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# MULTIFACTAL CHARACTERIZATION OF SEISMIC ACTIVITY IN THE PROVINCES OF ESMERALDAS AND MANABÍ, ECUADOR

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## ABSTRACT

*Due to the enormous impact of the seismic activity and the need to deepen more in the detailed knowledge of its behavior, in this research work we describe an analysis of the multifractal nature of the magnitude, interdistance and interevent time series of the earthquakes that occurred in Ecuador, during the years 2011-2017, in the provinces of Manabí and Esmeraldas, which is an area with a high seismic activity. For this study we use the Multifractal Detrended Fluctuation Analysis or (MF-DFA), which allows the detection of multifractality in non-stationary series as well as a series of parameters of non-linear characterization. Analyzing the results, it´s revealed that the interevent time serie presents the higher degree of multifractality than the two previously mentioned; added to this that the Hurst exponent is in a non-proportional function to (q), which is a weight value, and that indicates the multifractal behavior of the dynamics of the earthquakes analyzed in this work. A study is made of the statistical values resulting in each series and its relation is obtained according to the asymmetry (r), resulting in all the series skewed to the right, consistent with r > 1; this indicates that small variations in the series are more dominant than large fluctuations.*

**Keywords:** *Fluctuation, magnitude, earthquake, seismic activity, multifractal, Hurst, Ecuador.*

## 1. Introduction

As a product in the interaction between the continental and oceanic plates, subduction is activated, which is when a low density plate enters below the one that produces the highest density in a subduction zone, the plane of friction produced in earthquakes, volcanism, magmatism, origin fault systems and sutures. (Agustín Paladines & John Soto, 2010).

This is why Ecuador is considered seismically active, during the last 110 years there have been earthquakes that have been studied for their magnitude and their origin, a clear example is the earthquake of Esmeraldas in 1906, whose magnitude was 8.8 on the Richter scale and which has been one of the largest recorded in history. Earthquakes are natural catastrophes that can not be accurately predicted at present or avoided, which is why this research work focuses on previous data that serve as a basis for a map of susceptibility and demonstration of seismic scenarios, all this depending on of the land, geographical location, load on the ground, etc. Compaction, subsidence, liquefaction, landslides, settlements, cracks, balance, faults, cracks, etc.

They are some of the effects of an earthquake and that are associated with shaking or vibrations (Sánchez, 2000), all these damages can be estimated with probabilistic methods and therefore reduce their damage by reinforcing structures in common seismic zones or implementing safe zones within the reach of all the people that may be affected.

This research work addresses the seismic hazard of the areas indicated above and determines the size of the forces or the set of deparadas actions that affect the soil in a specific place during future earthquakes and that, therefore, act on the constructions, which implies a possible production of damages or collateral effects, a very important factor to take into account is seismic irrigation, which is not more than the estimate of damages or expected losses, to cover the study and analysis that is explained later, the definition, generation and different parameters that must be taken into account in relation to earthquakes.

## 2. Art State: Fractals

The concept of "Fractal" was improved by Mandelbrot in 1975, refers to a geometric figure, basic structure, fragmented, repeated at different scales, that is, self-similar. Due to its irregular nature it does not allow it to be described in traditional geometrical terms. Taking this into account as an initial step, from the theoretical development of fractality, we could analyze many manifestations that presented chaos and order at the same time in their characteristics. Fractals became a set of new rules to know and describe nature. A different perspective of approach to reality (Mandelbrot, 1983). There are two types of fractals, the "ideal", which is a geometrical figure that mathematicians create by means of an iterative algorithm or repetitive rule that has a shape, either highly irregular, highly interrupted or fragmented, and remains so at any scale that observation occurs. Mathematical fractals fulfill the property of exact self-similarity. In addition to the "ideal" fractals, there is the "natural" fractal, this is an element of nature that can be described by fractal geometry. Earthquakes, mountains, the circulatory system, coastal lines or snowflakes are natural fractals. This representation is approximate, because the properties attributed to ideal fractal objects, such as infinite detail, have limits in the physical world.

