure 5.** Experiment and analyzed data for load with arbitrary mean stress: (**a**) Time-stress variation; (**b**) Result by STFT method.

## 4. Conclusion

We applied a time-varying sinusoidal load to a round bar. We tried to evaluate a magnitude and a cycle of the stress of the bar by a natural frequency. To obtain a natural frequency, we gave impacts to the bar by an exciter to bring about a free vibration and measured a deflection of the bar. We could obtain a time-stress variation pattern by spectrum analyses but could not evaluate the magnitude of the stress definitely. We expect we will be able to obtain more detailed information on time-stress variation by accumulating up our knowledge and making elaborate use of time-frequency analyses.

**Author Contributions:** T.Yoshida conceived, designed the experiments and wrote the paper; T.Yoshida and K. Shinko performed experiments and analyzed data; K. Sakurada and T. Watanabe made up experiment tools.

**Conflicts of Interest:** The authors declare no conflict of interest.

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