

1 Article

# 2 Copula bias correction for extreme precipitation in re- 3 analysis data over a Greek catchment

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6 Received: date; Accepted: date; Published: date

7 Academic Editor: name

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11 **Abstract:** The projection of extreme precipitation events with higher accuracy and reliability, which  
12 engender severe socioeconomic impacts more frequently, is considered a priority research topic in  
13 the scientific community. Although large scale initiatives for monitoring meteorological and  
14 hydrological variables exist, the lack of data is still evident particularly in regions with complex  
15 topographic characteristics. The latter results in the use of reanalysis data or data derived from  
16 Regional Climate Models, however both datasets are biased to the observations resulting in non-  
17 accurate results in hydrological studies. The current research presents a newly developed statistical  
18 method for the bias correction of the maximum rainfall amount at watershed scale. In particular,  
19 the proposed approach necessitates the coupling of a spatial distribution method, namely Thiessen  
20 polygons, with a multivariate probabilistic distribution method, namely copulas, for the bias  
21 correction of the maximum precipitation. The case study area is the Nestos river basin where the  
22 several extreme episodes that have been recorded have direct impacts to the regional agricultural  
23 economy. Thus, using daily data by three monitoring stations and daily reanalysis precipitation  
24 values from the grids closest to these stations, the results demonstrated that the bias corrected  
25 maximum precipitation totals (greater than 90%) is much closer to the real max precipitation totals,  
26 while the respective reanalysis value underestimates the real precipitation totals. The overall  
27 improvement of the outputs, shows that the proposed Thiessen-copula method could constitute a  
28 significant asset to hydrologic simulations.

29 **Keywords:** copula; thiessen polygons; extreme; precipitation; bias correction

30 **PACS:** J0101

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## 32 1. Introduction

33 Last decades extreme precipitation episodes have been observed more frequently and their intense  
34 is greater compared with the past [1]. The impacts of these extremes are obvious in many different  
35 fields such as economy, society, agriculture, hydrology, resulting on the need of reliable projections.  
36 According to Mao et al. [2] in order to achieve higher accuracy in the hydrological results, is  
37 mandatory to correct the bias between model's used real values of several climate parameters and  
38 especially precipitations. In cases where lack of observed data is occurred, then proxy but reliable  
39 techniques, such as the use of reanalysis data, is proposed as scientifically proven solution. Bastola  
40 and Misra [3] in particular, demonstrate that reanalysis data is useful in simulating realistic  
41 hydrological response at watershed scales. At the same time, although precipitation estimates from  
42 global reanalysis are dynamically consistent with the large-scale circulation, on the other hand, when

43 precipitation is compared with rain gauge estimates the output is poor, since reanalysis products are  
 44 forecast by the reanalysis system and precipitation is not assimilated [4].

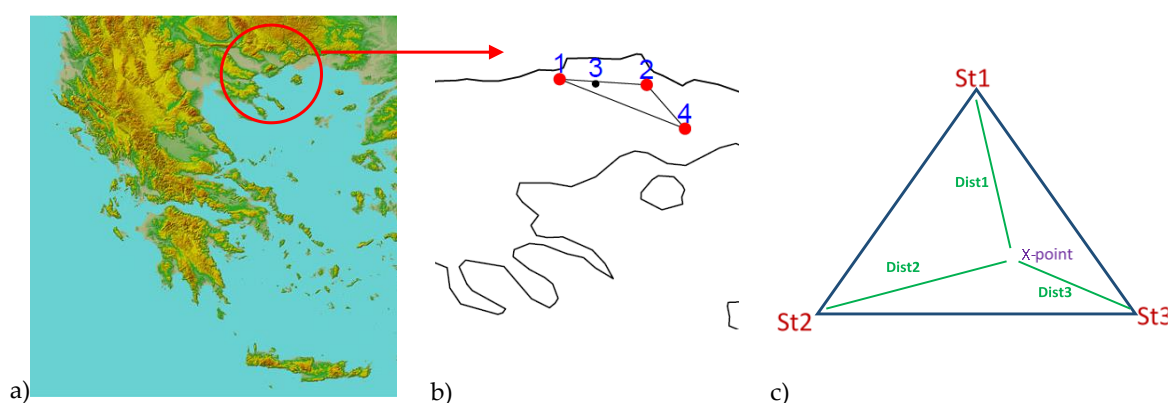
45 Numerous studies in the fields of insurance and finance have attempted using the Copula method  
 46 [5,6] technique. The utility of copula method based on its ability to analyze the dependence of two or  
 47 more random variables that have not necessarily the same distribution [7]. Additionally, copulas  
 48 have the advantage to capture the features of the dependence [8] and to examine this dependence not  
 49 only linearly, as the other indices do [2]. Consequently, last decades copulas have been used widely  
 50 in hydrology. Shiau [9] suggest copulas for drought analysis in order to overcome the problem that  
 51 drought characteristics have different distributions. Several scientists use copulas for analyzing  
 52 drought characteristics (e.g. severity and intense) [10, 11] or for correlating drought with other climate  
 53 parameters such as precipitation [12]. Furthermore Golian [13] used a bivariate copula function for  
 54 studying the rainfall–runoff simulations of a watershed in the region of Iran, while Perera [14] used  
 55 this method for studying the interdependence between the Kelani River and Kotte Canal in Sri-Lanka.