With the appearance and development of the computer, the graphic production of fractals that require highly complex calculations is possible. In this way, mathematicians and artists found a new means of expression and research.

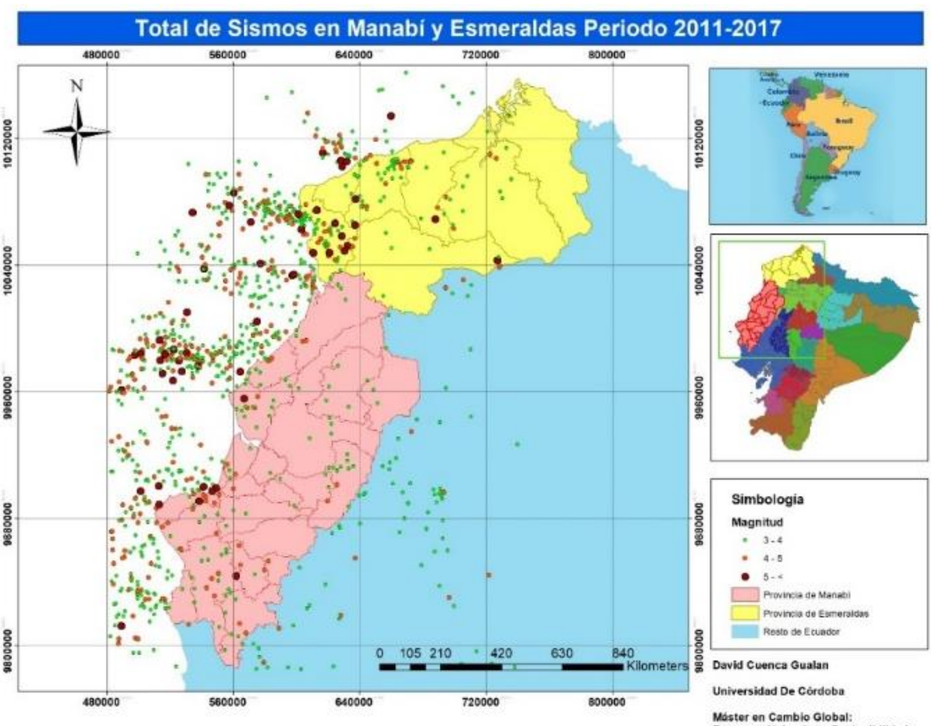
## 3. Data Source

Data collection is the first step to perform an analysis of them, all data is provided by the Military Geographic Institute of Ecuador through a request for information from the author to this institution which has been given all values The results of the year 2011 and the year 2017 in all of Ecuador, after obtaining this information, a filter is made in the area of interest.

It started with a data size equal to 2190 (Map 1) earthquakes produced between 2011 and 2017, of which 1020 are used in this investigation, according to their coordinates they are positioned and selected.

The data provided by the Military Geographic Institute of Ecuador are:

- Date and hour
- Deep
- Duration time
- Coordenates
- Magnitude




Map 1: Total earthquakes in the study area

The interevent series is determined by comparing each of the precise moments that an earthquake occurs, that is, it is the time difference between each of the earthquakes. As for the interdistance series, a similar analysis is carried out, determining each one of the coordinates in which each earthquake occurs, a distance difference is made. Subsequently each of the series is divided into sets (q) that takes positive and negative values, in this case 5 y -5.

For an appropriate characterization of the selected provinces, the multifractal analysis has been carried out by means of the Rescaled Range analysis, which is defined below as well as other concepts and values of importance in this study.

## 4. Metodology

### 4.1. Multifractal Detrended Fluctuation Analysis or (MF-DFA)

This method is described by (Kantelhradt, Zschiegner, & Koscielny-Bunde, 2002) and it is a general procedure consisting of five steps. When we start, we assume a series xk When starting we assume a series with a length N finite, which is a set of k- indexes with non-zero values of xk. The series is compact if the values xk = 0, that is, it is a negligible fraction compared to the total length of the series. For the null components of the series, values are not assigned to the index k.

