Quantum thermodynamics

An introduction to the thermodynamics of quantum computers

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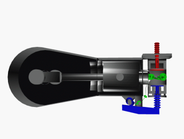
Classic thermodynamics

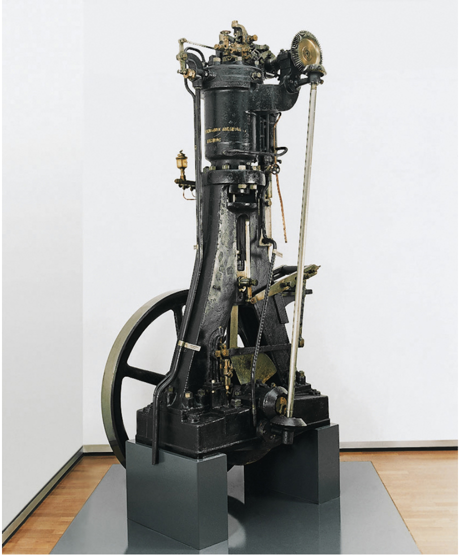
- phenomenological theory for average values of heat and work
- many applications on all length scales:
  phase transitions, chemical reactions, astrophysics...
- only quasistatic processes completely describable
- real processes: characterized by irreversible entropy production Σ

Purpose:

- understand and improve thermodynamic devices
- minimize dissipation in heat engines

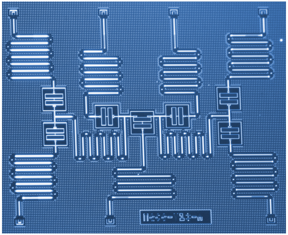


What we usually think of:

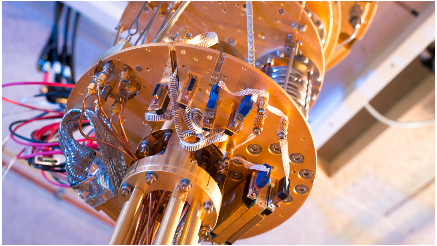


First operating Diesel engine (MAN Museum, Augsburg, Germany)

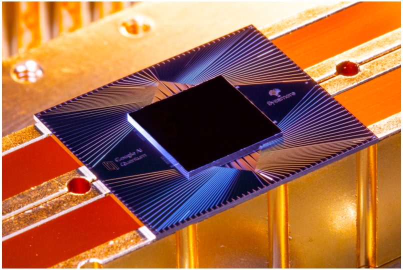
Rise of the quantum information age



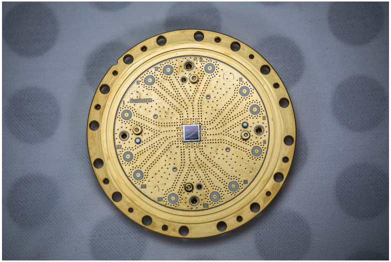
IBM Q Experience



Microsoft – integrative hardware/software approach



Google Sycamore



Rigetti – hybrid classical/quantum approach

Outline

Quantum thermodynamics and quantum work

- Quantum work and the two time measurement approach
- Jarzynski equality and work fluctuations

Thermodynamics and quantum information

- Stochastic thermodynamics in quantum computers
- New paradigm for error correction and thermodynamic cost

Quantum thermodynamics: Issues

- Mathematical description with operators, e.g.

energy	→	Hamilton operator
probability distribution	→	density operator

- Operators not commuting
- NO trajectories
- Work not a state function → no work operator
- Work complicated to measure

Thermodynamics of isolated quantum systems

- Driven Schrödinger dynamics

$$-i\hbar \frac{\partial}{\partial t}|\Psi\rangle = \left[\frac{p^2}{2m} + V(\alpha_t, x)\right] |\Psi\rangle$$

- External control parameter α_t

  Volume of piston, length of RNA molecule, frequency of trap, ...

- Isolated system, thus no heat exchanged with environment

$$\Delta E = \langle W\rangle \quad \text{and} \quad \langle Q\rangle = 0$$

- System initially prepared in Gibbs state,

$$\rho_0 = \sum_n \frac{e^{-\beta\, E_n(\alpha_0)}}{Z(\alpha_0)}\, |n(\alpha_0)\rangle\langle n(\alpha_0)|$$

Quantum Jarzynski equality

Problem: Notion of classical trajectory not applicable!