56 The present study investigates the combination of the copulas probabilistic distribution method with  
 57 the Thiessen polygon spatial distribution method to tackle bias correction of extreme precipitations  
 58 reanalysis data in the important hydrological region of Nestos river basin in Greece. Thiessen  
 59 polygons are widely used in hydrology for spatial interpolation since 1980 when Lee and Schachter  
 60 [15] present it. The evaluation of the result was achieved using statistical tools such as Taylor  
 61 diagrams as well as the relative operating characteristics curves (ROC). The latter are popular in  
 62 clinical epidemiology as they test the accuracy of a diagnosis [16] and in this study is checked the  
 63 accuracy of the bias corrected values.

## 64 2. Materials and Methods

### 65 2.1 Data

66 The present study uses daily precipitation data from four meteorological stations located in the  
 67 Nestos catchment (Figure 1a). Apart from the observed precipitation records, Era-Interim reanalysis  
 68 data with spatial resolution of  $12.5 \times 12.5$  km, were retrieved by the European Centre for Medium-  
 69 Range Weather Forecasts (ECMWF) for the selected case study area. Thus, for every station the closest  
 70 continental grid point that presented similar topographic characteristics was selected. Both reanalysis  
 71 data and observed records cover a time period of 9 sequential years, i.e. from 1987 to 1995.  
 72



73  
 74  
 75 **Figure 1.** a) The map of Greece – the red circle includes the studied region b) zoom of Nestos region  
 76 and the four studied stations with the studied triangle (1= Achladia, 2= Prasinada, 3= Sidironero, 4=  
 77 Toxotes), c) St1 to St3 indicate the three stations at the triangle vertices. x\_point is the unknown station  
 78 and dist1 to dist 3 are the distances between x\_point and the three stations.  
 79  
 80  
 81  
 82

## 83 2.2 Methodology

84 According to Nelsen [17] copulas are multivariate cumulative distribution functions which have the  
 85 ability to model mathematically the dependence between two or more variables using their marginal  
 86 distributions. Assuming that  $X$  and  $Y$  are two random variables with  $F$  and  $G$  marginal distributions  
 87 respectively, the joint distribution function of the initial variables is  $H$  and is equal to the copula  
 88 function of their marginal ( $H(x,y)= C(F(x), G(y))$ ). The centered theorem of copulas is the Sklar's  
 89 theorem [18]. According to that, if the marginals of the studied variables are continuous, then the  
 90 Copula function can uniquely be defined. Otherwise,  $C$  is unique on  $\text{Ran}F \times \text{Ran}G$  ( $\text{Ran}$  is the range).  
 91 The importance of this theorem is that every joint distribution function can be decomposed into the  
 92 marginal of the variables and into a Copula function which describes their dependence completely.