Step 1: The next profile to be analyzed is determined with the equation (1)

$$Y(i) \equiv \sum_{k=1}^{i} [x_k - \langle x \rangle], \qquad i = 1, \ldots, N. \tag{1}$$

where <x> represents the arithmetic mean:

$$\langle x \rangle = \frac{1}{N} \sum_{k=1}^{N} x(k) \tag{2}$$

The subtraction of the mean <x> is not mandatory since it will be eliminated later with the equation (2)

- Step 2: Divide the obtained profile with the equation (1) in $N_s \equiv int\ (^N/_S)$, segments, these are not overlapping in length s. Because the length N of the series is not always a multiple of s, in order not to neglect the remaining interval of the end, the same procedure is repeated from the opposite side of the series. Thus, they are obtained $2N_s$ total segments.

- Step 3: By adjusting the least squares, the local trend of each of the segments is determined $2N_s$. Subsequently, the covariance is calculated with the equation (2).

$$F^2(v,s) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[(v-1)s + i] - y_v(i)\}^2 \tag{3}$$

for each of the segments $v$, $v$ =1,…, $N_s$ and also for $v = N_s + 1, \ldots, 2N_s$. The $y_v(i)$ value is a polynomial fit in the segment $v$-th. Depending on the order of polynomial adjustment (you can use linear, quadratic, cubic or higher-order polynomials in the fitting procedure), the order m of the detrended fluctuation analysis is defined respectively (DFA) generalized (DFA1, DFA2, . . . ).

$$F^2(v,s) \equiv \frac{1}{s} \sum_{i=1}^{s} \{Y[(N-(v-1)s + i] - y_v(i)\}^2 \tag{4}$$

- Step 4: It is determined with the equation (5) the average of all the segments to obtain the function of the q-th order fluctuation.

$$F_q(s) \equiv \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(v,s)]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \tag{5}$$

where (q) can take any real value except zero. For different values for example q = 2, the scalar exponent h(2) provides information on the fluctuations of the data series. By repeating the process described above, the s time scales vary if the function $F_q(s)$ increases as s increases, it is also highlighted that $F_q(s)$ depends on the order m of the analysis DFA, this is defined as s ≥ m+2.

- Step 5: Through the graphical representation in logarithmic scale of $F_q(s)$ in front s, the scalar behavior of the fluctuation function is determined for each value of q. The function $F_q(s)$ increases for large values of s, when the $x_i$ series presents a long-range correlation, this as a power law, represented in equation (6).

$$F_q(s) \sim s^{h(q)} \tag{6}$$

In order to quantify the multifractal character in time series, the multifractal spectrum is used $f(\alpha)$ as a relationship between the generalized exponent of Hurst H(q) and the classical exponent $\tau$ (q), which is a scalar exponent of Renyi, when it depends linearly on q, the set is monofractal, otherwise the set is multifractal, to calculate this exponent the equation is used (7)

$$\tau\,(q) = qH(q) - 1 \tag{7}$$

By Legendre transformation you get:

$$\alpha = \tau\,(q) \qquad \text{f}(\alpha) = q(\alpha) - \tau\,(q) \tag{8}$$

where $\alpha$ is the Hölder exponent, and f($\alpha$) determines the dimension of the subsets of the series, which is based on $\alpha$, the same that is an exponent that measures the strength of the multifractal structure (Kantelhradt et al., 2002). Broadly speaking, a small value of $\alpha$ indicates indicates that the process loses a fine structure and that its appearance becomes more regular, but if the value is large, a complex structure is ensured. In that aspect it is possible to relate the Hurst exponent with $\alpha$ in the equation (9).

$$\alpha = H(q) + qH'(q) \quad \text{y} \qquad \text{f}(\alpha) = q[\alpha - H(q)] + 1 \tag{9}$$

Calculating a polynomial fit of the second order around the position of $\alpha$ max o $\alpha_0$ the equation results (10).

$$\text{f}(\alpha) = A(\alpha - \alpha_0)^2 + B(\alpha - \alpha_0) + C$$

(10)

where C is a constant equal to 1 and the coefficient B refers to the asymmetry of the multifractal spectrum, this is zero or equal to 0 for a symmetric spectrum, when it is higher than 0, the multifractal structure is quite solid; on the other hand, when it is less than 0, the multifractal structure is more regular and smooth, this indicates lower fractal exponents.