Solution: Two-time energy measurements

Campisi, Hänggi, & Talkner, RMP **83**, 771 (2011)

Quantum work:

$$W_{\mathrm{qm}}[|m(\alpha_\tau)\rangle;|n(\alpha_0)\rangle] = E_m(\alpha_\tau) - E_n(\alpha_0)$$

Work distribution:

$$\mathcal{P}_{\mathrm{qm}}(W) = \sum\!\!\!\!\!\!\int_{m,n} \delta\left(W - W_{\mathrm{qm}}[|m(\alpha_\tau)\rangle;|n(\alpha_0)\rangle]\right) p^\tau_{m,n}\, p^0_n$$

Consequences:

- Jarzynski equality: $\langle \exp\left(-\beta H_{\mathrm{H}}(\tau)\right) \exp\left(\beta H(0)\right)\rangle = \exp\left(-\beta \Delta F\right)$
- Conceptually simple notion of quantum work

Two-time measurement approach: Success story

Analytical calculation of work distribution:

- **Simple systems:**

  driven harmonic oscillator, particle in time-dependent box, Landau-Zener model,. . .

- **Many particle systems:**

  quantum Ising chains, non-interacting bosons and fermions,. . .

- **Relativistic systems:**

  Dirac equation, quantum field theories,. . .

Bartolotta & Deffner, PRX **8**, 011033 (2018) [and references therein]

Experimental developments:

- **Verification of quantum Jarzynski:**

  ion traps, NMR,. . .

- **Quantum engines:**

  single ion heat engine, quantum optomechanics,. . .

Where do we go from here?

What we have:

- Emerging framework for thermodynamic of quantum systems
- First experimental implementation of novel technology

What we want:

- Quantify resources for quantum computing
- Thermodynamic control strategies for error correction

Where we start:

- Apply stochastic thermodynamics to quantum information
- Tailor conceptual framework for available hardware

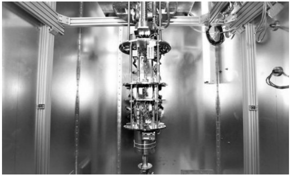
DWave – a “quantum” annealer

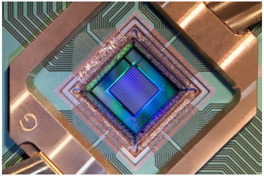
Paradigm: Adiabatic quantum computing

- ➙ Prepare quantum system in ground state of simple Hamiltonian
- ➙ Drive adiabatically: solution from “final” ground state

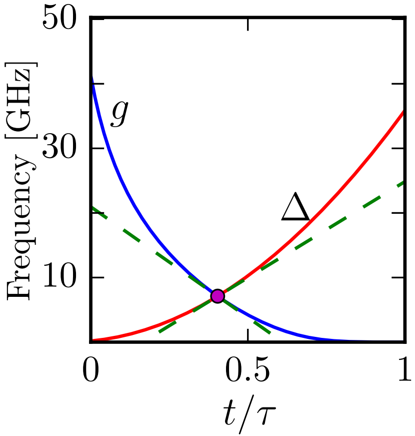
Problems and issues:

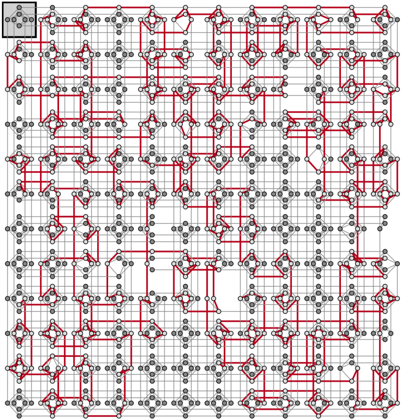
- ➙ Driving infinitely slowly (much slower that largest gap)
- ➙ Need unitary dynamics (very good insulation)





Experimental protocol on DWave





Quantum Ising model in transverse field

Hamiltonian:

$$H(t)/(2\pi\hbar) = -g(t)\sum_{i=1}^{L}\sigma_i^x - \Delta(t)\sum_{i=1}^{L-1} J_i\sigma_i^z\sigma_{i+1}^z$$

