ECEA 2019
5th International Electronic Conference on Entropy and Its Applications
18–30 November 2019

sciforum

Conference Proceedings Paper

Reverse Weighted-Permutation Entropy: A Novel Complexity Metric Incorporating Distance and Amplitude Information

Yuxing Li *

School of Automation and Information Engineering, Xi'an University of Technology, Xi'an 710048, Shaanxi, China; liyuxing@xaut.edu.cn

Abstract: Permutation entropy (PE), as one of the effective complexity metrics to represent the complexity of time series, has the merits of simple calculation and high calculation efficiency. In view of the limitations of PE, weighted-permutation entropy (WPE) and reverse permutation entropy (RPE) were proposed to improve the performance of PE, respectively. WPE introduces amplitude information to weigh each arrangement pattern, it not only can better reveal the complexity of time series with a sudden change of amplitude, but also has better robustness to noise; by introducing distance information, RPE is defined as the distance to white noise, it has the reverse trend to traditional PE and has better stability for time series of different lengths. In this paper, we propose a novel complexity metric incorporating distance and amplitude information and name it reverse weighted-permutation entropy (RWPE), which incorporated the advantages of both WPE and RPE. Three simulation experiments were conducted including mutation signal detection testing, robustness testing to noise based on complexity, and complexity testing of time series with various lengths. The simulation results show that RWPE can be used as a complexity metric, which has the ability to accurately detect the abrupt amplitudes of time series and has better robustness to noise. Moreover, it also shows greater stability than other three kinds of PE for time series with various lengths.

Keywords: permutation entropy (PE); weighted-permutation entropy (WPE); reverse permutation entropy (RPE); reverse weighted-permutation entropy (RWPE); time series; complexity

---

1. Introduction

PE was firstly brought forward by Bandt and Pompe in a seminal paper [1]. It introduced a simple and robust symbolic method that takes into account arrangement patterns of time series by comparing neighboring values of time series. The theoretical advantages of PE promote its application in practical problems [2]. Some typical applications of PE can be found in medical area [3], where it has been used to represent different states of human organs, including neuron, brain and heart. In addition, more applications can also be found in the fields of economics [4,5], mechanical engineering [6,7] and underwater acoustics [8,9]. The development of PE includes two aspects: one is the expansion of its application in different fields, the other is the improvement of PE theory. Many researchers have proposed improved PEs based on the limitations of PE.

In 2013, weighted-permutation entropy (WPE) was proposed and applied to electroencephalogram (EEG) data analysis by Fadlallah [10]. In 2017, Bandt proposed a new version of PE with a reverse trend to existing PEs, and I name it reverse PE (RPE) [11] in this paper. RPE is defined as the distance to white noise and has better stability for time series of different lengths. WPE and RPE have their own edges in indicating the complexity of time series from different angles. In order to keep and enhance the advantages of WPE and RPE, we propose a novel complexity metric incorporating distance and amplitude information and name it reverse weighted-permutation entropy (RWPE). Three simulation experiments have been carried out to demonstrate validity of RWPE by analysis and comparison with PE, WPE and RPE.

2. Reverse Weighted-Permutation Entropy

The specific steps of RWPE are as follows:

- Step 1: phase space reconstruction.

For a time series $Y = \{y(i), i = 1, 2, \cdots, T\}$ with T values, we reconstruct Y into L embedding vectors with the time delay τ and embedding dimension m, respectively. The matrix consisting of all embedding vectors can be represented as follows:

$$\begin{bmatrix} \{y(1), y(1+\tau), \cdots, y(1+(m-1)\tau)\} \\ \vdots \qquad\qquad \vdots \\ \{y(j), y(j+\tau), \cdots, y(j+(m-1)\tau)\} \\ \vdots \qquad\qquad \vdots \\ \{y(L), y(L+\tau), \cdots, y(L+(m-1)\tau)\} \end{bmatrix} \tag{1}$$

In formula (1), each row vector of the matrix corresponds to aembedding vector successively, and the number of embedding vectors L is $T-(m-1)\tau$.

- Step 2: ascending order.

