# Influence of the Pump Control System in the Selection of the Number of Fixed Speed and Variable Speed Drives Pumps in Water Pumping Stations

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**Abstract:** Proper design of a pumping system requires the use of the pump curve and the system setpoint curve. Both have to be as close as possible so that the energy used is optimal. This is achieved by means of the control systems, in which the type of control to be used (flow or pressure) and the combination between Fixed Speed Drives (FSD) pumps and Variable Speed Drive (VSD) pumps are involved. The objective of this work is to determine the optimal number of FSD and VSD pumps for each flow rate range in order to discuss the classic design of pumping stations and their control systems. For this, a methodology consisting in defining a parametric form of the pump curve, the efficiency curve and the set-point curve in relation of the best efficient point is applied. In this way, dimensionless expressions are obtained and the influence of the set point parameters in the design of the control system can be analyzed. Additionally, the method includes to get an expression to estimate the performance of the frequency inverter based on the load and pump speed rotation. The application of the methodology to different cases studies allows questioning many classic methods procedures for pumping stations. In summary, it can be concluded that the appropriate number of variable speed pumps for each contrl system cannot be established in advance but requires an in-depth study of the different options available.

**Keywords:** pumping control system; set-point curve; best efficient point; variable speed driver

## 1. Introduction

The growth of the population and the amount of urban areas entails the water distribution networks increase their water demands and their requirements of the energy for pumping stations. Therefore, it is important to optimize the operations of pumping station in order to save energy[1]. In fact, approximately 95% of energy consumption is related to pumping energy costs and approximately the 90% of the total cost of the pumps cycle life corresponds to operational costs which it is closely related with energy consume[2].

The efficiency of pumping stations is related with its operation and the system sizing requirements such as pipelines size, the flow demand and the minimum head pressure required to satisfy the consumption at nodes. In fact, pump optimization can be achieved by changing the control system and stablishing a new pump scheduling, adding pumps in a parallel configuration with variable speed drivers (VSDs) and using flow and pressure controls [1].

Recently researches have been worked about pump scheduling with fixed speed pumps (FSPs), using advanced mathematical models (evolutive algorithms) in order to optimize the reduction of energetic consumption by controlling the frequency of starting and stoppings pumps as it is showed in Yu et al. [3]. Similarly, Wu and Gao [4]use a multi objective optimization in pumping stations, including energy costs, treatment costs, the minimization of the number of pump switches and the maximization of the hydraulic service level.

Later, several works concluded the use of VSDs in pumping stations can achieved better results intermf of saving energy consumption. First Planells et al. [5] and Lamanddalena and Khila [6] developed several control systems with VSDs. These mechanisms sought to configure the pumping station so that head and flow values followed the system setpoint curve. In this way, as the driving curve is as close as possible to the resistant curve, the greatest energy savings are achieved. In the same way, Hashemi et al. [7] proposed an optimization of pump scheduling cost with variable speed pumps (VSP) by using an ant colony optimization model. Furthermore, Wu et al [8], worked on an optimization of parallel pump system with several configurations of VSDs and positions of control valves in order to provide a suitable operating mode with a balance between pumping system efficiency and reliability.

Walsky and Creaco [9] developed a comparision between different configurations of pumping station with FSPs and VSPs for a water distribution system without storage. They concluded that the use of more fixed speed pumps (FSPs)than the minimum required by the system, gives that pumps operate closer to their best efficient point (BEP), so the operational costs are lower. Another benefits in terms of operational costs can be achieved in the configuration where large pumps are flanked by small pumps or by adding VSD in the pumping station, especially in cases with small flow demands.

For a properly design and dimension of a pumping station, it is necessary the use of three curves; the set-point curve, the pump head curve and the pump efficiency curve. The set-point curve and the head curve should be as close as possible and make sure that the pump curve works closed to the BEP in order that consumed energy be optmimal. The set-point curve is defined as the minimum head required in a water source to satisfy the flow demand while the minimum pressure required at the critical consumed node of the network is guaranteed.

León Celi et al [10][11], developed a methodology to optimize the flow rate of multiple pumping stations in a closed water distribution network. This methdology is based on the use of the set-point curve as a reference for the pumping station control system in order to use the lowest quantity of energy. Moreover, this study was extended to cases with storage tanks. In this case another parameters such as the maximum and minimum of water level in the storage tanks [12] are considered.

