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BAYESIAN ANALYSIS OF THE MAGNITUDE OF EARTHQUAKES LOCATED IN A SEISMIC REGION OF ITALY

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Abstract: Bayesian statistical procedures are well known for estimating the probability distribution of the occurrence of an event. In this study, we applied the Bayesian statistical method to estimate the distribution of magnitudes of earthquakes occurred in central Italy in two different time periods. Using the Monte Carlo sampling we recovered the real magnitude distribution using just a small amount of available data, randomly selected by the seismic catalogue.

Keywords: Magnitude; Bayesian inference; Probability Density Function

1. Introduction

The seismic hazard assessment by using the Bayesian probability theory was introduced by Cornell [1] and was especially applied in earthquake engineering problems [2]. The Bayesian discrete approach was performed to forecast the interevent times for large earthquakes along the western and eastern Hellenic arc [3], and to predict probabilities of occurrence of moderate and strong earthquakes in the main seismogenic zones in Greece [4]. Stavrakakis and Drakopoulos [5] used the Bayesian extreme-value distribution to assess the seismic hazard in some seismogenic sources in Greece. Egozcue and Ruttener [6] applied Bayesian approaches for assessing seismic hazard with imprecise data; introducing the uncertainties inherent in the inaccuracy and heterogeneity of the measuring systems by which the data were recorded, they gave special attention to 'priors' in the Bayesian estimation. Wang et al. [7] developed a Bayesian algorithm to predict the next major event induced by the Meishan fault in central Taiwan, based on one magnitude observation of Mw 6.4 from the last event, along with the prior data including fault length of 14 km, rupture width of 15 km, rupture area of 216 km2, average displacement of 0.7 m, slip rate of 6 mm/yr, and five earthquake empirical models.

In this work, we apply the Bayesian probability theory to estimate the magnitude distribution in one of the most seismically active areas of Italy (Fig. 1), struck recently by several strong earthquakes with local magnitude larger than 5.5 [8].

Figure 1. The marked box represents the area under investigation delimited by longitude 12.3°E to 13.6°E and latitude 41.6°N to 44°N.

It is well known that the magnitude distribution of an earthquake sequence is given by the so-called Guttenberg-Richter law [9], which relates a threshold magnitude M_{th} with the number of earthquakes with magnitude $M > M_{th}$ in a form of semi-logarithmic power law as $\log_{10}(N) = a - bM_{th}$, where N is the number of earthquakes of magnitude $M > M_{th}$, a is the productivity of the earthquake, and b is a value that indicates the proportion of small events with respect to the large ones. The value b is a critical parameter that can inform about the conditions of the stress state of the investigation area and is generally estimated by the method of maximum likelihood [10].

In this work, our aim is to estimate by using the Bayesian approach the magnitude distribution of earthquakes occurred in central Italy on the base of a small percentage of real data.

2. Bayesian analysis of the magnitude of earthquake

2.1 Bayesian analysis

The inferences in the Bayesian analysis are based on the calculation of the conditional posterior probabilities calculated by the Bayes theorem [13]. The root of the calculation of Bayesian inference is the Bayes' theorem

$$P(parameters|data) = \frac{P(data|parameters) \times P(parameters)}{P(data)} \propto Likelihood \times prior \qquad (4)$$

So, if we have a set of observations $X|\theta \sim N(\theta, \sigma^2)$ where θ has a priori distribution $\theta \sim N(\mu, \tau^2)$ where μ and τ are known parameters, we have n number of Monte Carlo samples to calculate the posterior distribution as

$$\theta|x \sim N\left(\frac{\tau^2}{\frac{\sigma}{n} + \tau^2}\bar{X} + \frac{\sigma^2/n}{\sigma^2/n + \tau^2}\mu, \frac{(\sigma^2/n) \times \tau^2}{\sigma^2/n + \tau}\right) \qquad (5)$$