We can rearrange each embedding vector in ascending order as follows:

$$y(i+(j_1-1)\tau) \le y(i+(j_2-1)\tau) \le \cdots \le y(i+(j_m-1)\tau) \tag{2}$$

When two values of the same embedding vector are equal as follows:

$$y(i+(j_a-1)\tau) = y(i+(j_b-1)\tau) \tag{3}$$

Their arrangement depends on their own temporal order as follows:

$$y(i+(j_a-1)\tau) < y(i+(j_b-1)\tau)\ (a<b) \tag{4}$$

Hence, we can obtain original patterns of embedding vectors as follows:

$$\pi_g = (j_1, j_2, \cdots, j_m)\ (g = 1, 2, \cdots, m!) \tag{5}$$

Where π_g is one of the $m!$ original patterns.

- Step 3: entropy calculation.

(1) Probability calculation.

Since an original pattern corresponds to several possible patterns, we introduce amplitude information to calculate the probabilities of original patterns on the basis of WPE. For example, Figure 1 is an original pattern and its corresponding three possible patterns ($m=3$).

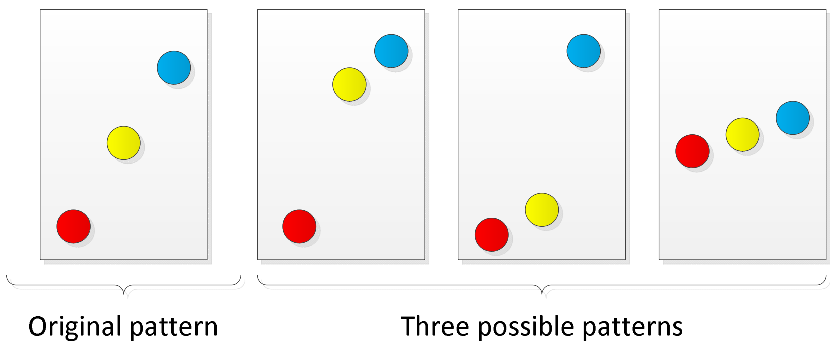


Figure 1. An original pattern and its corresponding three possible patterns ($m=3$).

We introduce amplitude information by weighted each embedding vector, weight value of the embedding vector Y_j can be expressed as follows:

$$\begin{cases} \omega_g(\pi_s) = \dfrac{1}{m}\sum_{k=1}^{m}(y_{(j+(k-1)\tau)} - \bar{Y}_j)^2 \\ \bar{Y}_j = \dfrac{1}{m}\sum_{k=1}^{m} y_{(j+(k-1)\tau)} \end{cases} \quad (6)$$

Where Y_j corresponds to the s-th possible pattern of the g-th original pattern, $\bar{Y}_j$ is the mean of Y_j, and $\omega_g(\pi_s)$ is the weight value of Y_j by calculating the variance of Y_j. The frequency for the g-th original pattern can be expressed as:

$$f(\pi_g) = \sum_{s}^{S} f(\pi_g(s))\ \omega_g(\pi_s) \quad (s = 0, 1, \cdots, S) \quad (7)$$

Where S is the number of possible patterns of the g-th original pattern, and $f(\pi_g(s))$ is the frequency of the s-th possible pattern. Then, the probability of the g-th original pattern can be represented as:

$$P(\pi_g) = \frac{f(\pi_g)}{\sum_{g=1}^{m!} f(\pi_g)} \quad (8)$$

(2) Calculation formula.

We define RWPE as the distance to white noise based on RPE by introducing distance information. Therefore, RWPE can be expressed as:

$$H_{RWPE}(m) = \sum_{g=1}^{m!}\left(P(\pi_g) - \frac{1}{m!}\right)^2 = \sum_{g=1}^{m!} P(\pi_g)^2 - \frac{1}{m!} \quad (9)$$

When $P(\pi_g) = 1/m!$, the value of $H_{RWPE}(m)$ is 0 (minimum value).

3. Simulations

3.1. Simulation 1

Mutation signal detection is carried out to verify the performance of RWPE. The synthetic signals are as follows:

$$\begin{cases} y = 50*(t == 0.498) + 0*(t >= 0\,\&\,t <= 1) \\ s = randn(t) \\ ys = y + s \end{cases} \quad (10)$$

Where the synthetic signal ys consists of impulse signal y and white Gaussian noise s with the sampling frequency of 1 KHz. The time domain waveform of synthetic signal ys is shown in Figure 2. We compute the values of PE, WPE, RPE and RWPE using sliding windows of 80 samples with 70 overlapped samples, the embedding dimension and time delay are 3 and 1, respectively. Figure 3 shows the four entropies of synthetic signal ys. As seen in Figure 3, the values of WPE and RWPE decrease and increase significantly in the windows containing impulse signal, the values of PE and RPE remain steady in all windows. Table 1 is the four entropies in the windows from 42 to 51. As seen in Figure 3 and Table 1, the impulse signal is in the windows from 43 to 50, the entropies incorporating amplitude information are obviously different from the ones without amplitude information, and WPE change in the opposite direction from RWPE. The simulation results show that WPE and RWPE have better performance than PE and RPE in mutation signal detection.