## 5. Results and Discussion

The first variable analyzed was magnitude (Figure 1). The Scaling function Fq is represented in Figure 1A for different q values, estimating de slope Hq. With the exception of some fluctuations, it can be seen that the fluctuation function Fq follows a linear trend, in logarithmic coordinates. Thus, we can identify in Figure 1B the generalized exponent of Hurst, H (q) with the slope, for each order of q, according to the potential law. The values given to (q) are -5 and 5, considering a set of events ranging from a scale of 5 to a total of N / 4 (N is the total value of our samples). According to Telesca & Lapenna (2006) the different slopes of the fluctuation curves indicate that the fluctuations of small and large events are scaled differently.

Likewise, if we compare each of the figures 1A, 2A and 3A, we can differentiate that the result of the variable magnitude and interdistance are similar (almost parallel to each other) in comparison with the results of the interevent variable that has steeper slopes, resulting in a different and strongly dependent Hurst exponents of (q).

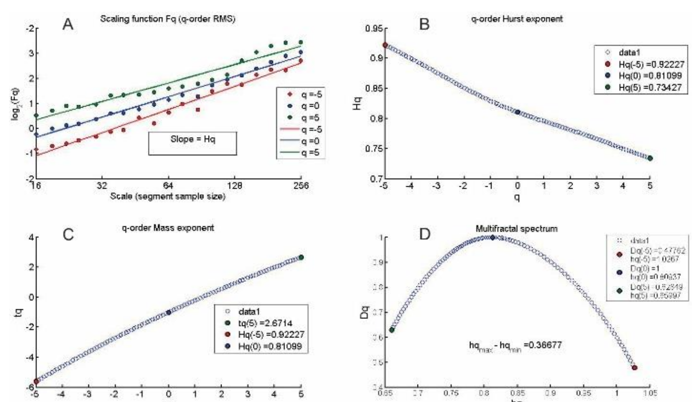


Figure 1: Results from Multifractal Analysis of Magnitude

Figures 1B, 2B and 3B relate each of the q values with its corresponding Hurst exponent for Magnitude, Interdistance and Interevent variables, respectively. As it can be observed, the less the value of (q) the greater the Hurst exponent and vice versa. There is a clarification regarding the curves: in the study of Interevent, a multifractal behavior was not detected in scales less than 32, because it has a linear curve by the massive exponent.

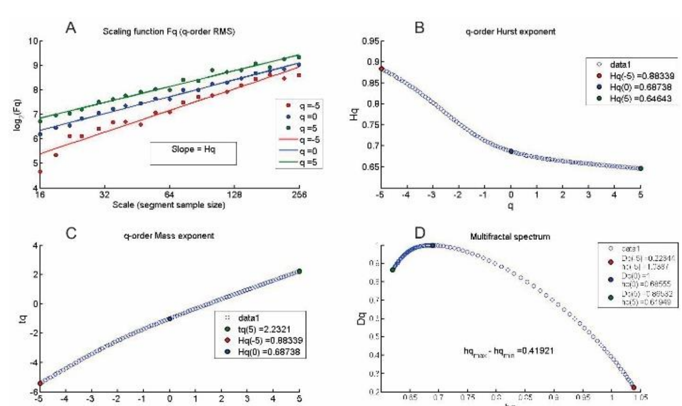


Figure 2: Results from Multifractal Analysis of Interdistance

In figures 1D, 2D and 3D the multifractal spectrum obtained by applying the Legendre transformation (Equation 8) is represented for each variable. The multifractal spectrum allows to describe qualitatively and quantitatively the multifractality of a time series considering its width (W), that determines the richness of the multifractal structure.