Initial and final observables (energy):

$$\Omega^{\mathrm{i}} = \sum_{i=n}^{L}\sigma_n^x - \mathbb{I} \quad \text{and} \quad \Omega^{\mathrm{f}} = \sum_{n=1}^{L-1}\sigma_n^z\sigma_{n+1}^z\,,$$

Ideal probability distribution: $\mathcal{P}\left(\Delta\omega\right) = \sum_{m,n}\delta\left(\Delta\omega - \Delta\omega_{n,m}\right)p_{m\to n}$

$$p_n = \mathcal{P}(|\omega_n|) = \begin{cases} 1 & \text{if} \quad |\omega_n| = L-1, \\ 0 & \text{otherwise.} \end{cases}$$

Implementation on DWave

Step 1:

- "Believe" initial state: $\rho_0 = |\rightarrow\rangle\langle\rightarrow|$
- "Believe" that DWave is described by quantum Ising model

Step 2:

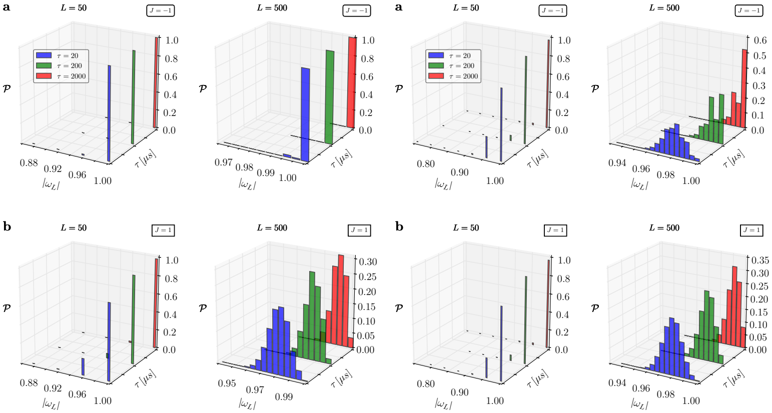
- Choose connections on the chimera graph randomly
- Run $N = 10^6$ times for different τ and L

Step 3:

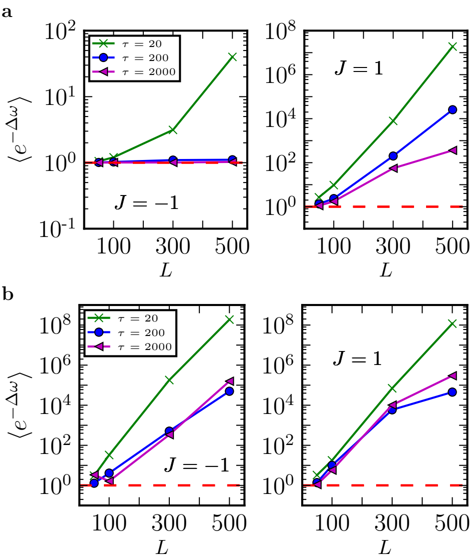
- Compare histogram of outcome with ideal distribution
- Compute average exponentiated quantum work

Results: DWave 2X and 2000Q

Gardas & Deffner, Sci. Rep. **8**, 17191 (2018)



Results: fluctuation theorem

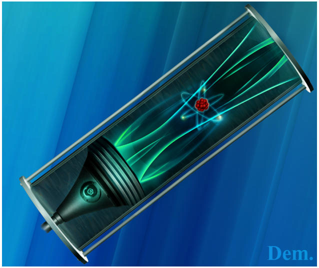


Gardas & Deffner, Sci. Rep. **8**, 17191 (2018)

- Jarzynski equality violated
  (not thermodynamically optimal)
- Dynamics not unitary
  (or rather not unital)
- Environmental noise
  (decoherence and dissipation)
- Finite-time excitations
  (need shortcut to adiabaticity)

Take-home-message

- First experimental systems with potential for quantum supremacy
- Emerging framework for thermodynamics of quantum information
- Quantum thermodynamics: exciting field with many open questions



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The book has arrived!!