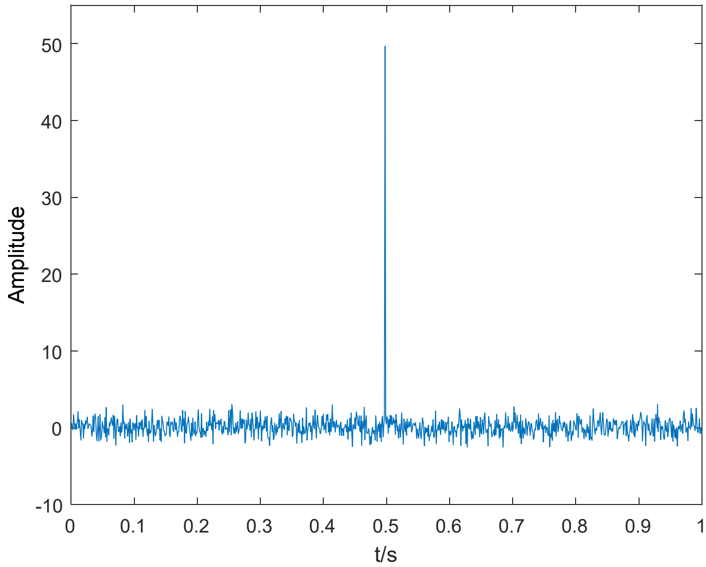


Figure 2. The time domain waveform of synthetic signal ys .

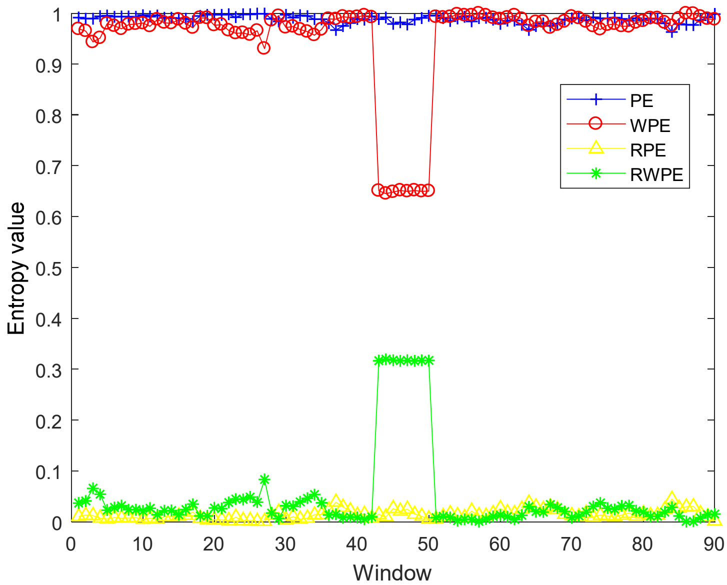


Figure 3. The four entropies of synthetic signal ys .

Table 1. The four entropies in the windows from 42 to 51.

Window	42	43	44	45	46	47	48	49	50	51
PE	0.994	0.988	0.992	0.979	0.982	0.978	0.987	0.990	0.995	0.995
WPE	0.992	0.651	0.645	0.648	0.651	0.649	0.652	0.649	0.650	0.993
RPE	0.007	0.014	0.010	0.026	0.021	0.025	0.015	0.011	0.006	0.005
RWPE	0.009	0.317	0.319	0.318	0.317	0.317	0.316	0.317	0.317	0.009

3.2. Simulation 2

Robustness testing to noise is carried out to verify the performance of RWPE. We use the same impulse signal y in the simulation 1, the synthetic signals ys with different signal-to-noise ratios (SNRs) can be obtained by adding white Gaussian noise to y . The embedding dimension and time delay are 3 and 1, respectively. Figure 4 shows the four entropies of synthetic signal under different

SNRs, each entropy value is the average of 1000 calculations. As seen in Figure 4, the values of PE and RPE are relatively stable under different SNRs and close to 1 and 0, respectively; the entropies incorporating amplitude information respond to changes in SRN, the influence of noise on complexity decreases with the increase of SNR, the values of WPE and RWPE are monotonically decreasing and increasing. The testing results indicate that RWPE and WPE have better robustness to noise than PE and RPE.