To ensure that the pumping station curve is as close as possible to the setpoint curve, different control systems have been considered. These control systems depend on two main factors: the type of pumps to be used (FSP, VSP) and the type of control that operates the pumps (pressure controls, flow controls). In the case of FSP, pressure controls are related to pumps starts and stops according set levels of pressure (maximum pressure level stops the pumps and minimum pressure level starts the pumps). For FSP, flow controls are related to start or stop pumps according to the demand flow. In the case of VSP, operation is more general since both types of controls not only determine the start and stop of the pumps, but also the speed of rotation of the pumps with VSDs[13].

In previous works, Leon Celi et al. [10][11] optimize the energy consumption only considered a theorical constant minimum pump efficiency, so it is not considered the design of the pump station and the selection of control system to operate the pumping station. Thus, The main of this study is to determine the influence that the number of VSPs used in the control system may have on operating costs.

## 2. Methodology

An optimized design of a pumping station requires the driving curve (consequence of the selection of the pumping groups and the control system) be as close as possible to the setpoint curve. In this way the minimum energy consumption is achieved that satisfies the demands and requirements of the water distribution network [14]. To achieve this statement, different control strategies are used in water pumping stations. Consequently, the methodology of this paper is based on the set-point curve, the head curve and the BEP of the pump in order to analyze the influence that the number of VSDs have on control systems in terms of energy saved in water pumping stations.

The methodology consists in defining all the equations in a dimensionless way, taking as reference the values of the pump in the BEP. The variables flow ($Q$), height ($H$), speed of rotation ($N$), efficiency ($\eta$), power (P) and torque (M) are transformed into their dimensionless equivalents (q, h,$\alpha$, $\theta$, $\pi$, $\beta$):

$$q = \frac{Q}{Q_0};\ h = \frac{H}{H_0};\ \alpha = \frac{N}{N_0}; \theta = \frac{\eta}{\eta_0};\ \pi = \frac{P}{P_0};\ \beta = \frac{M}{M_0} \tag{1}$$

In the previous equation (1), variables with subscript 0 refer to the value in the BEP.

Usually pump curves and efficiency curves are expressed in a quadratic polynomial expression of three terms [15]. Nevertheless, in this methodology is assumed that the head and efficiency curve and the set point curve have a quadratic polynomial expression of two terms. When a pumping system have VSD, affinity laws are used in order to predict the operation points of the pump Therefore, pump curves are the head curve and the efficiency curve using the affinity law are expressed as showed in the equations (2) and (3), while the set-point curve is expressed in the equation (4).

$$H = A \cdot \alpha^2 - B \cdot Q^2 \tag{2}$$

$$\eta_b = E \cdot \frac{Q}{\alpha} - F \cdot \frac{Q^2}{\alpha^2} \tag{3}$$

$$H_c = \Delta H - R \cdot Q^2 \tag{4}$$

In the previous equations; the terms *A* and *B* are constant parameters related to the characteristic of the model of the pump curve; *E* and *F* are constant parameters in the efficiency curve and ΔH and *R* are parameters related to the setpoint curve. *ΔH* is related to the minimum pressure required and *R* refers to the losses produced in the water distribution network. Besides, the term A of the equation (2) corresponds to the maximum head of pump. For the development of this formulation it is admitted that A is 4/3 of the head value in the BEP ($H_0$) [16]. Another assumption of the methodology, the pump flow (*Q*) is maximum when the head (*H*) and the efficiency (η) of the pump are 0. If it is solved the derivative of the equations (2) and (3) in function of the flow (*Q*), it gets that maximum flow ($Q_{max}$ ) is two times the flow in the BEP ($Q_{max} = 2 \cdot Q$). If the equations (2), (3) and (4) are represented in a parametric form in relation of the BEP, the pump curve, the set point curve and the efficiency curve are established in a dimensionless form as are showed in the following expressions (5), (6) and (7).

$$h = \frac{4}{3} - \frac{1}{3} \cdot q^2 \tag{5}$$

$$\theta = 2 \cdot q - q^2 \tag{6}$$

$$h_C = \lambda + r \cdot q^2 \tag{7}$$

In the above equations (8) and (9), $\lambda$ and *r* represent the reduced form of parameters *ΔH* and *R*.

$$\lambda = \frac{\Delta H}{H_0} \tag{8}$$

$$r = \frac{R \cdot Q_o^2}{Ho} \tag{9}$$

$$\pi = \frac{q \cdot h}{\theta} \tag{10}$$

Additionally, the expressions for the power ($\pi$)and for the mechanical torque ($\beta$) of the pump can be written in reduced variables as it is expressed in equation (11):

$$\pi = \frac{q \cdot h}{\theta} = \beta \cdot \alpha \tag{11}$$

It is important to remind that the aim of a properly pumping station design consists of ensuring a minimum pressure at every node in the system. In other words, for the proposed metohodology, for every flow the pump head pressure must be equal to the minimum head pressure required.