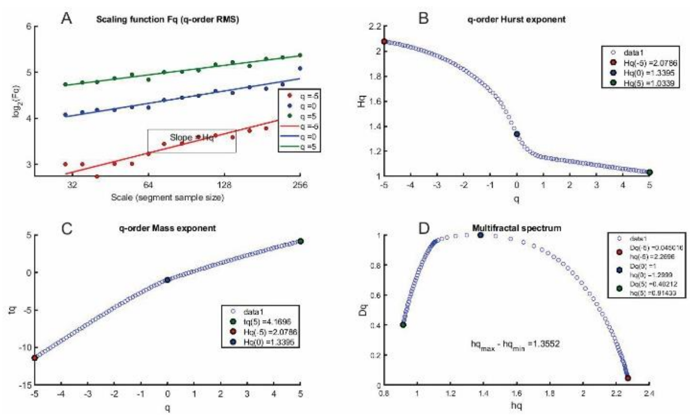


Figure 3: Results from Multifractal Analysis of Interevent

Regarding Hurst exponents (Hq), in the series of Magnitude a value for $H2 = 0.78$ was obtained, which indicates a persistence or the presence of a long-range correlation in the series, such that large values are preceded by the same type of values, as in the case of series of Interdistance ($H2$ = 0.66). However, Interevent series presents $H2$ = 1.16 which indicates that the dynamics of this variable is fluctuating noise, that is, it is common in critical self-similar systems.

| | | | | |
|---|---|---|---|---|
| **Magnitude** | | | | |
| Width | Asymetry r | Average h(q) | St. Desv. h(q) | Max. Delta. |
| 0.55 | 1.23 | 0.8283 | 0.1106 | 0.2396 |
| **Interdistance Km** | | | | |
| Width | Asmetry r | Average h(q) | St. Desv. h(q) | Max. Delta. |
| 0.56 | 1.94 | 0.7649 | 0.1464 | 0.3579 |
| **Interevent (min)** | | | | |
| Width | Asymetry r | Average h(q) | St. Desv. h(q) | Max. Delta. |
| 1.44 | 2.04 | 1.5562 | 0.5527 | 0.4584 |

Table1: Characteristics parameters of Multifractal Spectrum

Based on the above data and the table 1, it is determined that by comparing the three multifractional spectra (Figure 4), it is observed that the Interevent series is characterized by having a greater degree of multifractality than the other two series, and the series of Interdistance slightly presents a multifractality greater than the series of Magnitude. All series appear biased to the right, consistent with r> 1; this indicates that small variations in the series are more dominant than large fluctuations. It seems, also, that such dominance is more intense for the Interevent series than the other two series.

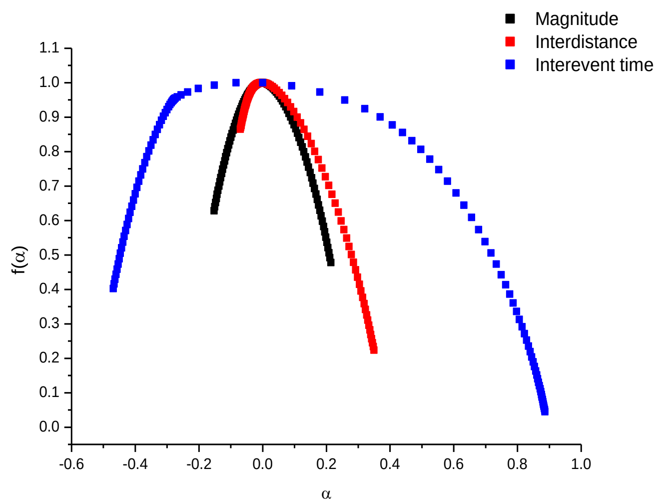


Figure 4: Multifractal Spectrum of the three variables studied

## 6. Conclusions

- Due to its enormous importance and the need to deepen its knowledge was carried out a characterization by multifractal analysis of the seismic activity of the provinces of Esmeraldas and Manabí in Ecuador obtaining as a result the multifractality curves and each of the parameters with their respective Multifractal spectra.