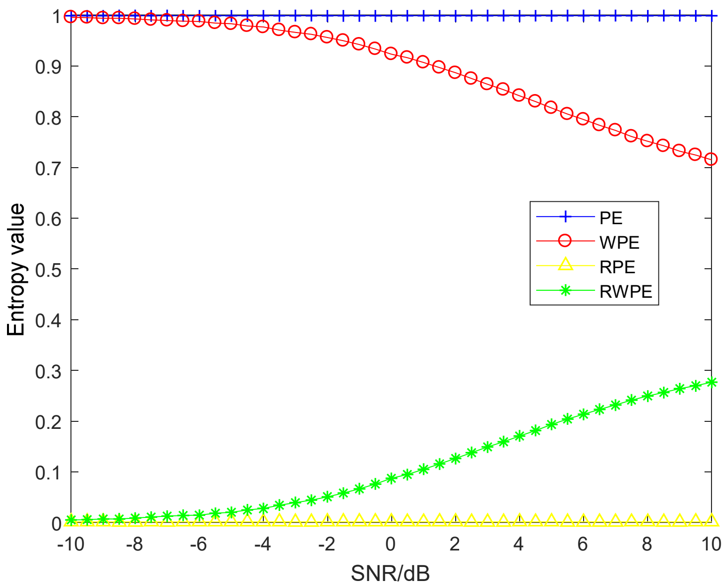
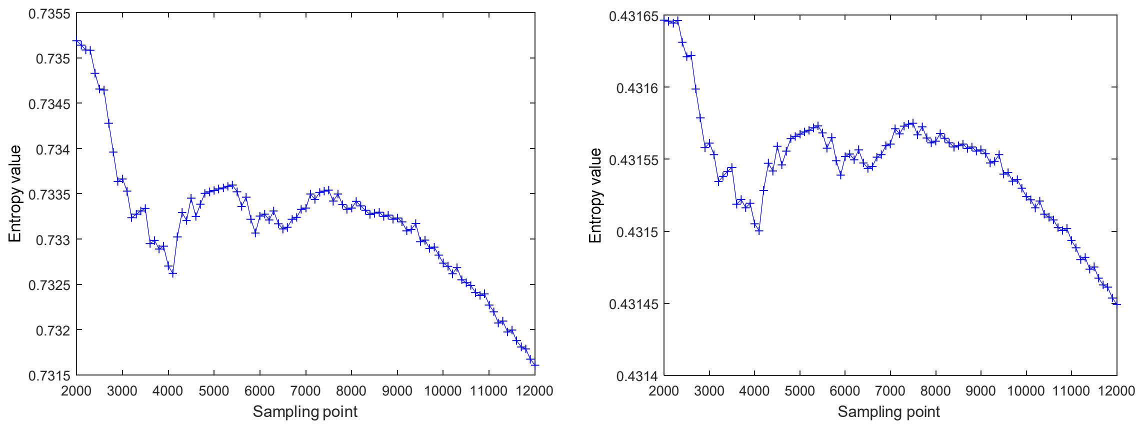


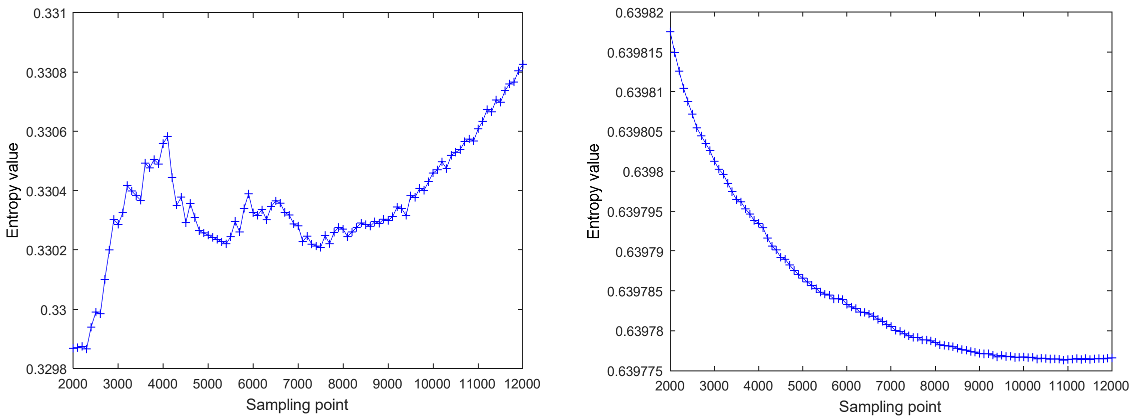
Figure 4. The four entropies of synthetic signal under different SNRs.