In the proposed methodology it is admitted that the selected control system uses VSP and FSP, in the same way it is admitted that a flow control is performed.

The regulation mode with VSP is based on a continuous modification of the rotational speed of the pumps in order that the pumping system follows the set-point curve. If fact, the pumping system is measured with flow and pressure controls and these variables are constantly sent to the programmable logic controller (PLC). This device does a comparison with the measured variables with the previously set variables ($Q$ and $H$) of the set-point curve for every time step defined and finally the PLC sends orders to the pumping system so the rotational speed of the pumps changes [17].

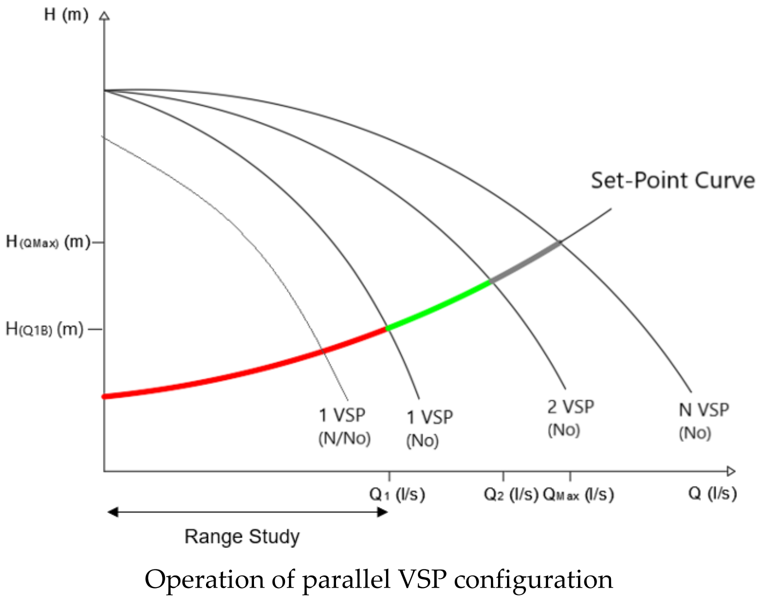



**Figure 1.** Operation squeme of parallel VSP following the set-point curve with pressure and flow controls.

The figure 1 represents a classical operation of VSPs configuration. When demand flow ($Q$) is lower than the $Q_1$, only one VSP operate at $N$ rotational speed. On the other hand, when de $Q$ is in the range between $Q = Q_1$ to $Q = Q_2$, starts to operate a second VSP at ($N$) rotational speed and the first VSP operates at the nominal speed ($N_0$). Finally, when Q is in the last range beween $Q = Q_2$ to $Q = Q_N$, the last pump starts and the other pumps operate at nominal speed ($N_0$). The presented methodology discusses about this classical methodology of mode regulation. Thus, this study is focused of the first range of operation that corresponds from $Q = 0$ to $Q = Q_{1b}$ (Maximum flow that one VPS supplies the system maintaining the head pressure of the set-point curve). The objective of this work is to determine the optimal number of VSP over the range study so that the consumed energy is optimal.

In this study it is set n VSPs in the pumping system. The dimensionless power of the pumping system is calculated for every number of VSPs by every set flow step in the range study. The decision variables of this study correspond to the dimensionless terms in the parametric form of the set-point curve.

In fact, the control system that is going to be focused in this work is the regulation system with variable speed drives incorporated in the pump stations. Besides, it is assuming that all the pumps have the same characteristic and the operating points are also the same. If the pump head in a parametric form ($h$) in the equation (5) is equal to head system in a parametric form ($h_c$) in the expression (7), it is obtained the following equations.