2.2 Modeling the magnitude of earthquakes in central Italy with Bayesian analysis

We analysed the seismic catalogue (magnitude larger or equal to 1.5) from 1995 and 2018 of the area shown in Figure 1. We selected two groups of events, between 1995 and 2008 and from 2009 to 2018. Figure 2 shows the two sequences of events.
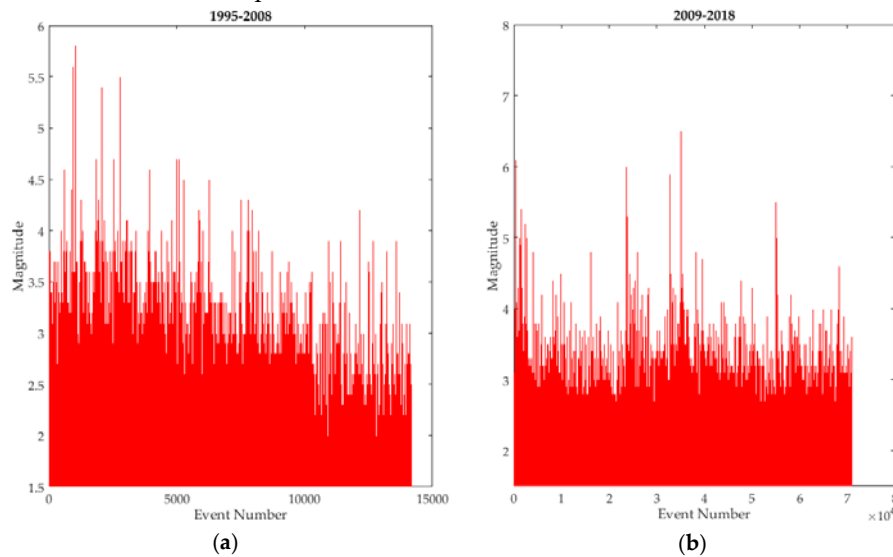

Figure 2. Magnitudes of the events for the selected periods: (**a**) between 1995 and 2008; (**b**) between 2009 and 2018.

Figure 3 shows the probability distribution of the magnitudes of the two groups of events shown in Figure 2. The first set of events is characterized by a semi-Gaussian probability distribution, while the second group by a power-law.
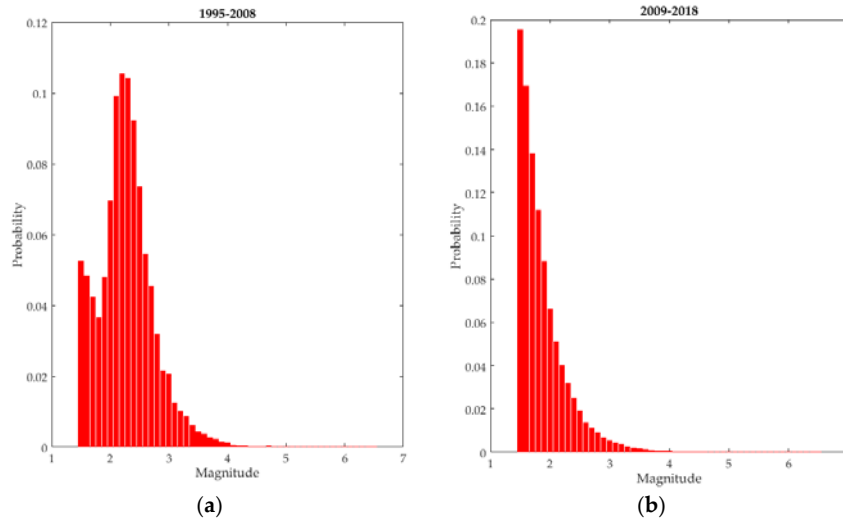
Figure 3. Probability distributions of magnitudes for the periods: (a) 1995 to 2008, (b) 2009-2018.

Using the Bayesian inference, we obtained the probability distribution functions of the magnitudes of the 2 sets of data, using 100 Monte Carlo samples for the Bayesian Method.
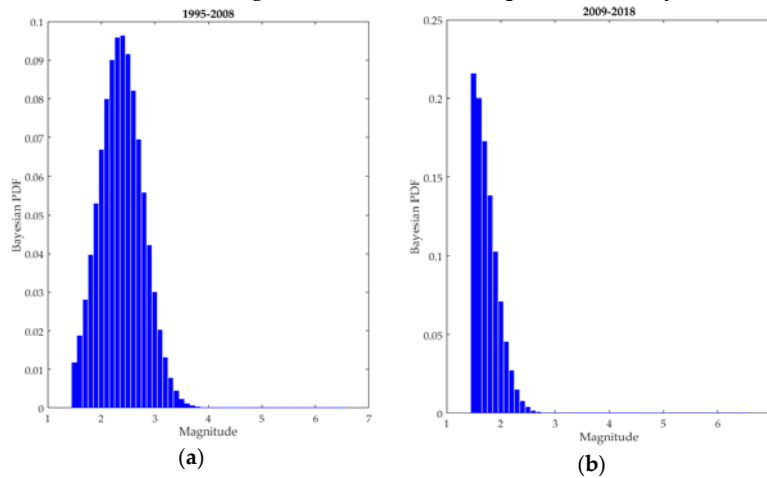

Figure 4. Probability density functions of magnitudes obtained by the Bayesian method: (a) from 1995 to 2008, (b) from 2009-2018.