3.3. Simulation 3

Complexity testing of time series with various lengths is carried out to prove the stability of RWPE. We calculate four entropies of cosine signals with frequencies of 100 Hz. The embedding dimension and time delay are set to 3 and 1. The initial data length is 2000 sampling points, and 100 sampling points are added each time until the data length reach 12000 sampling points. Figure 5 show the four entropies of cosine signals with frequencies of 100 Hz.



(**a**) PE (**b**) WPE



(**c**) RPE (**d**) RWPE

Figure 5. The four entropies of cosine signals with frequencies of 100 Hz.

As seen in Figure 5, the four entropies of two cosine signals change in varying degrees with the increase of data length, the variation ranges of WPE are one order of magnitude lower than ones of PE and RPE, and the variation ranges of WRPE are the smallest than ones of WPE. The complexity testing results indicate that RWPE has better stability than the other three entropies in the case of different length data.

4. Conclusions

This paper proposes a novel complexity metric incorporating distance and amplitude information and names it RWPE. Three simulation experiments were carried out to validate this approach. Firstly, like WPE, RWPE has the ability to accurately detect the abrupt amplitudes of time series and has the opposite trend with WPE. Secondly, RWPE has better robustness to noise than PE and RPE. Lastly, RWPE has better stability for time series with different lengths than PE, WPE and RPE. RWPE, as an effective complexity metric, could be used to solve practical problems of different fields in future work.

Funding: Authors gratefully acknowledge the supported by National Natural Science Foundation of China (No. 11574250).

Conflicts of Interest: The author declares no conflict of interest.

References

1. Bandt, C.; Pompe, B. Permutation entropy: A natural complexity measure for time series. *Phys. Rev. Lett.* **2001**, *88*, 174102.
2. Zanin, M.; Zunino, L.; Rosso, O.A.; Papo, D. Permutation Entropy and Its Main Biomedical and Econophysics Applications: A Review. *Entropy* **2012**, *14*, 1553-1577.
3. Nicolaou, N.; Georgiou, J. Detection of epileptic electroencephalogram based on Permutation Entropy and Support Vector Machines. *Expert Systems with Applications* **2012**, *39*, 202-209.
4. Zunino, L.; Zanin, M.; Tabak, B.; Perez, D.; Rosso, O. Forbidden patterns, permutation entropy and stock market inefficiency. *Physica A* **2009**, 388, 2854-2864.
5. Hou, Y.; Liu, F.; Gao, J.; Cheng, C.; Song, C. Characterizing Complexity Changes in Chinese Stock Markets by Permutation Entropy. *Entropy* **2017**, *19*, 514.
6. Zhang, X.; Liang, Y.; Zhou, J.; Zang Y. A novel bearing fault diagnosis model integrated permutation entropy, ensemble empirical mode decomposition and optimized SVM. *Measurement* **2015**, *69*, 164-179.
7. Yan, R.; Liu, Y.; Gao, R. Permutation entropy: A nonlinear statistical measure for status characterization of rotary machines. *Mechanical Systems and Signal Processing* **2012**, *29*, 474-484.

8. Li, Y.-X.; Li, Y.-A.; Chen, Z.; Chen, X. Feature Extraction of Ship-Radiated Noise Based on Permutation Entropy of the Intrinsic Mode Function with the Highest Energy. *Entropy* **2016**, *18*, 393.
9. Li, Y.; Li, Y.; Chen, X.; Yu, J. A Novel Feature Extraction Method for Ship-Radiated Noise Based on Variational Mode Decomposition and Multi-Scale Permutation Entropy. *Entropy* **2017**, *19*, 342.
10. Fadlallah, B.; Chen, B.; Keil, A.; Principe, J. Weighted-permutation entropy: A complexity measure for time series incorporating amplitude information. *Phys. Rev. E* **2013**, *87*, 022911.
11. Bandt, C. A New Kind of Permutation Entropy Used to Classify Sleep Stages from Invisible EEG Microstructure. *Entropy* **2017**, *19*, 197.