$$Q_{1b} = \sqrt{\frac{4 - 3 \cdot \lambda}{3 \cdot r + 1}} \tag{9}$$

$$\alpha_n = \sqrt{\frac{3 \cdot \lambda + \left(3 \cdot r + \frac{1}{n^2}\right) \cdot q^2}{4}} \tag{10}$$

$$\pi_n = \frac{\alpha^2}{3}(2 \cdot \alpha \cdot n - q) \tag{11}$$

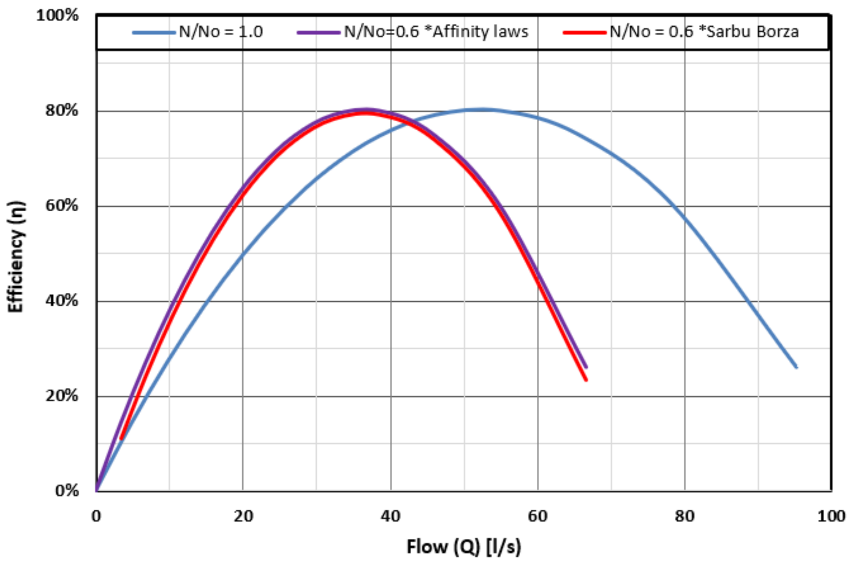
where $Q_{1b}$ in the equation (9) is defined as the maximum flow of a pump at nominal rotational speed ($N_O$) that can supply the system and guaranteeing the minimum head pressure required in the set-point curve. The term $\alpha_n$ in the equation (10) is defined as the rotational speed of each pump that has to operate in order to follow the head of the set-point curve, $n$ represents the number of the variable speed pumps operating in system and $\pi_n$ in the equation (11) is the reduced power in relation of the optimal power consumed by the n pumps that are operating in the system.

Even though, is usually to use the efficiency-flow curve and the affinity lows to estimate the efficiency of the pumping system, this approximation can have imperfections in the results of the efficiency pump, especially when the rotational speed has low values. Thus, this methodology has also used the formula proposed by Sarbu and Borza [18] tha provides reasonable good results. It is showed in the following expression (12) and (13).

$$\eta_2 = 1 - (1 - \eta_1) * (1/\alpha)^{0.1} \tag{12}$$

$$\theta_2 = 1 - (1 - \theta_1) * (1/\alpha)^{0.1} \tag{13}$$

where $\eta_2$ corresponds the efficiency of the variable speed pump at $N$ rotational speed and $\eta_1$ is the BEP at the nominal rotational speed ($N_O$) and the expression (13) is the same at the las expression but in a parametric form.



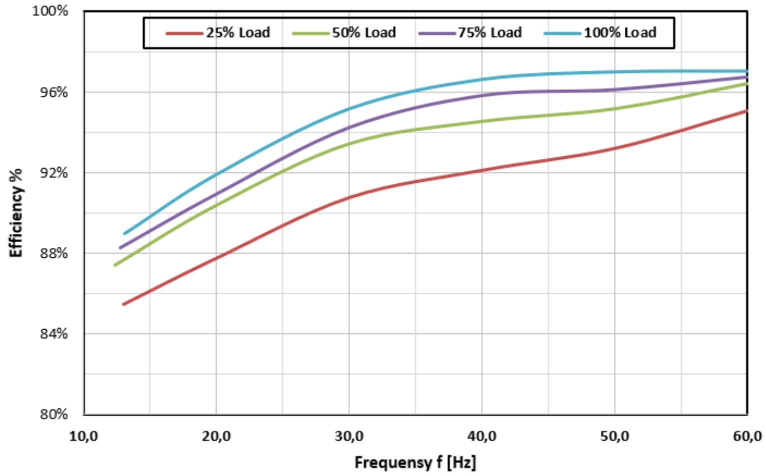

*η*-*Q* Curve

**Figure 2.** Comparison of the affinity laws and the Sarbu Borza expression when the rotional speed of the pump change.

When the rotational speed of the pump decrease in relation of the nominal rotational speed, the best efficiency point of the pump is affected and it can be proved using the Sarbu and Borza expression, but when it is only used the affinity laws the best efficiency point of the pump do not change as it is described Simpson and Marchi work [19] (figure 2).