The Bayesian method represented in equation (5) reproduces the probability density functions of the two groups rather well. Figures 5 and 6 show the errors between the probability density function of the data and that estimated by the Bayesian inference method.
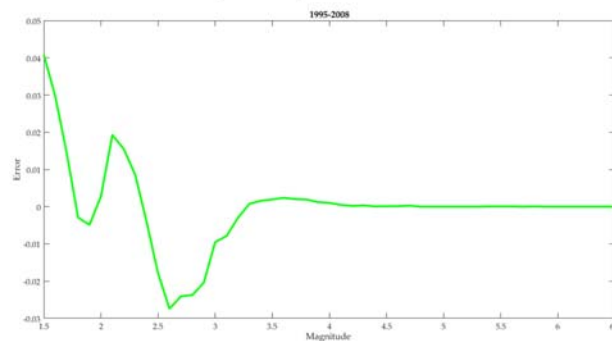

Figure 5. Error between the probability density function of the data (shown in Figure 3) and that estimated by the Bayesian method for the period from 1995 to 2008.
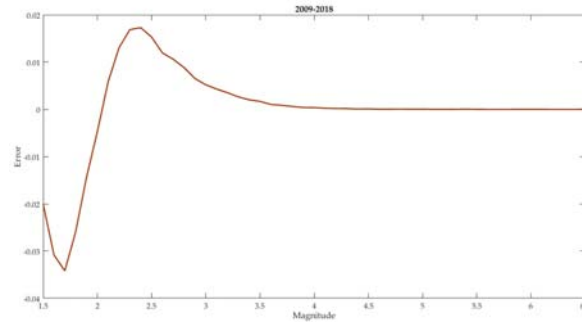
Figure 6. Error between the probability density function of the data (shown in Figure 3) and that estimated by the Bayesian method for the period from 2009 to 2018.

2.3 Prediction of the magnitude with Bayesian method

In the following example, the data from 2009 to 2017 are considered to calculate the distribution of the magnitudes of the events and then to make a prediction of this distribution in 2018, using 100 samples for the Bayesian method. Figure 7 shows the distribution of magnitude for the period from 2009 to 2017 and the distribution obtained through the Bayesian prediction in 2018.
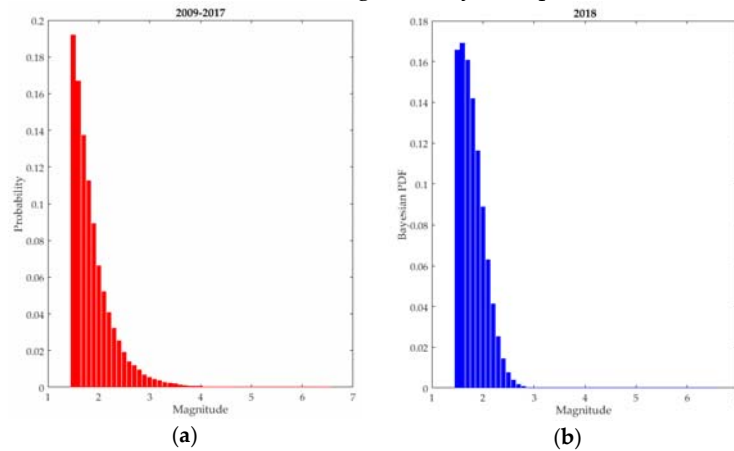

(**a**)   (**b**)

Figure 7. (a) Distribution of probability of magnitudes for the period from 2009 to 2017, (b) Bayesian probability distribution for the prediction in 2018.

The error of the prediction made in the year 2018 using the Bayesian method, with the information obtained during the years 2009 to 2017, and the probability of the magnitude of the events in the year 2018 is observed in Figure 8.
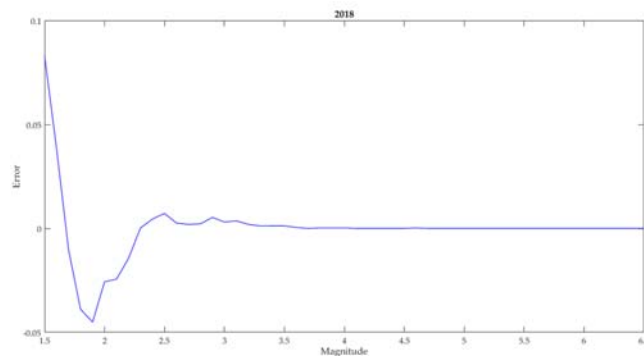

Figure 8. Error between the real distribution and the Bayesian prediction of magnitudes for the   2008.

3. Conclusions

The Bayesian inference method used in this paper can correctly reproduce the probability distributions of magnitudes of earthquakes occurred in central Italy from 1995 to 2018. In addition, with just a few observations the probability distributions of magnitudes of the events can be reproduced with a rather small error between the distribution of the data and that one obtained by Bayesian inference.

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