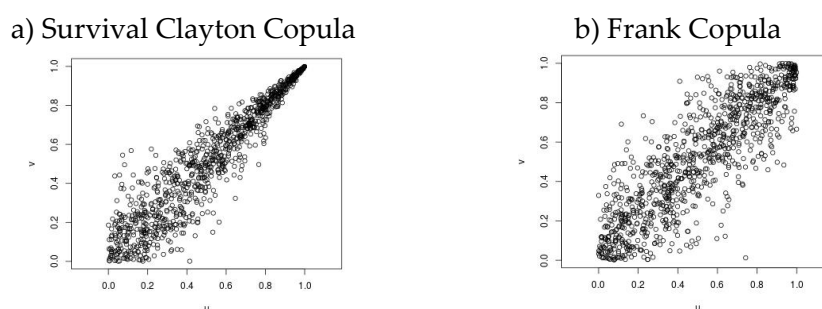
93 In the present study, Copula method was combined with Thiessen triangles, which is an alteration of  
 94 the Thiessen polygons method, to achieve a bias correction of total extreme precipitation between real  
 95 observations and reanalysis data. Firstly, three (stations 1, 2 and 4) of the four available stations, were  
 96 used for the analysis while the other one (station no 3) for evaluation. Specifically, the three selected  
 97 stations formed a triangle that includes the tested station (Figure 1b).

98 For the stations located at the triangle vertices, the absolute maximum and the monthly precipitations  
 99 were used to model the dependence among them. Twelve copula families (Gaussian, Student  $t$ ,  
 100 Clayton, Gumbel, Frank, Joe, BB1, BB6, BB7, BB8, Tawn type 1 and 2) coming from both Archimedean  
 101 and Elliptical categories were tested, in order to select the one which can describe the dependence  
 102 more satisfactory. The final selection was based on Akaike's information criterion [19] and Bayesian  
 103 information criterion. Thereafter, using the copula results by the stations located at the triangle  
 104 vertices and taking also into account the distance between them and the evaluated stations ( $x$ -point)  
 105 a newly copula family was defined (Figure 1c). The new defined copula family can describe  
 106 mathematically the dependence between the mean and maximum precipitation at the  $x$ -point.  
 107 Consequently, using the new copula family and the reanalysis mean precipitation data, the maximum  
 108 extreme bias corrected precipitations have been calculated.

## 109 3. Results

### 110 3.1. Total and Extreme precipitation analysis

111 The dependence between absolute maximum extreme and monthly precipitation was assessed using  
 112 copulas to three of the four studied stations. After testing several copula families, it was found that  
 113 the Survival Clayton family reflects the dependence for both 4 and 1 stations, while Frank copula for  
 114 station 2. The dependence is more power for Prasinada station, as the Kendall's tau correlation index  
 115 is almost 0.87, while for the other two stations is almost 0.78. Figure 2 visualize that relationship,  
 116 confirming the fact that Survival Clayton copula presents upper tail dependence (stations 1 and 4)  
 117 while Frank presents no upper or lower tail dependence (station 2).



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119 **Figure 2.** Structure of values which dependence is modelled by Survival Clayton or Frank Copulas.

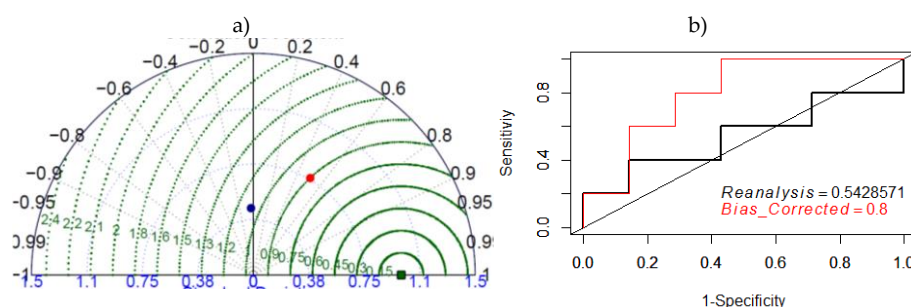
## 120 3.2 Bias correction results

121 Reanalysis data present important biases from real observations especially for precipitation  
 122 parameter and especially for extreme events. As a consequence the main purpose of this study  
 123 is to reduce extreme rainfall biases between real and reanalysis data at the scale of a hydrological  
 124 basin. Table 1 presents the observed, the reanalysis and the bias corrected extreme precipitation  
 125 indices, for the station of Sidironero. Extreme precipitation indices defined as the 90<sup>th</sup>, 95<sup>th</sup> and  
 126 99<sup>th</sup> percentile of the monthly precipitation [20].