VSD is an electrical device in the pumping system that controls the motor speed and torque. This device converts the simi-wave power from the power supply into the variable frequency power and sent to the motor, which it finally converts the variable frequency power into the mechanical power and this process generate some losses[20]. Therefore, the efficiency of the variable speed driver is defined as the relation between the input power to the motor and the input power of the variable speed drive.

According to the concepts of VSD. Another aim of this methodology is to define an expression that allows to estimate the performance of this device. The expression is obtained as the result of adjusting a curve to lab tests of VSD taken of Europump work [21] as it can showed in the figure 3. These tests consist on to measure the efficiencies with different combinations of rotational speed and the load of the motor, where the frequency or the rotational speed are kept constant and the load of the motor are changed.




Efficiency-Frequency curve of VSD

**Figure 3.** Laboratory test results of VSD performance (Source: Europump and Hydraulic Institute 2004).

The VSD efficiency is the defined as the following equations (14) and (15).

$$\eta_V = \eta o_V * [\beta_V^{0.0253} - 0.1 * (1-\alpha)^{2.705}] \tag{14}$$

$$\beta_V = \frac{3}{4} * \frac{\Pi}{\alpha} \tag{15}$$

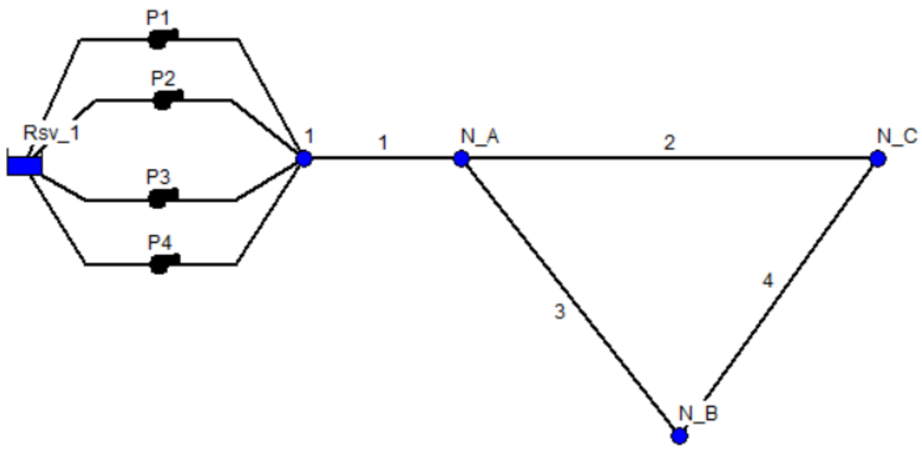
Where, $\eta_V$ is the VSD efficiency, $\beta_V$ is the dimensionless load of the VSD in relation to the optimal load and the $\beta_V$ is also defined as 4/3 of the maximum load of the motor ($\beta o$).

Once, it is defined the use of the affinity laws, Sarbu and Borza formulation and the VSD efficiency. The operation of a VSP can be determined in a better approximately way in order to analyze the influence of the number of VSD has over the regulation mode configurated with parallel VSP.

The reduced power for every number of VSPs is analyzed by different combinations of the terms $\lambda$ and $r$. The minimum value of $\lambda$ and $r$ is 0. In theory the maximum value of $\lambda$ cannot be greater than maximum head of the pump, but in this analysis the maximum value of $\lambda$ is 1 and the maximum value of r is 1, so the possible values of decision variables are between 0 and 1.

## 3. Case study

It has been defined an academic water distribution system to apply the previously described methodology. It has one pumping station equipped with 4 VSPs and supplying three consume nodes. It is assumed that the nodes have non-pressure driven demands. The layout of the network is showed in the figure 4 and the values of its elements are represented in the Table 1.