- The maximum and minimum delta parameters, asymmetry and width of the curves propose an analysis of the temporal evolution of the multifractality, that is, the studied seismic phenomenon is characterized by a dynamic change of heterogeneity towards homogeneity, from varying to becoming constant during the activation of the replica, being revealed by a loss of multifractality after a main event.

- After this study, the results revealed a persistent behavior of the magnitude and series of Interdistance, while the Interevent series showed a behavior of fluctuating noise.

- When determining a relationship between the resulting curves, all series appear biased to the right, consistent with r> 1; this indicates that small variations in the series are more dominant than large fluctuations. It seems, also, that such dominance is more intense for the Interevent series than the other two ones, due to the resulting values for each of the series and their respective exponent of Hurst.

**Funding:** This work was partially funded by the Spanish Ministry of Science, Innovation and Universities with Project No. AGL2017-87658-R.

**Acknowledgments:** The authors acknowledge the valuable help of the researcher Luciano Telesca

**Conflicts of interest:** The authors declare no conflict of interest.

## 7. Bibliographic References.

1. Brito, S., & Instituto Nacional de Investigación Geológico Minero Metalúrgico, E. (2017). *Atlas geológico minero del Ecuador*. Quito, Ecuador: INIGEMM

2. Cruz Atienza, V. (2013). *Los sismos, una amenaza cotidiana*. Ciudad de México, México: La caja de cerillos ediciones.
3. Gordillo Granda, D. (2008). *El conflicto socioambiental de La Propicia, visto con una mirada de género.* FLACSO, Quito, Ecuador. Retrieved from http://www.manabi.gob.ec/datos-manabi/datos-geograficos
4. Guartan, J. A., & Tamay, J. V. (2003). *Optimización del proceso de recuperación de oro contenido en los relaves de molienda de la planta "Vivanco" por el método de flotación-cianuración.* (Investigation), UTPL, Loja, Ecuador.
5. Instituto Geográfico Militar, I. (Cartographer). (2008). Carta Topográfica: Esmeraldas
6. Kantelhradt, J., Zschiegner, S., & Koscielny-Bunde, E. (2002). *Multifractal Detrended Fluctuation Analysis of Nonstationary Time Series.* Justus-Liebig-Universitat
7. Mandelbrot, B. B. (1983). *The fractal geometry of nature* (Updated and augm. ed.). New York: W.H. Freeman.
8. Mandelbrot, B. B., & Novak, M. M. (2004). *Thinking in patterns : fractals and related phenomena in nature*. River Edge, N.J.: World Scientific.
9. Morejón, J. (2012). *EsProvinciaVerde*. Esmeraldas, Ecuador: Fundación Naturaleza Viva
10. Paladines, A., & Soto, J. (2010). *Geología y Yacimientos Minerales del Ecuador*. Loja, Ecuador: Universidad Técnica Particular de Loja.
11. Prefectura de Manabí. (2014). *Plan de desarrollo y ordenamiento territorial de la provincia de Manabí, diagnostico estratégico*. Manabí: Prefectura de Manabí
12. Senesi, N., & Wilkinson, K. J. (2008). *Biophysical chemistry of fractal structures and processes in environmental systems*. Chichester, West Sussex, England ; Hoboken, NJ: Wiley.
13. Spalla, M. I., Marotta, A. M., & Gosso, G. (2010). *Advances in interpretation of geological processes : refinement of multi-scale data and integration in numerical modelling*. London ; Williston, VT: Geological Society.
14. Sánchez, F. V. (2000). *Los Terremotos y sus Causas*. España: Instituto Andaluz de Geofísica y Prevención de Desastres Sísmicos
15. Telesca, L., & Lapenna, V. (2006). *Measuring multifractality in seismic sequences.*