127 **Table 1.** Extreme precipitation indices (mm) from observed, reanalysis and bias corrected data

	90%	95%	99%
Observations	32.5	48	67
Reanalysis	26.3	31.5	37.4
Bias Corrected	36.3	42.5	49

128 As it can be seen from Table 1 for all indices the bias corrected values are closer to the observed ones  
 129 compared with reanalysis data. Additionally as the percentile became higher, reanalysis present  
 130 higher differences compared to observe while the bias corrected were closer to the observed ones.  
 131 This is also proved by the Taylor diagram Figure 3a of the studied data sets. Particularly, Taylor  
 132 diagram shows that the correlation between observed and reanalysis data was almost zero while after  
 133 the bias correction the correlation increased to 0.5. Additionally, the Root Mean Square Error has been  
 134 reduced while there is an increase of the variation.



135

136 **Figure 3.** a) Taylor diagram of extreme precipitation at Sidironero. Blue circle presents the reanalysis  
 137 data and red circle the bias corrected data b) ROC curves of extreme precipitation at Sidironero. Black  
 138 line concerns the observed extreme values and red line the bias corrected values.

139 An additional evaluation of the results was conducted with the use of the ROC curves.  
 140 According to Figure 3b the area under curve –which is an effective measure of accuracy [21] - is  
 141 bigger for the bias corrected values proving that the Thiessen- Copula method can be used for  
 142 the bias correction of extreme precipitation.

143 **4. Discussion- Conclusions**

144 The bias correction of extreme rainfall with the coupling of copula method and an alteration of the  
 145 Thiessen polygon method is presented. The proposed method adjusts the extreme reanalysis  
 146 precipitation data to observed data in a selected river basin in Greece. The method's evaluation was  
 147 achieved after the comparison of the bias corrected values with observed datasets using different  
 148 statistical and optional methods.

149 Lafon et al.[22], Teutschbein and Seibert [23] have studied and compared different bias correction  
 150 methods in hydrology. Methods such as the linear scaling approach, delta change method [24] or the  
 151 local intensity scaling [25] use simple mathematical equation for bias correction. However, they  
 152 mainly focus on estimating mean values without expanding to the whole distribution [2].

153 Additionally as Yang et al. [26] mention, the accuracy for extremes is much lower even with more  
154 dynamical methods. At the same concept, Berg, et al. [27], indicate the importance of bias correction  
155 to reanalysis products, regarding soil moisture, runoff, and snow water equivalence, at simulations  
156 covering the geographic area of N. America.

157 The results of the present study shows that the Copula method combining with Thiessen triangles  
158 can be an accurate tool for rainfall extremes bias correction. Copula method present also satisfying  
159 results at Mao's [2] study, who used it for the bias correction of model's precipitation values in  
160 Germany. Additionally, Piani [28] propose the use of Copula for a two dimensional bias correction  
161 for the parameters of temperature and precipitation in climate models. In accordance with this study,  
162 the success of the method derived from the ability of copula to represent the dependence structure of  
163 the studied variables satisfactorily. Consequently, the dependence structure is not the same in every  
164 station in a specific region, as it is also observed in the three studied stations of the presented case  
165 study. Furthermore the bias corrected extreme precipitations not only were much closer to the real  
166 ones but they also have higher correlation with the observed extremes, as well as the Root Mean  
167 Square Error is lower compared with the reanalysis data. The need of bias correction is more obvious  
168 in the case of climate change, where the models outputs need to be bias corrected before it can be  
169 used for climate change impact studies. In that case various methods, such as the quantile mapping,  
170 the cumulative distribution function transform (CDF-t), equidistant quantile matching are presented  
171 in the literature [29]. Concluding, the resent investigation propose the copula method in combination  
172 with Thiessen triangles technique as a useful tool for the bias correction of extreme precipitations. It  
173 is also believed that this method would be a fruitful area for further work and it is proposed its  
174 application in large river basins.

175 **Acknowledgments:** This work was supported by a grant by the State Scholarship Foundation (IKY) in Greece

176 **Author Contributions:** All authors conceived and initiated the study. Georgia Lazoglou analyzed the data. All  
177 authors contributed to the discussion and interpretation of the results and the writing of the manuscript.

178 **Conflicts of Interest:** The authors declare no conflict of interest

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## 245 Abbreviations

246 The following abbreviations are used in this manuscript:

247

248 ROC: Relative Operating Characteristics curves



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