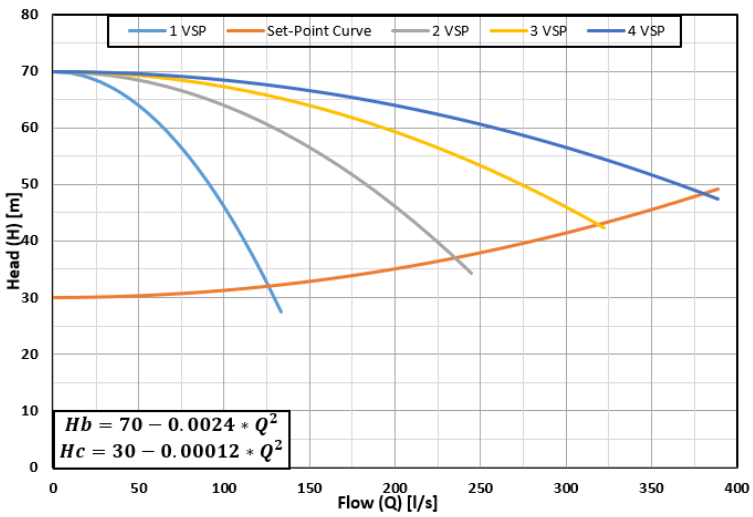

Network example

**Figure 4.** Esqueme of the example network.

**Table 1.** Information of nodes and pipe lines of the network example.

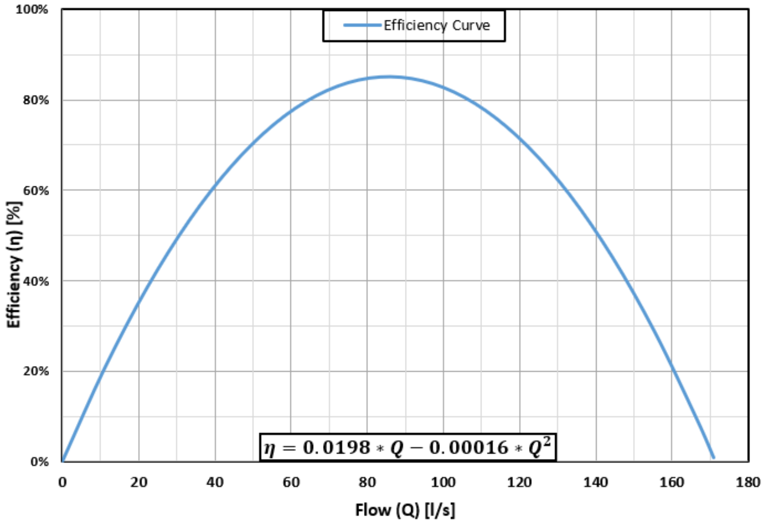
| I. Node | Elev (m) | F. Node | Elev (m) | Pipe | Diameter (mm) | Length (m) |
|---|---|---|---|---|---|---|
| Rsv 1 | 105 | 1 | 105 | PUMP | | |
| 1 | 105 | NA | 100 | 1 | 700 | 5000 |
| NA | 100 | NB | 102 | 3 | 400 | 1000 |
| NB | 102 | NC | 105 | 4 | 250 | 3000 |
| NC | 105 | NA | 105 | 2 | 300 | 3000 |

In order to show the methodology, a pump manufacturer model has been selected. It is the so-called GNI 100-20/75 model. Its BEP has the following values: the optimal flow ($Q_0$) is 85,5 l/s, the optimal head is 52.5 m and maximum efficiency is 83 %, so the head pump curve is defined as the figure 5., the efficiency curve is defined as figure 6. A certain setpoint curve is also represented in the figure 5.




*H-Q* Curve

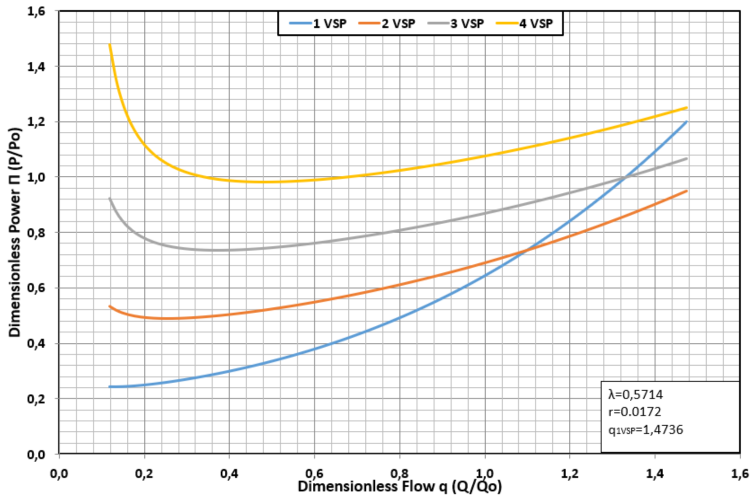
**Figure 5.** Corresponds to the model GNI 100-20/75 of the *H-Q* curves and the set-point curve of the system.




*η*–*Q* Curve

**Figure 6.** Represents the efficiency curve of model GNI 100-200/75 of the pump.

For this example network, the power of the pumping system is calculated in a dimensionless way. That is, the relation between the current power ($P$) and the optimal power ($P_0$) is calculated. The evaluation of the represents the efficiency curve of model GNI 100-200/75 of the pump.pumping station performance takes into consideration the affinity laws, the Sarbu and Borza expression and the efficiency of VSD, as it was explained previously. The results are summarized in the figure 6 and show the dimensionless power when different number of VSPs are operating. In this case, the maximum number of VSPs considered is four.



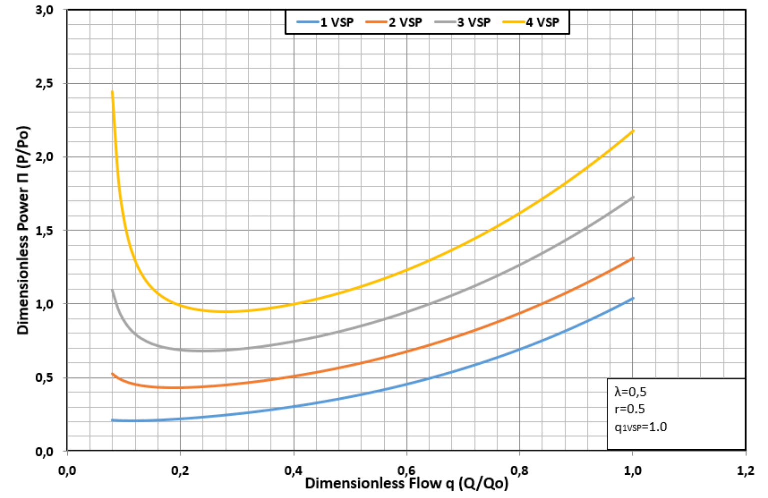

π-q Curve

**Figure 7.** Dimensionless power (π) for every number of VSPs in the flow range study (q=0 to q=1.47). λ and r are the terms of the set-point curve in a parametric form and are the decision variables of the analysis to evaluate consumed power in the pump station.

A preliminary analysis of Figure 7 shows that regardless of the number of VSPs, the more flow the greater the power consumed. There is only one exception: when the number of pumps grows and the flow rates are low, the power curve is decreasing. This happens because at very low flows, the efficiency of the pump has very low values, so it has made that the power has high values. This effect is specially significant when the control system is operating with more than one VSP.

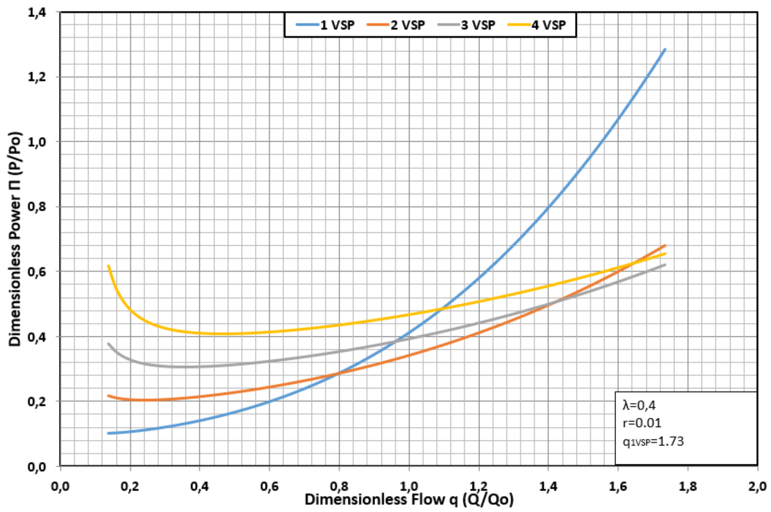
For this example, network, the maximum dimensionless flow ($q$) of one VSP is 1.47 (figure 7). This means that the maximum flow of one VSP is greater than the nominal flow. On the other hand, the decision variables of the set point curve in a parametric form are $\lambda$ = 0.57 and $r$ = 0.0172. (figure 5). In these conditions, it can be said that in the rage study, the best solution in terms of energy is not always the use of one VSP. In fact, when the flow is greater than 1.1 times the optimal flow, the optimal solution is two VSPs operating. A sensitivity analysis was performed varying the parameters $\lambda$ and $r$. The objective is to know how the parameters of the setpoint curve affect the optimal number of VSPs.

With the objective to know how the parameters of the set-point curve affects to determine the optimal number of VSPs in a regulation mode. It has tested two different combinations of $\lambda$ and in a pumping system assuming that it is equipped with four VSPs. The results are reported in the following figures 8 and 9.




$\pi$-q Curve ($\lambda$ = 0.5 and r = 0.5)

**Figure 8.** First combination of set-point parameters ($\lambda$=0.5 and r=0.5).




$\pi$-q Curve ($\lambda$ = 0.4 and r = 0.01)

**Figure 9.** Second combination of set-point parameters ($\lambda$=0.4 and r=0.01).

Figure 8 and 9 shows that it has set two different combination of decision variables of the set point curve that are $\lambda$ and r. The first combination corresponds to $\lambda$ = 0.5 and $r$ = 0.5, applying the proposed methodology, the maximum dimensionless of one VSP ($q_{1VSP}$) gets a value of 1. The second combination corresponds to $\lambda$ = 0.4 and $r$ = 0.01, so $q_{1VSP}$ gets a value of 1.73.

In the first combination of set-point variables, when $\lambda$ = 0.5 and $r$ = 0.5, the maximum dimensionless flow of one VSP ($q_{1VSP}$) is equal to the optimal dimensionless flow (q = 1). In this case, always one VSP operating is the best option so that the consumed energy be optimal as it can be evidenced in the figure 8. In contrast, in the second case, when $\lambda = 0.4$ and r = 0.01 and the maximum dimensionless flow of one VSP ($q_{1VSP}$) is greater than the optimal flow ($q = 1$). There are three adequate pumping configuration. When the dimensionless flow ($q$) is lower than 0.80, one VSP operating is the best configuration. In the second range of dimensionless flow ($q$) between values of 0.80 and 1.42, the best pumping configuration is with 2 VSP operating. Finally, when the dimensionless flow ($q$) between values of 1.42 and the maximum flow of one VSP ($q_{1VSP} = 1.73$), three VSPs operating get the best results of consumed energy (figure 9).

In the range study analyzed between ($Q = 0$ and $Q= Q_{1VSP}$). When the maximun flow of one VSP ($Q_{1VSP}$) is equal or lower than the nominal pump flow ($Q_0$) or ($q_{1VSP}$), the only best selection of pumping configuration is with one VSP operating in order that the energy be optimal. Even though, when the maximum flow of one VSP is greater than the nominal flow ($Q_0$) or ($q_{1VSP} >= 1$), n VSPs operating would have better results than only one VSP operating.

The parameters of the set-point curve ($\lambda$ and $r$) have a great influence in the selection of the optimal number of VSP in pumping control systems in order to obtain the optimal consumed energy. In fact, as $\lambda$ and $r$ have lower values, the possible options of the optimal number of VSPs operating tend to increase. The dimensionless terms $\lambda$ and r are related with the size of the pumps and the generated losses in the set-point curve. As a result, the dimensionless term $q_{1VSP}$ (the maximum flow of one VSP) have greater values if the system has lower losses or when the selectioned pumps have more capacity. In the opposite case the values of $q_{1VSP}$ decrease as the losses are greater or as the selectionded pump are smaller.

In summary, when the water distribution network have lower losses or in other words when the set-point curve is flat. Also, when the selectioned model of the pumps has a great capacity in terms of flow, the optimal number of VSPs operating would be more than only one pump in the range study ($Q =0$ to $Q_{1VSP}$) in order that the consumed energy be optimal.

## 5. Conclusions

It has been developed a methodology to analyse the optimal number of VSPs that consume the lowest energy in a parallel VSPs configuration. Besides, a correction of the inaccuracy for affinity laws and its effect on the VSD efficieny has been included.

There are numerous possibilities of adequate configuration on control systems with VSP to obtain the optimal consumed energy in the range study ($Q = 0$ to $Q_{1VSP}$). These possibilities depend of capacity of the pumps (small pumps or large pumps) and the losses of the set-point curve. If the pumps have a large capacity and the slope of the set-point curve have lower values, there would be several optimal numbers of VSPs in the range study.

In general, for low flows in the range study the optimal number of VSPs is one pump operating. However, as the flow increases in the range study, the optimal number of VSPs would be more than one pump, especially when the maximun flow of one VSP ($Q_{1VSP}$) is greater than the nominal flow pump ($Q_0$).

The present study proves that not always one VSP operating is the optimal in energy terms as the classic method states in parallel VSPs configuration, so it requires a depth analysis of the parameters of the pump and the set-point curve parameters to determine the optimal number of VSPs in the range flow.

It is important to mention that this proposed methodology can be applied for the nexts range flows ($Q = Q_{1VSP}$ to $Q = Q_{2VSP}$), ($Q = Q_{2VSP}$ to $Q = Q_{NVSP}$) as it showed in the figure 1. Besides, this study can be extended including opertational costs and investment costs in the pumping stations.

**Conflicts of Interest:** The authors declare no conflict of interest.

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